Extended Feynman rule and the two-slit experiment

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An extension of Feynman's rule, relating probability amplitudes and probabilities in two-photon interferometry, has been proposed recently [G. Jaeger *et al.*, Phys. Rev. A **48**, 1023 (1993)]. It would be interesting to analyze the validity of this rule in other physical situations not considered by these authors. In particular, we show in this paper that the extended rule cannot be applied to the gedanken two-slit experiment. This result suggests the existence of two quantum regimes, differentiated by the validity of the generalized rule. The distinguishability of one particle and the visibilities of one-particle and two-particle interferences are calculated. [S1050-2947(96)02507-3]

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I. INTRODUCTION

Feynman's rule is a general statement that provides a simple and concise description of the behavior of probability amplitudes and probabilities in quantum mechanics [1]. In the past the techniques of one-particle interferometry have been extended by employing correlated two-particle systems. The experimental verifications of the theoretical analyses carried out in this new field have always been performed with photons pairs. Two-photon interferometry has already exhibited new nonclassical optical phenomena and confirmations of quantum mechanics [2–7]. The most striking feature of two-photon interferometry is the possibility of simultaneously studying the variation of both, single- and joint-detection probabilities as functions of arbitrary phase shifts of the beams. In general, the rules for the sum of probability amplitudes and probabilities will be different in one- and two-particle interferometry. A two-particle generalization of Feynman's rule was proposed by Jaeger *et al.* [3]:

''When an event can occur in several alternative ways, the probability amplitude for the event is the sum of the probability amplitudes for each way considered separately. There is interference. If an experiment could be performed without disturbing the system, which is capable of determining whether one or another alternative is actually taken, the probability of the event is the sum of the probabilities for each alternative. The interference is lost.''

This is the extended Feynman rule (EFR) that generalizes in a natural way the usual Feynman rule from one- to two-particle systems.

As remarked earlier, the analyses and experiments that have suggested this generalization are restricted to the field of two-photon interferometry. The source of the photons emits the particles into a number of well-defined beams that are recombined after phase shifting. These experimental arrangements yield two-photon fringes, in the sense that there is a trigonometric dependence of the probabilities of joint detections upon the variable phase angles.

This type of analysis does not cover some interesting situations, for instance, the interference patterns generated by diffraction gratings. The mathematical analysis of this problem is very difficult. Then, in order to simplify the mathematical tools and present the physical ideas in a simple way, we restrict our considerations to the gedanken two-slit experiment, in which two correlated particles are incident on two spatially separated screens with two slits. This experiment is the generalization to two-particle systems of the gedanken one-particle two-slit experiment that has played a central role in the discussions about the meaning of probability in quantum theory. This conceptual experiment is adequate for the study of the EFR because, in principle, we could perform an observation on a particle without disturbing the correlated particle and determine whether one or another alternative is actually taken (we can determine the slit through which the particle passes). The situation is similar to the experiment considered by Jaeger *et al.*

The plan of the paper is as follows. In Sec. II we calculate the single- and joint-detection probabilities in the two-slit experiment. In order to obtain an analytical expression, we restrict the study to the semiclassical approximation. In Sec. III, the distinguishability of the one-particle interference and the visibilities of one-particle and two-particle interferences are calculated. In Sec. IV, we discuss the main physical ideas involved in the former results. We shall suggest the convenience of distinguishing between two quantum regimes.

II. THE TWO-SLIT EXPERIMENT

We initiate this section by presenting a brief description of the experimental arrangement. The arrangement that we propose is shown in Fig. 1. A source emits pairs of correlated particles that impinge on two screens ($S1$ and $S2$) that are parallel and placed at the same distance L from the source. Every screen has two slits, A and B in $S1$ and C and D in $S2$. We suppose a symmetric arrangement with A and B in front of C and D, respectively. The distance between the slits is $2a$. Finally, behind $S1$ and $S2$ we place, at a distance d, two screens $D1$ and $D2$ that serve to detect the arrival of the particles. If the intensity of the source is very low, the detectors will record pulses representing the arrival of individual pairs of particles.

Let us suppose that the particles are emitted by the source in the following entangled state:

