Autler-Townes effect for an atom in a 100% amplitude-modulated laser field. I. A dressed-atom approach

M. F. Van Leeuwen, S. Papademetriou, and C. R. Stroud, Jr.
The Institute of Optics, University of Rochester, Rochester, New York 14627
(Received 27 September 1994; revised manuscript received 12 October 1995)

A dressed-state rate-equation model is used to describe the Autler-Townes effect that is produced by a resonant 100% amplitude-modulated field driving a two-level resonance. The purpose of using this model is to give an intuitive interpretation of the rather complex absorption spectra that are produced when a weak probe field couples this strongly driven transition to a third atomic state. We obtain analytic expressions for the positions, strengths, and widths of the components of the absorption spectra, which are in excellent agreement with experimentally obtained spectra presented in the accompanying paper [S. Papademetriou, following paper, Phys. Rev. A **53**, 997 (1996)].

PACS number(s): 42.50.Hz, 32.80.−t

I. INTRODUCTION

When a sufficiently intense monochromatic laser field drives a two-level atomic resonance the resonant coupling changes the atomic structure by strongly mixing the upper and lower levels. This change is evident in the well-known three-peaked fluorescence spectrum of such an atom [1–4]. If one of the resonantly coupled levels is probed by tuning a second low-intensity laser in the vicinity of the transition frequency between this resonantly coupled level and a third level in the atom, the Autler-Townes doublet spectrum is observed [5–12], similarly demonstrating the effect of the intense resonant laser field on the atomic levels.

The three-peaked fluorescence spectrum and the doubly peaked Autler-Townes spectrum are conveniently understood in terms of a dressed-state formalism [13–15] in which one finds the eigenstates of the atom coupled to the strong resonant field and then calculates the spectrum for transitions both among and to these dressed states. The resonant coupling splits the bare or uncoupled states by an amount proportional to the strength of the field, the Rabi frequency.

The nature of the dressing is quite dependent on properties of the strong resonant field. Some experiments have been performed studying the effects, on the driven atom, of finite laser bandwidth [16–19] and phase jumps [20]. Recently, an experiment has been carried out studying the fluorescence spectrum in a case in which the intense resonant field is not monochromatic but instead is bichromatic in the form of an amplitude-modulated (AM) field with the resonant carrier suppressed [21]. The spectrum was much more complex with many lines of various strengths separated by the modulation frequency. The number of peaks in the spectrum increased with increasing field strength and the widths of neighboring peaks in the spectrum were observed to alternate between narrow and broad.

Subsequent theoretical analyses of the fluorescence spectrum [22], using the Bloch-vector formalism [23–25] and the dressed-state approach [26], have been able to qualitatively account for the observed phenomena. The dependence of the strength of individual lines and the number of components in the spectrum on the single-field or time-averaged Rabi frequency was reproduced. The nonlinearities introduced by the intense driving field were seen to cause harmonics of the modulation frequency to manifest themselves in a nontrivial fashion in variables related to atomic observeables, such as the fluorescence spectrum.

The widths of neighboring peaks in the spectrum of the inelastically scattered light were predicted to alternate as functions of the field strength. In addition, at every odd peak in the spectrum an elastically scattered subharmonic of the modulated driving field [27] contributes to the experimentally observed linewidths. The experiment of Zhu *et al.* was unable to resolve these contributions to the linewidths.

The Autler-Townes spectrum is a direct probe of the strongly driven atomic resonance that avoids the complication of the elastically scattered spectral components. In the following paper we report a measurement of the Autler-Townes absorption spectrum for an intense 100% AM driving field [28]. In the saturating regime the observed linewidths may be interpreted solely in terms of transition rates between dressed levels.

In this article we present a dressed-state calculation of the Autler-Townes absorption spectrum that offers an intuitive interpretation of the location and widths of the many peaks in the spectrum. We calculate an analytic expression for the spectrum for a number of three-level atomic configurations.

II. ATOM-PLUS-FIELD SYSTEM

Initially, we consider a beam of atoms with two nondegenerate energy eigenstates, a ground state $|a\rangle$ and a dipole-connected excited state $|b\rangle$, which has a natural width Γ_b and is situated $\hbar\omega_{ba}$ above $|a\rangle$ in energy. This beam is irradiated at right angles by an intense bichromatic laser with equal amplitude modes of frequency $\omega_L \pm \delta$, which henceforth we shall refer to as the $\pm$ modes (see Fig. 1). The atoms interact with the laser field for a time T that is much greater than the natural lifetime Γ_b^{-1} of the excited atomic state, i.e., $\Gamma_b T \gg 1$.

