# Spin-orbit coupling in free-space Laguerre-Gaussian light beams

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The expression for the azimuthal force on an atom in a circularly polarized Laguerre-Gaussian beam is shown to contain a term that depends upon the coupling of the intrinsic spin and orbital angular momenta of the light. Consequently, the state of polarization of the light affects the gross motion of the atom and not simply its internal dynamics. The effect arises from the spin-orbit coupling exhibited in the linear momentum density of the free-space mode. [S1050-2947(96)50505-6]

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The concepts of electron spin and orbital angular momentum are a commonplace in the theory of atomic structure. The detailed origin and labeling of the fine structure of the energy-level scheme arise from the coupling between them. These joint concepts are not so well known for light. The spin of the photon is well understood and explains the polarization of light beams, but although the orbital angular momentum of the photon is known as a concept, it is rarely cited in discussions of dipole radiation and is more commonly associated with multipole radiation [1]. It is not customary, moreover, in this regime to think of a beam of light with a discrete, quantized, amount of orbital angular momentum. More important, there is no evidence of spin-orbit coupling in a beam of free-space light.

A good deal of activity has followed the prediction [2] that free-space Laguerre-Gaussian laser modes possess quantized orbital angular momentum. Theoretical activity includes an eigenfunction description of such beams, the way in which the orbital angular momentum of a beam of light may be analogous to the angular momentum of the harmonic oscillator, as well as studies of the properties of their Poynting vector [3]. In addition, the property has been shown to occur outside the paraxial approximation [4]. Experimental work has included the production of Laguerre-Gaussian beams in the visible, microwave and millimeter-wave regimes [5] culminating, for the moment at least, with the direct qualitative observation of the transfer of orbital angular momentum to absorptive particles [6].

There is also a significant body of work concerned with the interaction of Laguerre-Gaussian beams with atoms. We have shown that an atom moving in a light beam with orbital angular momentum $l$ experiences an azimuthal shift proportional to $l$ in addition to the usual axial Doppler shift and recoil shifts, that the atom is subject to a light-induced torque proportional to $l$, and that atom trajectories are strongly influenced by such a torque [7].

Specific discussion of the commutation rules and eigenvalues of spin and orbital angular momentum has been given by van Enk and Nienhuis [8]. But until now there has been no theoretical evidence of spin-orbit coupling involving the orbital angular momentum and polarization of Laguerre-Gaussian beams. This is not too surprising at first sight as both $\sigma_z$, the state of the spin, and $l$ relate to the $z$ components of the angular momenta. Consequently it is not obvious that there should be an interaction with a term involving a product of these quantities. We show that such a term does in fact exist, which in principle can manifest itself in the trapping and cooling of atoms.

Within the paraxial approximation, the field of an arbitrarily polarized Laguerre-Gaussian mode may be written as

$$\mathbf{E}=i\omega\left[(\alpha\hat{\mathbf{x}}+\beta\hat{\mathbf{y}})u-\frac{i}{k}\left(\alpha\frac{\partial u}{\partial x}+\beta\frac{\partial u}{\partial y}\right)\hat{\mathbf{z}}\right]\exp-ikz. \quad (1)$$

The coefficients $\alpha$ and $\beta$ are such that $\sigma_z=i(\alpha\beta^*-\alpha^*\beta)$ is the polarization operator with $\sigma_z=0$ for linearly polarized light and $\sigma_z=\mp1$ for right-hand and left-hand circularly polarized light. The function $u$ may be written in cylindrical coordinates $(r,\phi,z)$ as

$$u=C\frac{z_R}{(z_R^2+z^2)^{1/2}}\left[\frac{r\sqrt{2}}{w(z)}\right]^l L_p^l\left[\frac{[2r^2]}{w^2(z)}\right]\exp\left(\frac{-r^2}{w^2(z)}\right)\times\exp\left(\frac{-ikr^2z}{2(z^2+z_R^2)}\right)\times\exp(-il\phi)\exp\left(i(2p+l+1)\tan^{-1}\frac{z}{z_R}\right), \quad (2)$$

where $L_p^l$ is a Laguerre polynomial, $p$ and $l$ are the radial and azimuthal indices characterizing the mode, $C$ a normalization factor, $w(z)$ the radius of the beam, and $z_R$ the Rayleigh range.