$$\psi^0=\psi^0_{1A}\psi^0_{2D}+\psi^0_{1B}\psi^0_{2C}. \tag{1}$$

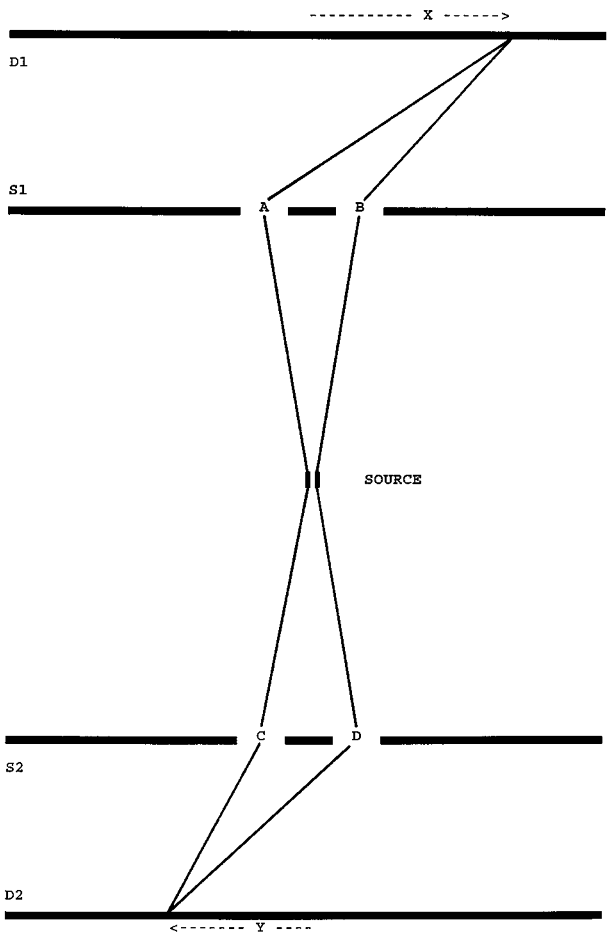

FIG. 1. Schematic two-particle, two-slit arrangement. The source emits two particles. In the semiclassical approximation, the action must be calculated along the classical trajectories that pass through the slits A, B, C, and D.

That is, if we determine that particle 1 passes through hole A (B), then particle 2 passes through hole D (C). The entangled state of Eq. (1) can be prepared using particles that, after their interaction, have opposite momenta

$$\vec{p}_1+\vec{p}_2=0. \tag{2}$$

It is understood that the state represented by Eq. (1) should be symmetrized or antisymmetrized since the particles are bosons or fermions, but the results that we obtain without explicit symmetrization or antisymmetrization would not be changed, provided that the subspace spanned by ψ_{1A}^0, ψ_{1B}^0 is orthogonal to that spanned by ψ_{2C}^0, ψ_{2D}^0.

From now on, we suppose that the particles are massive and obey Schrödinger's equation. To obtain the single- and joint-detection probabilities we must evaluate the wave function as a function of the spatial coordinates. This is done by using Feynman's approach of path integrals [8]. Let the vector $\vec{x}$ represent the coordinates of particle 1 and the vector $\vec{y}$ represent the coordinates of particle 2. Because the two particles do not interact after the preparation of the state, the action can be separated into two parts, $S(\vec{x},\vec{y})=S_1(\vec{x})+S_2(\vec{y})$, and the kernel becomes the product of two factors,

$$K(\vec{x}_0,\vec{y}_0,t_0;\vec{x}_f,\vec{y}_f,t_f)=K_1(\vec{x}_0,t_0;\vec{x}_f,t_f)K_2(\vec{y}_0,t_0;\vec{y}_f,t_f). \tag{3}$$

The subscripts 0 and f refer to the initial and final points. The two kernels are given by well-known expressions; for instance,

$$K_1(\vec{x}_0,t_0;\vec{x}_f,t_f)=\int_{\vec{x}_0}^{\vec{x}_f}\exp\left(\frac{i}{\hbar}S_1(\vec{x})\right)D^3\vec{x}(t), \tag{4}$$

with the usual notation in path integration, indicating the sum over all the paths.

If Eq. (1) represents the initial wave function, the final wave function, that is, that evaluated in the detectors will be

$$\begin{aligned}\psi(\vec{x}_f,\vec{y}_f)=&\int\int K(\vec{x}_0,\vec{y}_0,t_0;\vec{x}_f,\vec{y}_f,t_f)\psi^0(\vec{x}_0,\vec{y}_0,t_0)\\&\times d^3\vec{x}_0 d^3\vec{y}_0\\=&\int K_1\psi_{1A}^0 d^3\vec{x}_0\int K_2\psi_{2D}^0 d^3\vec{y}_0\\&+\int K_1\psi_{1B}^0 d^3\vec{x}_0\int K_2\psi_{2C}^0 d^3\vec{y}_0.\end{aligned} \tag{5}$$

The resulting two-particle wave function can be evaluated using one-particle wave functions.

The evaluation of the four integrals in Eq. (5) is very difficult. In order to obtain closed analytical expressions we use an approximate method of evaluation, the semiclassical approximation. If the potential is a sufficiently slowly varying function of time and space, the semiclassical approximation is adequate. In the particular case considered here, we cannot approach the problem by a single application of the free-particle law of motion, since the particles are constrained by the slits. So we break the problem up into two successive free-particle motions. The exact overall amplitude is an integral of the product of these two free-particle kernels. If we use the semiclassical approximation in each one of these free-particle kernels, we will have a very good approximation of the exact problem. The semiclassical method gives, for the first integral in Eq. (5),

$$\begin{aligned}&\int K_1(\vec{x}_0,t_0;\vec{x}_f,t_f)\psi_{1A}^0(\vec{x}_0,t_0)d^3\vec{x}_0\\&\approx C\exp\left(\frac{i}{\hbar}S_{\rm cl}(\vec{x}_s,t_0;\vec{x}_f,t_f)\right).\end{aligned} \tag{6}$$

$S_{\rm cl}$ is the classical action between the points $(\vec{x}_s,t_0)$ and $(\vec{x}_f,t_f)$, with $\vec{x}_s$ the position vector of the source and C a constant. Similar expressions hold for the other integrals and, because of the symmetry of the problem, all the constants are equal.