We set the ground-state energy of our system equal to zero for convenience and consider only the case in which the average laser frequency is equal to the atomic resonance frequency $\omega_L = \omega_{ba}$. We assume that the $\pm$ modes are large-amplitude coherent states for which the initial average num-

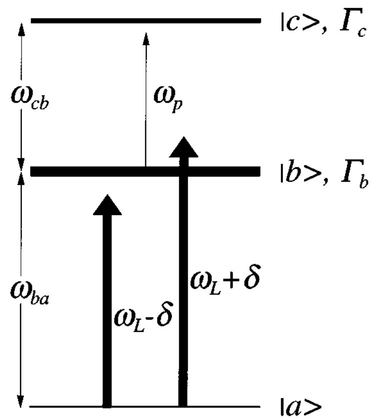

FIG. 1. Three-level atomic cascade $|a\rangle \leftrightarrow |b\rangle \leftrightarrow |c\rangle$ with an intense 100% AM field ($\omega_L \pm \delta$) resonantly pumping the lower $|a\rangle \leftrightarrow |b\rangle$ transition and a weak field (ω_p) probing the $|b\rangle \leftrightarrow |c\rangle$ transition.

ber of photons in both modes is N_0 and the width of the distribution of the number of photons about N_0 is $\sqrt{N_0}$ such that $N_0 \gg \sqrt{N_0} \gg 1$. We further restrict our problem by considering only modulation frequencies δ that are much smaller than the average laser frequency but still many times greater than the natural width of the excited atomic state

$$\omega_L \gg \delta \gg \Gamma_b , \tag{2.1}$$

such that the field amplitude experiences many modulation periods within an atomic lifetime. For optical transitions, where $\omega_{ba}/\Gamma_b \sim 10^8$, this condition is not very restrictive and includes regimes in which the atom responds nonadiabatically to the modulated field.

The bare states of this system are the eigenstates of the uncoupled atom-plus-field Hamiltonian

$$\hat{H}_0 = \hbar \omega_{ba} \hat{\sigma}_{bb} + \hbar(\omega_L + \delta)\hat{a}_+^\dagger \hat{a}_+ + \hbar(\omega_L - \delta)\hat{a}_-^\dagger \hat{a}_- , \tag{2.2}$$

where $\hat{\sigma}_{bb}$ is the atomic projection operator for the state $|b\rangle$ and $\hat{a}_\pm(\hat{a}_\pm^\dagger)$ are the annihilation (creation) operators for the quantized $\pm$ field modes. These states can be written as product states $|\alpha, N_+, N_-\rangle$, where $\alpha \in \{a,b\}$ labels the atomic state and $N_\pm \in \{0,1,2,\ldots\}$ designate the number of photons in the $\pm$ field modes. The bare eigenenergies can be found from the solutions of the eigenvalue equations

$$\hat{H}_0|a,N_+,N_-\rangle = \hbar[(N_+ + N_-)\omega_L + (N_+ - N_-)\delta]|a,N_+,N_-\rangle, \tag{2.3a}$$

$$\hat{H}_0|b,N'_+,N'_-\rangle = \hbar[(N'_+ + N'_- + 1)\omega_L + (N'_+ - N'_-)\delta]|b,N'_+,N'_-\rangle. \tag{2.3b}$$

It is evident from the above equations that the set of states $\{|a,N_+,N_-\rangle, |b,N'_+,N'_-\rangle\}$, where $N_+ + N_- = N'_+ + N'_- + 1 = N$, form a manifold of $2N+1$ states that are quasidegenerate in energy, with energies $E = N\hbar\omega_L + n\hbar\delta \equiv E_N + n\hbar\delta$, where $n \in \{0, \pm 1, \pm 2, \ldots, \pm N\}$. This manifold of states is separated from neighboring manifolds by an energy $\hbar\omega_L$, while neighboring states within a manifold are separated by $\hbar\delta$. The state space therefore separates naturally into subspaces $\mathscr{E}_N$ with dimension $2N+1$, which can be said to contain states representing N elementary excitations of the atom and the laser fields (see Fig. 2). It is convenient to relabel the bare states with angular-momentum-like notation $|N,n\rangle$ with $N \in \{0,1,2,\ldots\}$ and $n \in \{0, \pm 1, \pm 2, \ldots, \pm N\}$, such that

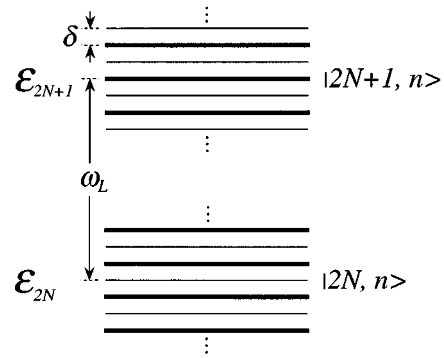

FIG. 2. Uncoupled eigenstates of the two-level atom-plus-AM-field system.

$$|2N+1, 2n+1\rangle \equiv |a, N+n+1, N-n\rangle, \tag{2.4a}$$

$$|2N+1, 2n\rangle \equiv |b, N+n, N-n\rangle, \tag{2.4b}$$

$$|2N, 2n\rangle \equiv |a, N+n, N-n\rangle, \tag{2.4c}$$

$$|2N, 2n+1\rangle \equiv |b, N+n, N-n-1\rangle, \tag{2.4d}$$

as suggested by the eigenvalue equation

$$\hat{H}_0|N,n\rangle = (E_N + n\hbar\delta)|N,n\rangle \tag{2.5}$$

and orthonormality condition

$$\langle N,n|N',n'\rangle = \delta_{n,n'}\delta_{N,N'} \tag{2.6}$$

of the uncoupled system. It is useful to note that manifolds with an even number of excitations ($2N$) and those with an odd number of excitations ($2N+1$) are distinct, i.e., $|2N,0\rangle = |a,N,N\rangle$ while $|2N+1,0\rangle = |b,N,N\rangle$, and we should anticipate the need to consider them separately in our treatment to follow.