Following the paper by Allen *et al.* [2] the linear momentum density $S$ for an arbitrarily polarized beam may be shown to be

$$\mathbf{S}=i\omega\frac{\varepsilon_0}{2}(u^*\nabla u-u\nabla u^*)+\omega k\varepsilon_0|u|^2\mathbf{z}+\omega\sigma_z\frac{\varepsilon_0}{2}\frac{\partial|u|^2}{\partial r}\hat{\boldsymbol{\phi}}. \quad (3)$$

The final term in (3) clearly relates to the spin or polarization part of the angular momentum. It may be seen that the term is only nonzero when there is a gradient in the intensity of the light. An attempt to repeat the Beth experiment and measure the spin of the photon would fail for a genuinely infinite plane wave, were such a wave to be possible; there has to be a gradient; see [9]. This is also why the investigation of the turning angle of the Poynting vector of Laguerre-Gaussian beams by Padgett and Allen [3] is independent of polarization, because at the peak of the beam intensity the gradient is zero.

The ratio of the angular momentum per unit length to the energy per unit length [2] is

$$\frac{J}{W}=\frac{l+\sigma_z}{\omega}, \tag{4}$$

where $J$ and $W$ are the angular momentum density and the energy density, respectively, each integrated over the cross section of the beam. Although at first sight the separation of total angular momentum into spin and orbital contributions might seem impermissible [10], this result has been subsequently confirmed beyond the paraxial approximation to within a small correction term, [4], of order $10^{-6}$. In this rigorous theory the full Maxwell equations are solved to yield modes that consist of superpositions of plane polarized modes. The paraxial approximation, the calculation for which was carried out in Lorentz gauge thus preserving the invariance of the Maxwell equations, is equivalent to a very slight truncation of the angular range of the plane-wave superposition. The small correction term in the full Maxwell solution [4] is proportional to $\sigma_z$; thus for linearly polarized light the result that the angular momentum in the Laguerre-Gaussian beam is orbital is entirely rigorous and not an artefact of any approximation. Similarily, when the orbital angular momentum is zero and the field is unfocused, the polarization result is also rigorous. The free-space solutions of the equations offered here are consequently Lorentz invariant. It has been argued elsewhere for the quantized radiation field [8] that the operators $\hat{L}$ and $\hat{\sigma}$ are both Hermitian and gauge invariant.

If no integration over the beam occurs, it is found locally that the ratio of the angular momentum density and the energy density is everywhere constant, namely $l/\omega$, for the nonpolarization-dependent part. However, the contribution of the last term in Eq. (1) to this ratio, the polarization-dependent part, is

$$\frac{\sigma_z}{2}r\frac{\partial|u|^2}{\partial r}\frac{1}{|u|^2}\frac{1}{\omega}, \tag{5}$$

which depends intimately on the position in the beam and involves both $\sigma_z$ and $l$.

The final term in the linear momentum density (3), after simple differentiation of the expression for $|u|^2$ using (2), may be written as

$$\frac{\omega\varepsilon_0}{2}\sigma_z\left\{\frac{l}{r}-\frac{2r}{w^2}+\frac{1}{L_p^l\left(\frac{2r^2}{w^2}\right)}\frac{\partial}{\partial r}L_p^l\left(\frac{2r^2}{w^2}\right)\right\}. \tag{6}$$

We see that locally there is a term that depends on the product $l\sigma_z$. It appears likely therefore that, in suitable circumstances where the local value of the momentum density is called into play, it should be possible to observe spin-orbit coupling arising from the intrinsic angular momentum properties of the light beam. A particularly simple form of (6) occurs for modes with $p=0$, when the final term is zero.