We now must calculate $S_{\rm cl}$ in the four trajectories. For instance, when particle 1 goes through slit A,

$$S_{\rm cl}(\vec{x}_s,t_0;\vec{x}_f,t_f)=\tfrac{1}{2}\int_{\vec{x}_s}^{\vec{x}_A}d\vec{x}\cdot\vec{p}_{\rm cl}+\tfrac{1}{2}\int_{\vec{x}_A}^{\vec{x}_f}d\vec{x}\cdot\vec{p}_{\rm cl}. \tag{7}$$

The classical momentum is given from $\vec{x}_s$ to $\vec{x}_A$ by $(p/2)(\vec{x}_A-\vec{x}_s)/|\vec{x}_A-\vec{x}_s|$, and from $\vec{x}_A$ to $\vec{x}_f$ by

$(p/2)(\vec{x}_f - \vec{x}_A)/|\vec{x}_f - \vec{x}_A|$, where p is the initial magnitude of the momentum vector. Note that we have used the same value of the magnitude of the momentum vector after passing the slit. This choice can be justified by the fact that the deflection of a particle in passing through a slit is actually a change in momentum. This change amounts to the addition of a small component of momentum in a direction approximately perpendicular to the original momentum vector. However, the change in energy, and then in the magnitude of the momentum vector, is completely negligible. The classical action becomes $S_{\rm cl} = (p/2)(|\vec{x}_A - \vec{x}_s| + |\vec{x}_f - \vec{x}_A|)$. If we denote by x the component of $\vec{x}_f$ parallel to the screen in the direction of the line that joins both slits (with $x=0$ the mean point between the slits and the positive axis in the direction of A), the classical action is $S_{\rm cl} = (p/2)\{(L^2 + a^2)^{1/2} + [d^2 + (x-a)^2]^{1/2}\}$. Note that considering infinitely large screens and slits in the direction perpendicular to x inside the detectors, we can disregard these coordinates.

Following the same steps, we can calculate the classical action of the other paths. Finally, the wave function is

$$\psi(x,y) \sim \exp\left(\frac{ip}{2\hbar} \{[d^2 + (x-a)^2]^{1/2} + [d^2 + (y+a)^2]^{1/2}\} \right) + \exp\left(\frac{ip}{2\hbar} \{[d^2 + (x+a)^2]^{1/2} + [d^2 + (y-a)^2]^{1/2}\} \right), \tag{8}$$

where two overall phase factors, one time-dependent and the other a function of $(L^2 + a^2)^{1/2}$, and the constant C^2 have been suppressed. We have introduced for y the same definition of x. The use of the semiclassical approximation can be justified because the two-particle wave function is evaluated using only one-particle wave functions. The semiclassical approximation describes correctly the one-particle wave function in the two-slit experiment because the problem can be broken up into two successive free-particle motions. As a matter of fact, since the free-particle Lagrangian is a quadratic form, the approximate expression for the kernel $K \sim \exp(1S_{\rm cl}/\hbar)$ is exact. Therefore, the semiclassical approximation is adequate for this problem. In the Appendix we present an explicit mathematical discussion of the semiclassical approximation dealing directly with two-particle wave functions.

Now we are in a position to evaluate the joint detection probability, $n(x,y)$. A simple calculation gives

$$n(x,y) = \psi^*(x,y)\,\psi(x,y) = \text{const}[1 + \cos(Z_x + Z_y)], \tag{9}$$

with

$$Z_x = \frac{p}{2\hbar} \{[d^2 + (x-a)^2]^{1/2} - [d^2 + (x+a)^2]^{1/2}\} \tag{10}$$

and

$$Z_y = \frac{p}{2\hbar} \{[d^2 + (y+a)^2]^{1/2} - [d^2 + (y-a)^2]^{1/2}\}. \tag{11}$$

The coincidence count rate given by Eq. (9) shows that the predicted fringe pattern describes simultaneously two- and one-particle interference phenomena. First, we shall establish the existence of one-particle interference phenomena. The count rates of the individual particles can be determined by adding up all the contributions of the other particles; that is, by integrating over the spatial variables of the correlated particles. For instance, the count rate of particle 1 is

$$n_1(x) = N^{-1} \int_{-\infty}^{\infty} n(x,y)\,dy. \tag{12}$$

The normalization factor N is

$$N = \int_{-\infty}^{\infty} dy. \tag{13}$$

After straightforward manipulations, the count rate becomes

$$n_1(x) = \text{const} \times \left(1 + N^{-1} \cos Z_x \int_{-\infty}^{\infty} \cos Z_y\,dy \right). \tag{14}$$

To obtain this result, we have taken into account that the integral of $\sin Z_y$ vanishes because Z_y is an odd function of y. The integration of $\cos Z_y$ is also simple. First, we note that for $y \gg d$, $[d^2 + (y+a)^2]^{1/2} \approx y + a$ and $[d^2 + (y-a)^2]^{1/2} \approx y - a$. Then, the function Z_y tends to the limit a/λ, with λ the wavelength of the particle and $\cos Z_y \approx \cos(a/\lambda)$. If we denote by $(-y^*, y^*)$ a finite interval of the real axis where $y \approx d$, the integral of $\cos Z_y$ can be approximated by

$$\int_{-\infty}^{\infty} \cos Z_y\,dy \approx \int_{-y^*}^{y^*} \cos Z_y\,dy + \alpha \int_{-\infty}^{\infty} dy - \alpha \int_{-y^*}^{y^*} dy, \tag{15}$$

with $\alpha = \cos(a/\lambda)$.