In the electric-dipole and rotating-wave approximations the atom and the laser fields are coupled by the interaction Hamiltonian

$$\hat{V}_{AL} = \frac{\hbar\Omega_0}{2}[\hat{a}_+^\dagger \hat{\sigma}_- + \hat{\sigma}_+ \hat{a}_-] + \text{H.c.}, \tag{2.7}$$

where $\hat{\sigma}_+(\hat{\sigma}_-)$ is the atomic raising (lowering) operator and Ω_0 is the vacuum Rabi frequency, which we assume to be real. $\hat{V}_{AL}$ represents energy conserving processes and therefore only couples states within a given manifold. The nonzero interaction matrix elements can be shown to couple only neighboring states

$$\langle 2N+1,2n\pm1|\hat{V}_{AL}|2N+1,2n\rangle=\frac{\hbar\Omega_0}{2}\sqrt{N\pm n+1}, \quad (2.8a)$$

$$\langle 2N,2n\pm1|\hat{V}_{AL}|2N,2n\rangle=\frac{\hbar\Omega_0}{2}\sqrt{N\mp n}. \quad (2.8b)$$

In the $\mathcal{E}_N$ manifold the interaction matrix elements range in magnitude from $\hbar\Omega_0/2$ to $\hbar\Omega_0\sqrt{N}/2$. If the number of photons scattered from the laser field, which is of the order $\Gamma_b T$, is much smaller than the range over which the average number of photons in the $\pm$ modes is relatively constant, i.e., $\sqrt{N_0}\gg\Gamma_b T$, then we may neglect the variation in the magnitude of the interaction matrix elements within and among neighboring manifolds and set them equal to $\hbar$ times half the classical single-field Rabi frequency $\hbar\Omega/2\equiv\hbar\Omega_0\sqrt{N_0}/2$. We further assume that the manifolds about the $\mathcal{E}_{2N_0}$, where the initial state of the system is centered, are effectively infinite dimensional.

In the $\mathcal{E}_N$ subspace the coupled atom-plus-field Hamiltonian $\hat{H}_{AL}=\hat{H}_0+\hat{V}_{AL}$ can therefore be represented in the bare basis by the matrix $\hat{H}_{AL}=E_N\hat{I}+\hbar\hat{M}$, where $\hat{I}$ is the identity matrix and

$$\hat{M}=\begin{bmatrix} \ddots & & & & & & \\ & 2\delta & \Omega/2 & & & & \\ & \Omega/2 & \delta & \Omega/2 & & & \\ & & \Omega/2 & 0 & \Omega/2 & & \\ & & & \Omega/2 & -\delta & \Omega/2 & \\ & & & & \Omega/2 & -2\delta & \\ & & & & & & \ddots \end{bmatrix}. \quad (2.9)$$

The dressed states and energies are the eigenstates and eigenenergies of $\hat{H}_{AL}$ that follow directly from the eigenvectors and eigenvalues of the symmetric tridiagonal matrix $\hat{M}$.

The eigenvalues of $\hat{M}$ are found by solving the characteristic equation $\det[\hat{M}-\lambda\hat{I}]=0$. The periodicity along the diagonal of the infinite dimensional matrix $\hat{M}$ suggests that $\det[\hat{M}-\lambda\hat{I}]=\det[\hat{M}-(\lambda+m\delta)\hat{I}]$, where $m\in\{0,\pm1,\pm2,\dots\}$. Therefore, if $\lambda^{(0)}$ is a given eigenvalue of $\hat{M}$, then $\lambda^{(m)}=\lambda^{(0)}+m\delta$ are also eigenvalues. The symmetry of $\hat{M}$ implies that $\mathrm{Tr}\hat{M}=0$ or, equivalently, that the sum of the eigenvalues of $\hat{M}$ is zero such that $\lambda^{(0)}=0$ and $\lambda^{(m)}=m\delta$. The dressed energies are therefore

$$E_N^{(m)}=E_N+m\hbar\delta, \quad m\in\{0,\pm1,\pm2,\dots\}. \quad (2.10)$$

The corresponding dressed states (represented with rounded kets) are found from the eigenvalue equation

$$\hat{M}|N,m)=\lambda^{(m)}|N,m), \quad (2.11)$$

where $|N,m)$ can be expanded in the bare basis

$$|N,m)=\sum_{n=-\infty}^{\infty}a_n^{(m)}|N,n\rangle. \quad (2.12)$$

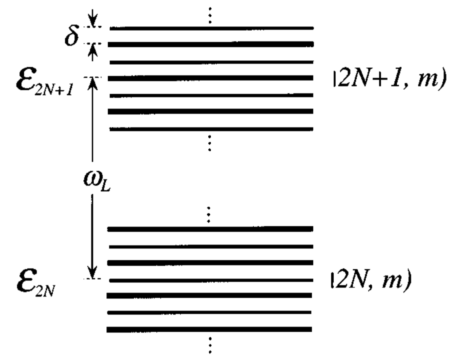

FIG. 3. Dressed eigenstates of the two-level atom-plus-AM-field system.