Just as previously we have investigated the role of Laguerre-Gaussian modes in the processes of cooling and trapping of atoms and ions, so may we investigate the potential coupling of spin and orbital angular momentum in this context. We have shown [7] that there is an azimuthal dissipative force due to the orbital angular momentum, given by

$$\langle\mathbf{F}\rangle_{\mathrm{diss}}=\frac{2\hbar\Gamma\Omega_{kpl}^2(\mathbf{R})}{\Delta^2+2\Omega_{kpl}^2(\mathbf{R})+\Gamma^2}\frac{l}{r}\hat{\boldsymbol{\phi}}, \tag{7}$$

where $\Omega_{kpl}(\mathbf{R})=\mathbf{D}\cdot\mathbf{E}/\hbar$ is the Rabi frequency, $\mathbf{D}$ the dipole matrix element, $\Delta$ the detuning of the light from resonance, and $\Gamma$ the decay rate of the excited state of the atom. It is easy to calculate $\Omega$ by use of Eqs. (1) and (2) and we note that the atom may have an induced dipole moment in the $x$, $y$, or $z$ direction, where $z$ is the direction of the light beam. If we write $u=v\ \exp i\theta$ where $v$ and $\theta$ are both real, it is easy to show that we may write the azimuthal component of dissipative force as

$$\langle\mathbf{F}\rangle_{\mathrm{diss}}=-\frac{2\hbar\Gamma\Omega_{k00}^2(\mathbf{R})}{\Delta^2+2\Omega_{kpl}^2(\mathbf{R})+\Gamma^2}vc^2\left[\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial v}{\partial x}\frac{\partial\theta}{\partial y}\right]\frac{\sigma_z l}{r}\hat{\boldsymbol{\phi}}, \tag{8}$$

where $\Omega_{k00}(\mathbf{R})$ is the equivalent plane-wave Rabi frequency and $\Omega_{kpl}(\mathbf{R})$ the positionally dependent Rabi frequency [11].

We see that the product of spin and orbital angular momentum, $\sigma_z l$, appears. It manifests itself in a $\phi$ component of force due to the dipole induced in the direction of the propagation of the light and is due to the $z$ component of electric field associated with the Laguerre-Gaussian mode. Clearly just as reversing the direction of the orbital angular momentum changes the direction of the force and its accompanying torque [7] so, very remarkably, changing the handedness of the circularly polarized light will change the direction of this force and torque. Normally the handedness of the polarization would only be expected to change the internal state of the atom not, as here, the gross motion.

It remains to consider the detailed form and magnitude of the effect. In general, the bracketed quantity in Eq. (8) is a function of Laguerre polynomials and their derivatives, but in the perfectly experimentally valid case of $p=0$ we find that

$$\left[\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial v}{\partial x}\frac{\partial\theta}{\partial y}\right]=lv\left(\frac{l}{r^2}-\frac{2}{w^2}\right). \tag{9}$$

Clearly when $r=w\sqrt{l/2}$ this term and the azimuthal force go to zero. This is as expected because, as discussed previously, this is when the beam intensity is at a peak [3] and $\partial|u|^2/\partial r$ in Eq. (3) is zero. The magnitude of the effect is best considered by comparing the azimuthal force due to spin-orbit coupling with that due to the transverse component of the electric field. To a good approximation for circularly polarized light when $\sigma_z=\pm1$, we see, assuming that the atom is equally polarizable in all directions,

$$\frac{F_{s-l}}{F_{\phi}}=\left|\frac{E_z}{E_x+iE_y}\right|^2\approx\frac{2l}{k^2w^2}\approx\frac{l}{2\pi^2}\left(\frac{\lambda}{w}\right)^2, \qquad (10)$$

where λ is the transition wavelength. This expression is not true when $r=0$, but for a $p=0$ mode the light intensity is in any case zero at that position.

It may be seen that the term is of order $1/k^2w^2$ and is comparable with many other terms that are usually ignored in trapping calculations, but there is a physically meaningful size of beam waist $w$ for which the term is $\sim 10^{-3}$. However, the existence of a spin-orbit term arising from the light is intriguing, and it may prove to be the case that the presently chosen milieu is not the most appropriate to display it. Evidence of its existence and of the increasing parity between certain aspects of the properties of electrons and of light beams creates an alternative frame of discussion for light-matter interactions. Something of the same spirit imbues the recent work [12] on the wave function of the photon. The evidence of quantized orbital angular momentum in free-space light beams, itself a surprising and very recent idea, has already led to experiment and new predictions. It should be noted that in waveguides the concept of orbital angular momentum is not new. While in optically inhomogeneous media such as an optical fiber with two refractive indices, effects such as the Rytov-Vladimirsky-Berry rotation have been ascribed to the interaction of the spin of the photon and its orbital motion [13].

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