Clearly, the first and third integrals of the above expression are finite. Thus, when the normalization factor N is taken into account, the count rate for particle 1 becomes

$$n_1(x) = \text{const} \times (1 + \alpha \cos Z_x). \tag{16}$$

Note that this result, because of the presence of the normalization factor N, is exact. A similar result holds for particle 2.

Finally we shall demonstrate that, simultaneously, there is a two-particle interference phenomenon. We say that there is a two-particle interference phenomenon when the coincidence count rate differs from the product of the count rates for one-particle interference, that is, $n(x,y) \neq n_1(x) n_2(y)$. The individual count rates are $n_1(x) \sim (1 + \alpha \cos Z_x)$ and $n_2(y) \sim (1 + \alpha \cos Z_y)$. Comparing with Eq. (9) we see that $n(x,y) \neq n_1(x) n_2(y)$ and, therefore, we deduce that there is a genuine two-particle interference phenomenon. We have established the simultaneous existence of one- and two-particle interference phenomena in the gedanken two-particle, two-slit experiment. This result shows that the EFR cannot be applied to this experiment.

There are also two alternatives for every particle, to pass by one or the other of the slits. However, the observation made about the correlated particle, which could be used to determine the alternative taken by the other member of the pair, is simultaneous to the interaction of the particle with the diffraction grating. This interaction is the physical mechanism that combines the amplitudes of the two alternatives. To destroy the interference, the determination of the alternative actually taken must be prior to the physical combination of the alternatives. Moreover, any measurement made upon the correlated particle in the interval between the emission of the particles and the arrival to the screens does not determine the slit through which a particle passes. In effect, the determination of the position of a particle, measuring the position of the correlated particle, is compatible with the two alternatives, because the wavelength of the particle is comparable to the separation between the slits.

The physical interpretation of the examples presented by Horne *et al.* [2] and Jaeger *et al.* [3] is very different. A source emits pairs of particles, which propagate in narrow beams, that is, beams whose extent in the two axes perpendicular to the direction of propagation is small. The typical longitude characterizing this extent is always much smaller than the typical separation of the different beams (with the exception of the region where beams intersect). We can speak about well-defined paths that remember closely classical trajectories. In the interval between the emission of the particles and the interaction with the macroscopic beam splitter or the intersection of the beams, it is possible to determine, using the correlated member of the pair, if a particle enters a particular beam. One can distinguish between the different paths, and use this information to determine the path of the other particle.

These differences can be seen more clearly by comparing the experiment presented here to those proposed by Ghosh and Mandel [7]. This experiment bears a strong resemblance to the two-slit experiment, presenting also a continuum of outcomes for each particle. Horne *et al.* [2] have discussed a more general class of similar experiments: several beams associated with two photons intersect in one or two small regions, where miniature detectors are placed at variable positions. An essential difference exists between these arrangements and the two-slit experiment. In the experiments of Refs. [2] and [7] the beams are described to good approximation by correlated plane waves. The possibility of using a description based on plane waves is related as much to the beams' propagation as narrow beams as to the form in which the beams are combined. As is well known, if the particles are described by plane waves at times previous to the overlap, the particles have well-defined directions and their paths can be defined. Moreover, except in the overlap regions, the beams are well separated. According to our former discussion, the EFR can be applied. At times subsequent to the overlap, as shown in Ref. [2], the one-particle count rate derived from a correlated plane-wave function is constant; according to the EFR there is no one-particle interference (as has been experimentally shown in Ref. [7]). On the other hand, in the two-slit experiment (in the semiclassical approximation used here) the wave functions, at times previous to the combination of amplitudes, are plane waves along the classical trajectories ($S_{\mathrm{cl}}=px/2$); you can define a path. However, according to our previous discussion, the classical trajectories are not well separated and the EFR cannot be applied. Now, the superposition of amplitudes is done in a way that is completely different from that of the former experiment. A macroscopic instrument, the diffraction grating, is used to combine the probability amplitudes, instead of the simple overlap of beams (without the aid of any macroscopic device) presented in Refs. [2] and [7]. After the combination, the wave function is no longer the superposition of two-particle plane wave functions. For instance, the first exponential in Eq. (8) can only be considered as locally plane when $x \gg d$ and $y \gg d$, which gives $\exp[ip(x+y)/2\hbar]$. In the points where this local two-particle plane-wave function can be used, the one-particle count rates are constant. In the rest of the points, we have one-particle fringes.