Equations (2.11) and (2.12) lead to the three-term recurrence relation

$$\Omega a_{n+1}^{(m)}+\Omega a_{n-1}^{(m)}-2(m-n)\delta a_n^{(m)}=0. \quad (2.13)$$

The Bessel function recurrence relation [30]

$$xJ_{n+1}(x)+xJ_{n-1}(x)-2nJ_n(x)=0 \quad (2.14)$$

suggests that

$$a_n^{(m)}\propto J_{m-n}(\Omega/\delta). \quad (2.15)$$

It can be verified with the use of well-known Bessel function identities [30] that

$$|N,m)=\sum_{n=-\infty}^{\infty}J_{m-n}(\Omega/\delta)|N,n\rangle \quad (2.16)$$

such that

$$\hat{H}_{AL}|N,m)=E_N^{(m)}|N,m) \quad (2.17)$$

and

$$(N,m|N',m')=\delta_{mm'}\delta_{NN'}. \quad (2.18)$$

It is evident from Eqs. (2.5) and (2.17) that the bare and dressed energy spectra are identical for a two-level resonance dressed by a bichromatic field with equal amplitude modes and a resonant average frequency (Fig. 3). If the mode amplitudes are imbalanced and/or the average frequency is detuned from resonance, then in the limit of effectively infinite-dimensional manifolds the periodicity of $\hat{M}$ suggests that the spectrum then consists of pairs of eigenvalues separated by 2δ, i.e., $\lambda^{(i,m)}\in\{\lambda^{(0)}+2m\delta,\lambda^{(1)}+2m\delta\}$ with $\lambda^{(0)}+\lambda^{(1)}=\delta$ [25,29]. $\lambda^{(0)}$ and $\lambda^{(1)}$ will in general depend upon the field strengths and detunings. Signatures of this eigenspectrum have been recently observed in the Autler-Townes absorption spectrum with a fully AM pump field and a detuned average frequency [28].

III. DRESSED-STATE TRANSITION RATES

We now consider the interaction of the dressed atom with the empty modes of the electromagnetic field in order to calculate the spontaneous transition rates from the dressed states. This coupling induces changes in the atomic state from $|b\rangle$ to $|a\rangle$ while leaving unchanged the number of photons in the laser fields. Given that the dressed states are nondegenerate $\delta \gg \Gamma_b$, the transition probability per unit time between two states is proportional to the square of the dipole moment connecting the states. This can be shown to be nonzero only between states in neighboring manifolds as the atom scatters laser photons into the empty modes of the field. With Eqs. (2.4) and (2.16) and Bessel function summation formulas [30] the dipole moment connecting $|2N+1,m)$ to $|2N,m')$ can be calculated to be

$$\boldsymbol{\mu}_{m'm} = (2N,m'|\hat{\mu}|2N+1,m) = \frac{\boldsymbol{\mu}_{ab}}{2}[\delta_{m'm} + (-1)^m J_{m'-m}(2\Omega/\delta)]. \quad (3.1)$$

The transition rate from $|2N+1,m)$ to $|2N,m')$ is therefore

$$\Gamma_{m'm} = \frac{\Gamma_b}{4}[\delta_{mm'} + (-1)^m J_{m'-m}(2\Omega/\delta)]^2. \quad (3.2)$$

Similarly, the dipole moment connecting $|2N,m)$ to $|2N-1,m')$ is

$$\boldsymbol{\mu}_{m'm} = (2N-1,m'|\hat{\mu}|2N,m) = \frac{\mu_{ab}}{2}[\delta_{m'm} - (-1)^m J_{m'-m}(2\Omega/\delta)] \quad (3.3)$$

such that the transition rate from $|2N,m)$ to $|2N-1,m')$ is

$$\gamma_{m'm} = \frac{\Gamma_b}{4}[\delta_{mm'} - (-1)^m J_{m'-m}(2\Omega/\delta)]^2. \quad (3.4)$$

The total transition rate out of $|2N+1,m)$ is then given by

$$\Gamma_m = \sum_{m'=-\infty}^{\infty} \Gamma_{m'm} = \frac{\Gamma_b}{2}[1 + (-1)^m J_0(2\Omega/\delta)] \quad (3.5)$$

and the total transition rate out of $|2N,m)$ is given by

$$\gamma_m = \sum_{m'=-\infty}^{\infty} \gamma_{m'm} = \frac{\Gamma_b}{2}[1 - (-1)^m J_0(2\Omega/\delta)]. \quad (3.6)$$

From the definition of the dressed states it can be seen that states with a greater contamination of the excited state $|b\rangle$ are more unstable than others, as expected.

It is clear from Eqs. (3.5) and (3.6) that, unlike in the case of a monochromatic laser field, the dressed-state transition rates differ between manifolds of even and odd numbers of excitations. The dynamics of even and odd manifolds should therefore be treated separately in order to properly characterize an atom dressed by a 100% AM field as we anticipated in Sec. II. This distinction has recently been made in the literature [26].

IV. DRESSED-STATE POPULATIONS

The rates of change of the dressed-state populations may be described by population rate equations in the secular limit ($\delta \gg \Gamma_b$) of the master equation describing the evolution of the dressed-atom density operator $\hat{\rho}_{AL}(t)$ [15]. If the populations of the dressed states are defined as

$$\Pi_m(N;t) \equiv (N,m|\hat{\rho}_{AL}(t)|N,m), \quad (4.1)$$

then the rates of change for the populations in manifolds with an odd and even number of excitations can be written respectively as

$$\frac{d}{dt}\Pi_m(2N+1;t) = -\Gamma_m \Pi_m(2N+1;t) + \sum_{k=-\infty}^{\infty} \gamma_{mk}\Pi_k(2N+2;t), \quad (4.2a)$$

$$\frac{d}{dt}\Pi_m(2N;t) = -\gamma_m \Pi_m(2N;t) + \sum_{k=-\infty}^{\infty} \Gamma_{mk}\Pi_k(2N+1;t). \quad (4.2b)$$

The above infinite set of coupled first-order differential equations appears rather intractable, but fortunately in our calculation of the absorption spectrum we will only require knowledge of the sums of all the populations of levels with the same total transition rates or natural widths in the dressed basis.