This analysis shows that when the problem is considered in the Schrödinger form, the possibility of describing the beams by well-separated plane waves is essential in order for the EFR to be applicable. We shall return to this point in the Discussion.

III. VISIBILITY AND DISTINGUISHABILITY

The simultaneous appearance of one-particle and two-particle interferences was already described by Jaeger *et al.* [3]. However, the results obtained by these authors relative to the complementarity between the two types of interference do not hold in the two-slit experiment. In order to justify this point, we shall calculate the visibilities in our example. Moreover, to properly compare the results of both studies, we shall also analyze the relationship between the distinguishability of one particle and the visibility of one-particle interference.

To carry out this analysis we use an initial entangled state that is more general than those given by Eq. (1). Let the initial state be

$$\psi^0 = A\psi^0_{1A}\psi^0_{2D} + B\psi^0_{1B}\psi^0_{2C}. \tag{17}$$

In order to simplify the mathematical analysis, we suppose that A and B are real and positive. Then, $A^2+B^2=1$. Following the steps of the previous section we can easily obtain the joint-detection probability,

$$n(x,y) = \mathrm{const}\times[1+2AB\cos(Z_x+Z_y)]. \tag{18}$$

On the other hand, the count rate for particle 1 is

$$n_1(x) = \mathrm{const}\times(1+2AB\alpha\cos Z_x). \tag{19}$$

The definition for the one-particle visibility is

$$V_i = \frac{n_i^{\max} - n_i^{\min}}{n_i^{\max} + n_i^{\min}}. \tag{20}$$

To evaluate Eq. (20) we need an explicit expression for $n_i^{\max}$ and $n_i^{\min}$. We restrict the study to the interval $0 \leqslant a/\lambda \leqslant \pi$. We must consider separately two cases, $\alpha \geqslant 0$ and $\alpha < 0$.

(i) $\alpha \geqslant 0$, $0 \leqslant a/\lambda \leqslant \pi/2$:

$$n_i^{\max} = \mathrm{const}\times(1+2AB\alpha). \tag{21a}$$

and

$$n_i^{\min} = \mathrm{const}\times(1+2AB\alpha^2). \tag{21b}$$

(ii) $\alpha<0$, $\pi/2<a/\lambda\leq\pi$:

$$n_i^{\max}=\text{const}\times(1+2AB\alpha^2) \tag{21c}$$

and

$$n_i^{\min}=\text{const}\times(1+2AB\alpha). \tag{21d}$$

To obtain these results we have taken into account that $|Z_x|\leq a/\lambda$. Combining Eqs. (20) and (21) we obtain the visibilities in the two intervals:

$$V_i=\frac{AB(\alpha-\alpha^2)}{1+AB(\alpha+\alpha^2)} \tag{22a}$$

for $\alpha\geq0$, and

$$V_i=\frac{AB(\alpha^2-\alpha)}{1+AB(\alpha+\alpha^2)} \tag{22b}$$

for $\alpha<0$.

Using these results we can show that in some cases the complementarity relation between visibilities derived by Jaeger *et al.* does not hold. For instance, if we take $A=B=1/\sqrt{2}$ and $a/\lambda=\pi$, we obtain $V_i=1$. Any realistic definition for the visibility of two-particle fringes must give a positive value ($V_{12}>0$) for the nonconstant pattern represented by Eq. (18). Therefore, $V_i^2+V_{12}^2>1$. That is incompatible with the complementarity relation of Jaeger *et al.*

The above example is not the only case where the complementarity relation does not hold. To continue the analysis we must define the two-particle visibility. First, we must define a corrected joint-detection probability n^*. As discussed in Ref. [3], the mathematical expression of n^* must be of the form $n-n_1n_2+cte$ [see Eq. (33) of Ref. [3]]. If we take const=0.475 in Eqs. (18) and (19) (this is equivalent to a normalization of the fringe pattern), n^* reads

$$n^*=1.4+[1+2AB\cos(Z_x+Z_y)]-(1+2AB\alpha\cos Z_x)(1+2AB\alpha\cos Z_y). \tag{23}$$

A rationale for the term $cte=1.4$ in the above equation is the fact that this value is the least real number, such that n^* is non-negative for all wave functions of the form of Eq. (17). Analogously to Eq. (20), visibility V_{12} can be defined in terms of n^*. We have studied numerically the relation between both visibilities. For instance, for $A=0.57$ and $a/\lambda=3.1$, $V_1^2+V_{12}^2=1.58$; for $A=0.53$ and $a/\lambda=2.5$, $V_1^2+V_{12}^2=1.01$; and for $A=0.5$ and $a/\lambda=3.1$, $V_1^2+V_{12}^2=1.49$. There is a large interval of values of A and a/λ in which $V_1^2+V_{12}^2\geq1$. For instance, for $a/\lambda=\pi$, this inequality holds for any A in the interval [0.36, 0.77].

The above analysis shows that, in general, the complementarity relation of Jaeger *et al.* is not valid in the two-particle two-slit experiment. This result indicates that the physical arrangement considered in this paper is not included in the analysis of these authors. However, the numerical study shows that a new complementarity relation holds, namely,

$$V_{12}^2+V_i^2<1.59. \tag{24}$$

The value of the upper bound is only valid for const=1. For other values of this constant we would have obtained different values of the bound. However, for any choice of the constant the bound is always larger than 1.

To understand the difference between the two complementarities, we must describe the relationship between entanglement and one-particle visibility. This relationship is based on two processes of interaction of the particle: (i) the interaction of the particle and the interferometer (passive lossless transducers, two-slit diffraction gratings, etc); (ii) the observation of the interference pattern by the interaction of particle and detector.

First, we describe process (i): the interaction of the particle and the interferometer is an event that can occur in several alternatives ways. The interference is generated because the probability amplitude is the sum of the probability amplitudes for each alternative. If the wave function of the particle is entangled and the EFR can be applied, some (or all) of these alternatives do not contribute to the probability amplitude. The interference is partially (or completely) lost.

Now, we describe process (ii): the phase information between the basis wave functions of every particle in an entangled state is carried by correlated wave functions of the companion particle. Therefore, observations made only upon the first particle cannot fully extract the phase information.

Processes (i) and (ii) explain the limitations on the values of the one-particle visibility. The more entangled the two-particle wave function is, the stricter is the bound on the one-particle visibility. On the other hand, a high degree of entanglement entails high two-particle fringe visibility.

The difference between the two complementarities now becomes clear. The EFR cannot be applied to the two-slit experiment, process (ii) is the only restriction entailed by entanglement. On the other hand, in the family of states considered by Jaeger *et al.* the EFR is valid, and processes (i) and (ii) restrict the visibility. The bound is more stringent and the complementarity relation is stricter than in the two-slit experiment.

Jaeger *et al.* [4] have also formulated and demonstrated another interferometric complementarity. This new complementarity relates the distinguishability D of the path of propagation of a particle to the fringe visibility V_1 when amplitudes from two paths are combined. According to the terminology of Jaeger *et al.* the state represented by Eq. (17) is a pure entangled state. Using Eqs. (17a) and (28) of Ref. [4] we obtain the distinguishability of these states $D(\Psi)=1$. This result is valid for any value of A and a/λ. Therefore, as $V_1\geq0$ we have

$$D(\Psi)^2+V_1^2\geq1. \tag{25}$$

This result indicates that if the definition of distinguishability proposed by Jaeger *et al.* is used, the relation between distinguishability and visibility obtained in the two-particle two-slit arrangement is in marked contrast with the complementarity relation obtained by these authors.

The definition of distinguishability proposed by Jaeger *et al.* is not adequate for problems in which the EFR cannot be applied. For the state represented by Eq. (17), $D(\Psi)=1$. A value of unity represents a situation in which the path of a particle can be predicted with certainty after observing the path of the companion particle. In the two-slit experiment, we cannot predict the alternative taken by a particle observing the companion particle. We must use a definition of distinguishability D_* such that $D_*(\Psi)<1$. Distinguishability is defined by a function for which the probability of a correct prediction of the alternative taken by the particle has the maximum value. We are considering systems in which no observations on the companion particle yield information for predicting the alternative of the particle of interest. Therefore, the strategy of prediction must be based entirely on the preparation. These are simple preparations in the terminology of Jaeger *et al.* Moreover, Eq. (17) represents a pure state of the two-particle system, with all the pairs of particles in the same quantum state. The optimum strategy to predict the alternatives of one of the particles must be the same as that used with pure simple states (to predict that particle 1 goes through slit A is equivalent to predicting that the two particles of the composed system go through slits A and D). Therefore, in this case the adequate definition of distinguishability is

$$D_*(\Psi)=|A^2-B^2|. \tag{26}$$

Using this definition of distinguishability, we recover a complementarity relation with the one-particle visibility. We write $D_*^2+V_i^2=(A^2-B^2)^2+(2AB+V_i-2AB)^2=1+V_i^2-(2AB)^2$. For $\alpha\geqslant 0$, we have the relation $\alpha-\alpha^2<1+AB(\alpha+\alpha^2)$ that implies $V_i<AB$ and $V_i^2<(2AB)^2$. On the other hand, for $\alpha<0$ we have $\alpha^2-\alpha\leqslant 2$, which implies $V_i\leqslant 2AB$ and $V_i^2\leqslant(2AB)^2$. Combining all these expressions we obtain

$$D_*(\Psi)^2+V_i^2\leqslant 1, \tag{27}$$

that is, the mathematical expression for the complementarity between distinguishability and one-particle visibility for the state represented by Eq. (17).

Note that Eq. (27) is not equivalent to the complementarity relation for the pure simple case, $D^2+V_i^2=1$, obtained by Greenberger and YaSin [9]. This difference is a new consequence of entanglement. As remarked earlier in the discussion of the complementarity between visibilities, process (ii) imposes bounds on the one-particle visibility, bounds that do not act on non-entangled systems.

IV. DISCUSSION

The most important result of this paper is the analysis of a gedanken experiment in which the EFR cannot be applied. In addition to the intrinsic interest of this result, we show that it influences the bounds imposed on the one-particle visibility; the complementarity relation between the visibilities is modified. The algorithm to compute the distinguishability of a particle in an entangled state must also be modified.