There are two characteristic dressed-state natural widths, which we define as $\Gamma_+ \equiv \Gamma_{2m} = \gamma_{2m+1}$ and $\Gamma_- \equiv \Gamma_{2m+1} = \gamma_{2m}$, such that

$$\Gamma_\pm = \frac{\Gamma_b}{2}[1 \pm J_0(2\Omega/\delta)]. \quad (4.3)$$

If we define the reduced populations

$$P_+(t) \equiv \sum_N \sum_{m=-\infty}^{\infty} \Pi_{2m}(2N+1;t) + \Pi_{2m+1}(2N;t), \quad (4.4a)$$

$$P_-(t) \equiv \sum_N \sum_{m=-\infty}^{\infty} \Pi_{2m+1}(2N+1;t) + \Pi_{2m}(2N;t), \quad (4.4b)$$

where $P_\pm$ are the total populations in states with widths $\Gamma_\pm$, then Eqs. (4.2) reduce to

$$\frac{d}{dt}P_+(t) = -(\Gamma_+ - \Gamma)P_+(t) + (\Gamma_- - \Gamma)P_-(t), \quad (4.5a)$$

$$\frac{d}{dt}P_-(t) = -(\Gamma_- - \Gamma)P_-(t) + (\Gamma_+ - \Gamma)P_+(t), \quad (4.5b)$$

where we have defined

$$\Gamma \equiv \frac{\Gamma_b}{8}[1 - J_0(4\Omega/\delta)]. \quad (4.6)$$

These equations can be solved simply in the quasistationary regime to which we have restricted our consideration ($\sqrt{N_0} \gg \Gamma_b T \gg 1$) to give

$$P_\pm^{ss} = \frac{\Gamma_\mp - \Gamma}{\Gamma_b - 2\Gamma} = \frac{1}{2}\left(1 \mp \frac{4J_0(2\Omega/\delta)}{3 + J_0(4\Omega/\delta)}\right). \quad (4.7)$$

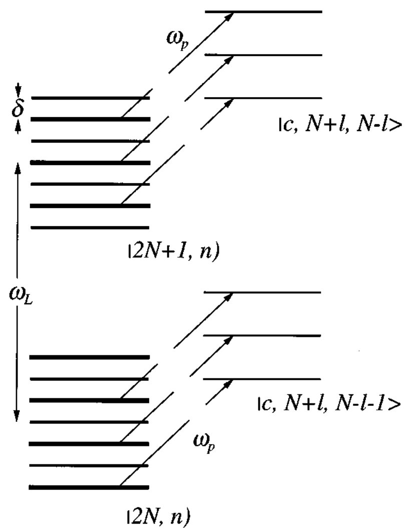

FIG. 4. Weak probe transitions from the dressed eigenstates to a third atomic level $|c\rangle$.

From Eqs. (4.5) we see that in the steady state the number of transitions from states with a width Γ_+ to those with a width Γ_- is balanced by the number of transitions in the opposite direction to achieve a condition of detailed balance. It can also be shown that in the steady state the total population in manifolds of an even or odd number of excitations approach 1/2, as do the sums of populations of dressed states $|N,m)$ with even and odd quantum numbers m.

V. ABSORPTION OF A WEAK PROBE

We now introduce a third atomic state $|c\rangle$ with a natural width Γ_c, which is situated $\hbar\omega_{cb}$ above $|b\rangle$. We assume that ω_{cb} is not degenerate with any frequencies in the dressed system and that the modulation frequency is large compared to Γ_c. If we perturbatively dipole couple $|c\rangle$ to $|b\rangle$ with a weak monochromatic probe laser field we can neglect any broadening or frequency shifts. The probe laser will induce changes in the atomic state from $|b\rangle$ to $|c\rangle$ while the number of photons in the AM field will remain unchanged.

The states $|c,N_+,N_-\rangle$, which are coupled to the dressed manifolds of states form degenerate manifolds of states themselves. It can be shown from dipole selection rules that only states $|c,N+l,N-l\rangle$, from what we shall label as the $\mathscr{E}_{c,2N+1}$ manifold, can couple to states within the $\mathscr{E}_{2N+1}$ manifold and only states $|c,N+l,N-l-1\rangle$ within the $\mathscr{E}_{c,2N}$ manifold can couple to states within the $\mathscr{E}_{2N}$ manifold, where $l\in\{0,\pm1,\pm2,\ldots\}$ (see Fig. 4).