The formulation of the EFR makes it clear that the limitation upon the visibility of one-particle interference patterns is not imposed by the information that the observer has extracted concerning the particles of interest, but lies in the information that could in principle be extracted (compatible with the constraints given by the preparation). Following this interpretation, the state or wave function reflects not only what is known, but also what could be known about the particle [5]. The two-slit experiment imposes some restrictions on this interpretation; the one-particle interference pattern does not disappear, although one can distinguish, in principle, between the two possible alternatives. The disappearance of the one-particle interference pattern can only be the result of a measurement made of the particle of interest and not from the information extracted on the companion particle. The interpretation based on the possible information is only valid when one can, in principle, distinguish between the different alternatives at a time prior to the combination of the probability amplitudes.

By no means does the possibility of applying the EFR or the interpretation of the possible information depend on the fact that the particles are massive or massless. Neutron interferometry provides an example of interferometry with massive particles in which the EFR holds. On the other hand, the interpretation of photon scattering by diffraction gratings is similar to the analysis of the gedanken two-slit experiment.

The former considerations suggest the convenience of introducing two quantum regimes in the study of interference patterns in two-particle systems; the dispersive and nondispersive regimes. The nondispersive regime is characterized by the fact that the alternatives of the particles are paths and, moreover, one can distinguish between these alternative paths. As remarked earlier, this is equivalent to describing the different paths by narrow, well-separated beams. There are interference phenomena, but without dispersion of particles. In this regime, the particles fulfill the conditions that assure the validity of the EFR. On the other hand, in the dispersive regime one can also distinguish between the different alternatives. However, this determination of the alternative is simultaneous with the combination of the amplitudes of the different alternatives by the macroscopic instrument. The information obtained in position measurements prior to this interaction is compatible with both alternatives and does not destroy the interference. The EFR cannot be applied. There is dispersion of particles and the description based on paths is not adequate.

The above considerations suggest an analogy between these regimes and the geometric approximation in classical optics. As is well known, see Ref. [10], geometric optics assumes that light travels in rays, that is, narrow beams of radiation with well-defined directions of propagation. This approximation partially disregards the wave nature of light, and is equivalent to describing the electromagnetic waves by plane waves (if this description is adequate it is possible to define, at least locally, the direction of propagation). The resemblance to the nondispersive regime, in which the beams are described by narrow beams that recall rays, is clear. The resemblance is even more compelling in the experiment of Ghosh and Mandel [7]. As remarked earlier, in this case the wave functions can be approximated by plane waves. The results obtained using this approximation (no one-particle interference, validity of the complementarity between visibilities) are completely different from those of the two-slit experiment.

The analogy between some experiments in quantum mechanics and the geometric approximation in classical optics can be viewed as an example of the convenience of using classical concepts to obtain a conceptual comprehension of microphysical phenomena (beyond the symbolic description provided by the mathematical formalism). This point has been frequently emphasized by Bohr [11].

Another analogy between quantum mechanics and optics has been frequently presented in the literature. The analogy can be summarized briefly by saying that classical mechanics (Hamilton-Jacobi equation) represents the geometrical limit of quantum mechanics (Schrödinger equation) as $\hbar\to 0$. As a matter of fact, this is the basis of the semiclassical approximation. However, this analogy is less stringent than those presented in this paper. In effect, in the semiclassical limit we obtain wave functions that, in general, are not plane waves (even locally), a necessary condition in geometric optics to define directions of propagation and rays. In order to avoid any misunderstanding between the two analogies, we shall not designate the nondispersive regime as the geometric regime, and shall maintain the names of dispersive and nondispersive regimes.

A final comment is in order. The method used in this work can suggest a relationship between the proposal of two quantum regimes and the use of the semiclassical approximation. However, such a relationship does not exist. This can be seen by showing that the results obtained using the semiclassical approximation and the exact Schrödinger equation are equivalent in relation to the definition of the two regimes. At any time previous to the combination of the probabilities of the alternatives, the semiclassical approximation gives plane waves along the classical trajectories, but these classical trajectories are not separated enough. In the case of the exact Schrödinger equation, as the separation between slits is of the same order of magnitude of wavelength of the particle, the probability distribution of the particle at points placed between the two slits and between the source and the slit is clearly different from zero. In any of the two cases, an observation of the companion particle does not give useful information about the alternative taken by the particle. In none of the two approaches can the EFR be applied. The introduction of the two regimes does not depend on the use of the semiclassical approximation.

We want to also emphasize the remarkable inequality of $n(x,y)\neq n_1(x)n_2(y)$. Because the two screens S_1 and S_2 operate independently, the correlated behavior of particles 1 and 2 manifested in these inequalities is an example of quantum nonlocality.

The mathematical analysis of the two-slit experiment has been based on the semiclassical approximation. The analysis presented in the Appendix shows that for particles that do not interact after the preparation, the two-particle semiclassical method can be easily applied and gives a good approximation. Then, the two-particle semiclassical method can be a useful mathematical tool for the class of problems considered in this paper, which can be decomposed into a set of free one-particle trajectories.