To first order in the probe interaction the steady-state cross section, as a function of the probe frequency, for absorbing a photon and causing a transition from a dressed state in the $\mathscr{E}_N$ manifold to a state in the $\mathscr{E}_{c,N}$ manifold is proportional to the product of the population difference between the two states and the square of the dipole moment between the two states with a width equal to the sum of the widths of the two states [31]. The cross section in the steady state as a function of the probe laser frequency is therefore

$$\sigma(\omega_p)=\frac{4\pi^2\omega_{cb}|\boldsymbol{\mu}_{bc}|^2}{\hbar c}\sum_N\sum_{m,l=-\infty}^{\infty}\Pi_m^{\rm ss}(2N)\times J^2_{m-2l-1}(\Omega/\delta)\mathscr{L}(2l+1-m;\gamma_m)+\frac{4\pi^2\omega_{cb}|\boldsymbol{\mu}_{bc}|^2}{\hbar c}\sum_N\sum_{m,l=-\infty}^{\infty}\Pi_m^{\rm ss}(2N+1)\times J^2_{m-2l}(\Omega/\delta)\mathscr{L}(2l-m;\Gamma_m), \tag{5.1}$$

where we have calculated the dipole matrix elements

$$(2N+1,m|\hat{\mu}|c,N+l,N-l\rangle=\boldsymbol{\mu}_{bc}J_{m-2l}(\Omega/\delta), \tag{5.2a}$$

$$(2N,m|\hat{\mu}|c,N+l,N-l-1\rangle=\boldsymbol{\mu}_{bc}J_{m-2l-1}(\Omega/\delta) \tag{5.2b}$$

and represented the normalized Lorentzian with a full width at half maximum (FWHM) of $\gamma+\Gamma_c$ centered about $\Delta_p\equiv\omega_p-\omega_{cb}=k\delta$ by

$$\mathscr{L}(k;\gamma)\equiv\frac{1}{2\pi}\frac{(\gamma+\Gamma_c)}{[\Delta_p-k\delta]^2+[(\gamma+\Gamma_c)/2]^2}. \tag{5.3}$$

The frequency separation of the states $|c,N+l,N-l\rangle$ and $|2N+1,m)$ is $\omega_{cb}+2l\delta-m\delta$ and the separation of the states $|c,N+l,N-l-1\rangle$ and $|2N,m)$ is $\omega_{cb}+(2l+1)\delta-m\delta$, where $m,l\in\{0,\pm1,\pm2,\ldots\}$. The probe laser will therefore be resonant with an infinite number of independent $(2\delta\gg\Gamma_b,\Gamma_c)$ transitions for any frequency $\omega_p\approx\omega_{cb}+k\delta$ where $k\in\{0,\pm1,\pm2,\ldots\}$ (see Fig. 4).

If we consider a given probe frequency $\omega_p\approx\omega_{cb}+2k\delta$, then the steady-state absorption cross section about this resonance can be written

$$\sigma_{2k}(\omega_p)=\alpha P_+^{\rm ss}J^2_{2k}(\Omega/\delta)\mathscr{L}(2k;\Gamma_+), \tag{5.4}$$

where we have made use of Eqs. (4.4) and represented the constant factors by α. Similarly for a probe frequency $\omega_p\approx\omega_{cb}+(2k+1)\delta$ the steady-state absorption cross section is

$$\sigma_{2k+1}(\omega_p)=\alpha P_-^{\rm ss}J^2_{2k+1}(\Omega/\delta)\mathscr{L}(2k+1;\Gamma_-). \tag{5.5}$$

The total absorption cross section as a function of the probe field frequency is therefore

$$\sigma(\omega_p)=\alpha\sum_{k=-\infty}^{\infty}P_-^{\rm ss}J^2_{2k+1}(\Omega/\delta)\mathscr{L}(2k+1;\Gamma_-)+\alpha\sum_{k=-\infty}^{\infty}P_+^{\rm ss}J^2_{2k}(\Omega/\delta)\mathscr{L}(2k;\Gamma_+). \tag{5.6}$$

We have used the condition that all of the transitions are well separated $(\delta\gg\Gamma_b,\Gamma_c)$ in order to incoherently sum the individual cross sections above. The above expression suggests a simple labeling scheme for the absorption resonances whereby the resonance at $\omega_p=\omega_{cb}+k\delta$ is labeled the kth resonance.

The system we have used above corresponds to a three-level cascade. In a similar manner the absorption cross section can be calculated for the analogous transition in a Λ or

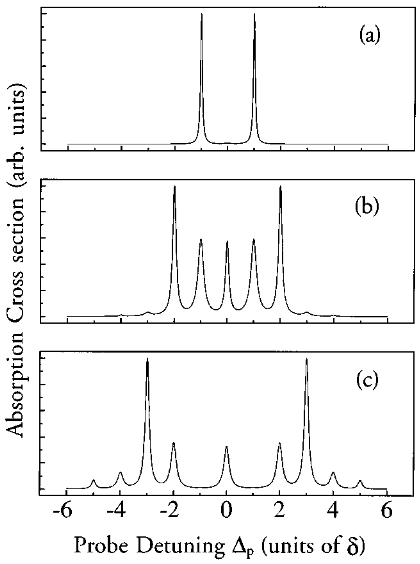

FIG. 5. Probe absorption cross section in arbitrary units versus detuning Δ_p in units of modulation frequencies δ with $\delta/\Gamma_b=4$ and $\Gamma_b/\Gamma_c=3$ for (a) $\Omega/\delta=1$, (b) $\Omega/\delta=15$, and (c) $\Omega/\delta=30$.

V system or for the lower transition of a three-level cascade, the upper transition of which is being resonantly driven by the AM field. The spectra of the absorption cross sections will retain the same form as Eq. (5.6), but the population differences, dipole moments, and linewidths will vary depending upon the particular configuration. For example, if the probe coupled the unstable state $|c\rangle$ to state $|a\rangle$ instead of to state $|b\rangle$, in a V configuration, then the even and odd resonances would switch widths and population weightings, while the oscillator strengths of resonances would remain unchanged. For the other two cases the population would be pumped into or remain in the third level so that all the resonances would have equal population weightings such that line strengths would be directly observable from the peak heights in the absorption spectrum.

VI. DISCUSSION

In Fig. 5 we have plotted the expression in Eq. (5.6) for the AM Autler-Townes absorption spectrum. We have used experimentally realistic parameters corresponding to the $3S_{1/2}\leftrightarrow 3P_{3/2}\leftrightarrow 4D_{5/2}$ three-level cascade in Na such that $\Gamma_b/2\pi=10$ MHz and $\Gamma_c/2\pi=3.15$ MHz with a modulation frequency $\delta/2\pi=40$ MHz. These spectra show excellent agreement with plots generated by a matrix continued-fraction steady-state solution of the three-level optical Bloch equations with a strong fully AM pump field and weak probe field. The agreement improves for large values of the ratio δ/Γ_b, which we have assumed in our calculation. We have also compared our solution to recently obtained experimental observations of the Autler-Townes effect for an atom in an AM field and have found very good qualitative and quantitative agreement [28].

In Fig. 4 we see that the absorption spectra consist of many resonances always separated by the modulation frequency. For increasing pump intensities the number of absorption resonances increases. In fact, for a given Rabi frequency all of the absorption resonances lie within a

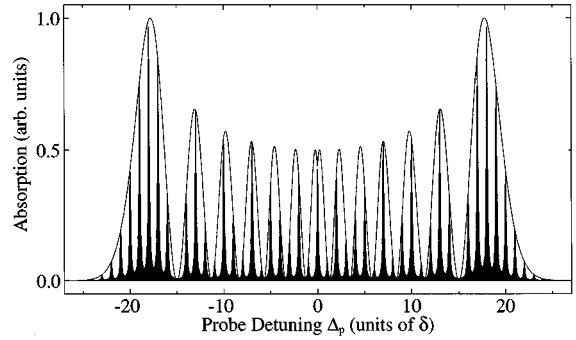

FIG. 6. Histogram of absorption cross section and a scaled plot of $J^2_{\Delta_p/\delta}(\Omega/\delta)$ in arbitrary units versus detuning Δ_p in units of modulation frequencies δ with $\delta/\Gamma_b=4$ and $\Gamma_b/\Gamma_c=3$ for $\Omega/\delta=20$.

frequency range approximately equal to what would have been the peak separation of the Autler-Townes spectrum for an unmodulated pump 2Ω. There will therefore be roughly $2\Omega/\delta$ resonances.

We have plotted the absorption spectrum for a large value of $\Omega/\delta=20$ along with the envelope of the values of the peak oscillator strengths $J^2_{\Delta_p/\delta}(\Omega/\delta)$ in Fig. 6. We see that in the limit of $\Omega\gg\delta$ the population weightings $P_\pm$ approach 1/2 and spectrum is well described by peaks of aproximately equal width at the integer values of $k=\Delta_p/\delta$ with strengths $J^2_k(\Omega/\delta)$, i.e., a comb of equally spaced peaks separated by δ under the envelope $J^2_{\Delta_p/\delta}(\Omega/\delta)$. This envelope has a modulated structure for values of $\Delta_p\leqslant\Omega$ that peaks at $\Delta_p\approx\Omega$ and falls off rapidly for $\Delta_p>\Omega$.

It is clear from our expression that the absorption spectrum directly reflects the alternation in width of the dressed states. Even resonances have a FWHM of $\Gamma_+ + \Gamma_c$ and odd resonances have a FWHM of $\Gamma_- + \Gamma_c$, where $\Gamma_\pm$ are oscillating functions of $2\Omega/\delta$. The relative widths of the even and odd components of the spectrum oscillate 180° out of phase with each other as a function of the modulation index (see Fig. 5 of Ref. 13) and approach the same value $\Gamma_c+\Gamma_b/2$ for large values of Ω/δ and at the zeros of $J_0(2\Omega/\delta)$. The alternation in the widths of the resonances can be interpreted in the dressed-state picture as being a consequence of selection rules that dictate that for the even reso-

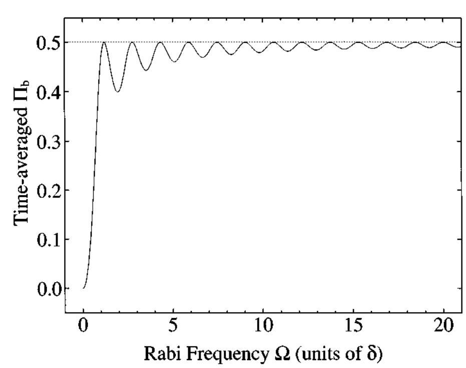

FIG. 7. Time-averaged population of level $|b\rangle$ versus Ω/δ. The steady-state integrated probe absorption cross section is proportional to the time-averaged population of the probed atomic state.

nances the probe only interacts with dressed states with widths Γ_+ and for odd resonances the probe interacts with Γ_- dressed states only.