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APPENDIX

We present in this appendix an explicit discussion of the semiclassical approximation in the context of two-particle interferometry. We follow closely the analysis of one-particle systems that is presented in many textbooks (see, for instance, Ref. [12]). For the sake of mathematical simplicity we shall only discuss the case of one-dimensional systems. The Schrödinger equation for two particles of masses m_1 and m_2 that do not interact after the preparation is

$$i\hbar\,\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m_1}\frac{\partial^2\Psi}{\partial x_1^2}-\frac{\hbar^2}{2m_2}\frac{\partial^2\Psi}{\partial x_2^2}+V_1(x_1)\Psi+V_2(x_2)\Psi. \tag{A1}$$

Let us now consider the case where the potentials are slowly varying and try a solution of the form $\Psi=N\exp(iS/\hbar)$, with N a constant. As $\hbar\to 0$ represents the classical limit, we must expand S in terms of Planck's constant:

$$S=S_0+\hbar S_1+\hbar^2 S_2+\cdots . \tag{A2}$$

Substituting this expression into Eq. (A1) we obtain the equations that determine the S_1's. The equations for S_0 and S_1 are

$$\frac{\partial S_0}{\partial t}+\frac{1}{2m_1}\left(\frac{\partial S_0}{\partial x_1}\right)^2+\frac{1}{2m_2}\left(\frac{\partial S_0}{\partial x_2}\right)^2+V_1(x_1)+V_2(x_2)=0 \tag{A3}$$

and

$$\frac{\partial S_1}{\partial t}+\frac{1}{2m_1}\frac{\partial^2 S_0}{\partial x_1^2}+\frac{1}{2m_2}\frac{\partial^2 S_0}{\partial x_2^2}+\frac{1}{m_1}\frac{\partial S_1}{\partial x_1}\frac{\partial S_0}{\partial x_1}+\frac{1}{m_2}\frac{\partial S_1}{\partial x_2}\frac{\partial S_0}{\partial x_2}=0. \tag{A4}$$

Equation (A3) is the Hamilton-Jacobi equation for a two-particle system. Therefore, S_0 is the classical action. In the stationary case this action becomes

$$S_0=\int p_1^{\rm cl}(x_1)dx_1+\int p_2^{\rm cl}(x_2)dx_2-E^{\rm cl}t. \tag{A5}$$

We can also obtain Eq. (A5) by direct calculation. In a stationary system the action is $S_0=-E^{\rm cl}t+S_0^*$. The energy is the sum of the energy of the two particles, $E^{\rm cl}=E_1^{\rm cl}+E_2^{\rm cl}$. Then, using Eq. (A3), we have

$$\frac{1}{2m_i}\left(\frac{\partial S_0^*}{\partial x_i}\right)^2+V_i(x_i)=E_i^{\rm cl}\quad (i=1,2). \tag{A6}$$

The two equations (A6) are equivalent to Eq. (A5). The semiclassical approximation is acceptable as long as the terms in Eq. (A3) are smaller than the terms in Eq. (A4) (and, of course, the terms in the equations for $S_2,\ldots$). We compare, for instance, $(\partial S_0/\partial x_1)^2/2m_1=(p_1^{\rm cl})^2/2m_1$ and $\hbar(\partial^2 S_0/\partial x_1^2)/2m_1=\hbar(dp_1^{\rm cl}/dx_1)/2m_1$. This condition is the usual one in one-particle systems

$$\left|\frac{d\lambda_i^*}{dx_i}\right|\ll 1, \tag{A7}$$

where $\lambda^*=\lambda/2\pi$. The two equations (A7) indicate that the wavelengths of the two particles must vary slowly in lengths of the same order of magnitude of the wavelength.

Now, however, we must also compare the crossed terms ($\partial S_0/\partial x_1$ and $\partial^2 S_0/\partial x_2^2 \ldots$). Two new conditions emerge from this analysis,

$$\left|\frac{d\lambda_i^*}{dx_i}\right| \ll \frac{E_j^{\mathrm{cl}}}{E_i^{\mathrm{cl}}}, \tag{A8}$$

where $i=1,2$ and $i \neq j$. When $E_1^{\mathrm{cl}} \approx E_2^{\mathrm{cl}}$ the two conditions (A8) are automatically fulfilled because of Eq. (A7). On the other hand, if one of the two energies is much larger than the other, for instance, $E_1^{\mathrm{cl}} \gg E_2^{\mathrm{cl}}$, we have a new condition $|d\lambda_1^*/dx_1| \ll E_2^{\mathrm{cl}}/E_1^{\mathrm{cl}}$. This new condition defines a new length scale for the variation of the wavelength.

In the system studied in this paper all the conditions, Eqs. (A8) and (A7), are fulfilled. Equation (A7) holds because the semiclassical trajectories can be decomposed into free-particle trajectories. Moreover, the two energies are equal, $E_1^{\mathrm{cl}} = E_2^{\mathrm{cl}}$, and Eq. (A8) is equivalent to Eq. (A7).

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