We can gain further insight into the effect of an intense fully AM pump field on a two-level resonance by integrating the expression in Eq. (5.6) over the frequency range of the probe laser to obtain the total integrated absorption cross section. This can be shown to be proportional to the steady-state population of the atomic state $|b\rangle$ averaged over a modulation period, i.e.,

$$\int \sigma(\omega_p)d\omega_p=\alpha_b=\alpha\frac{1}{2}\left(1-\frac{4J_0^2(2\Omega/\delta)}{3+J_0(4\Omega/\delta)}\right). \quad (6.1)$$

In this expression, which we have plotted in Fig. 7, we see evidence of the subharmonic Rabi resonances where the ability of the driven atom to absorb the probe is maximized to the saturation level [32]. These resonances occur at the zeros of $J_0^2(2\Omega/\delta)$ where dressed-state transition rates are equal $\Gamma_\pm=\Gamma_b$ and hence the populations $P_\pm^{ss}$ become equal. At each resonance another higher-order parametric absorption process becomes significant; another harmonic of the modulation frequency appears and more peaks arise in the probe spectrum.

VII. CONCLUSION

In conclusion, we note that we have obtained an analytic expression, via the dressed state formalism, for the 100% AM Autler-Townes spectrum in the intense field limit when the modulation frequency is much greater than the natural width of the strongly driven transition. Within this formalism we have found a simple interpretation of the absorption spectrum in terms of transitions between dressed states and uncoupled bare states.

ACKNOWLEDGMENT

This work was supported by the U. S. Army Research Office.

[1] B. R. Mollow, Phys. Rev. **188**, 1969 (1969).

[2] F. Schuda, C. R. Stroud, Jr., and M. Hercher, J. Phys. B **7**, L198 (1974).

[3] H. Walther, in *Proceedings of the Second Laser Spectroscopy Conference*, edited by S. Haroche (Springer, Berlin, 1975).

[4] F. Y. Wu, R. E. Grove, and S. Ezekiel, Phys. Rev. A **15**, 277 (1977).

[5] S. H. Autler and C. H. Townes, Phys. Rev. **100**, 703 (1955).

[6] B. R. Mollow, Phys. Rev. A **5**, 1522 (1972).

[7] A. Schabert, R. Keil, and P. E. Toschek, Opt. Commun. **13**, 265 (1975).

[8] Ph. Cahuzac and R. Vetter, Phys. Rev A **14**, 270 (1976).

[9] C. Delsart and J. C. Keller, J. Phys. B **9**, 2769 (1976).

[10] J. L. Picque and J. Pinard, J. Phys. B **9**, L77 (1976).

[11] J. E. Bjorkholm and P. F. Liao, Opt. Commun. **21**, 132 (1977).

[12] H. R. Gray and C. R. Stroud, Jr., Opt. Commun. **25**, 359 (1978).

[13] R. M. Whitley and C. R. Stroud, Jr., Phys. Rev. A **4** 1498 (1976).

[14] C. Cohen-Tannoudji and S. Reynaud, J. Phys. B **10**, 345 (1977).

[15] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions* (Wiley, New York, 1992).

[16] A. T. Georges and P. Lambropoulos, Phys. Rev. A **20**, 991 (1979).

[17] P. Zoller, Phys. Rev. A **20**, 1019 (1979).

[18] M. W. Hamilton, D. S. Elliott, K. Arnett, and S. J. Smith, Phys. Rev. A **33**, 778 (1986).

[19] M. W. Hamilton, K. Arnett, S. J. Smith, D. S. Elliott, M. Dziemballa, and P. Zoller, Phys. Rev. A **36**, 178 (1987).

[20] Y. S. Bai, T. W. Mossberg, N. Lu, and P. R. Berman, Phys. Rev. Lett. **57**, 1692 (1986).

[21] Y. Zhu, A. Lezama, D. J. Gauthier, and T. Mossberg, Phys. Rev. A **41**, 6574 (1990).

[22] A comprehensive list of references on the AM excitation of a two-level resonance can be found in Ref. [28].

[23] S. P. Tewari and M. K. Kumari, Phys. Rev. A **41**, 5273 (1990).

[24] G. S. Agarwal, Y. Zhu, D. J. Gauthier, and T. Mossberg, J. Opt. Soc. Am. B **8**, 1163 (1991).

[25] Z. Ficek and H. S. Freedhoff, Phys. Rev. A **48**, 3092 (1993).

[26] H. S. Freedhoff and Z. Chen, Phys. Rev. A **41**, 6013 (1990); **46**, 7328(E) (1992).

[27] W. M. Ruyten, J. Opt. Soc. Am. B **9**, 1892 (1992).

[28] S. Papademetriou, M. F. Van Leeuwen, and C. R. Stroud, Jr., following paper, Phys. Rev. A **53**, 997 (1996).

[29] G. S. Agarwal and Yifu Zhu, Phys. Rev. A **46**, 479 (1992)

[30] F. W. J. Olver, in *Handbook of Mathematical Functions*, edited by M. Abromowitz and I. A. Stegun (Dover, New York, 1972).

[31] R. W. Boyd, *Nonlinear Optics* (Academic, New York, 1992).

[32] S. Chakmakjian, K. Koch, and C. R. Stroud, Jr., J. Opt. Soc. Am. B **5**, 2015 (1988).