

## Continuum absorption spectra in the far wings of the Hg $1S_0 \rightarrow 3P_1$ resonance line broadened by Ar

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Absolute reduced absorption coefficients for the Hg resonance line at 253.7 nm broadened by Ar were determined between 390 and 430 K in the spectral range from 20 to 1000  $\text{cm}^{-1}$  on the red wing and from 20 to 400  $\text{cm}^{-1}$  on the blue wing. The resultant reduced absorption coefficients are in fair agreement with those obtained by Petzold and Behmenburg [Z. Naturforsch. Teil A **33**, 1461 (1978)]. The observed  $A^3O^+ \leftarrow X^1O^+$  spectrum in the spectral range from 80 to 800  $\text{cm}^{-1}$  on the red wing agrees remarkably well both in shape and magnitude with the quasistatic line shape calculated using the potential-energy curves of the HgAr van der Waals molecule given by Fuke, Saito, and Kaya [J. Chem. Phys. **81**, 2591 (1984)], and Yamanouchi *et al.* [J. Chem. Phys. **88**, 205 (1988)]. The blue-wing spectrum is interpreted as the  $B^31 \leftarrow X^1O^+$  free-free transition of HgAr by a simulation of the spectrum using uniform semiclassical treatment for the free-free Franck-Condon factor. The source of the satellites on the blue wing is attributed to the phase-interference effect arising from a stationary phase-shift difference between the  $B$ - and  $X$ -state translational wave functions. The stationary phase-shift difference arises owing to the existence of a maximum in the difference potential between the  $B$  and  $X$  states. The repulsive branches of the potential-energy curves of HgAr for the  $X$  and  $B$  states have been revised to give excellent agreement between the observed and calculated spectra, both in shape and magnitude.

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### I. INTRODUCTION

The line shape in the far wings of the rare-gas broadened Hg  $6s^1S_0 \rightarrow 6p^1P_1$  resonance line at 253.7 nm has for a long time been the subject of much experimental work [1–7]. In particular, the undulations or the satellite peaks superimposed on the far-wing continuum have attracted considerable attention. The nature of the blue-wing satellites of the Hg-Ar system has, however, been interpreted in contradictory ways: as a result of transitions from the free states [1,5] or from the bound states [2,7] of the Hg-Ar molecule.

The semiclassical line-shape theories have also been developed to describe the far-wing continuum including the satellite structures arising from the free-free transitions [8–10]. Petzold and Behmenburg [7] found, however, that their observed blue-wing spectra of Hg-Ar differ in shape and intensity level from the calculation by the unified Franck-Condon line-shape theory of Szudy and Baylis [10]. Petzold and Behmenburg therefore concluded that the Hg-Ar blue-wing spectra must be predominantly due to bound states and/or quasibound states existing within the rotational barrier of the effective potential.

On the other hand, laser spectroscopy combined with the free jet expansion technique has been applied to observe the bound-bound transitions of the Hg-Ar van der Waals (vdW) molecule in the vicinity of 253.7 nm [11–14]. These vdW studies have yielded much more accurate information about the potential wells for the ground  $X^1O^+$  and the excited  $A^3O^+$  and  $B^31^+$  states relevant to the Hg-Ar transitions near 253.7 nm. The Hg-Ar potentials given by the vdW studies have been tested recently by the alignment effects studies for the photodissociation of the Hg-Ar molecule excited near 253.7 nm [15–17]. In their analysis of the alignment experiments, Segal and Harris [15] have reviewed the potentials given by

the line-shape and the vdW experiments and proposed the potentials to be a combined form of the Morse potential from Fuke, Saito, and Kaya [11] and the  $C_6/R^6$  potential. Laser spectroscopy of the Hg-Ar vdW molecule has been followed up most recently by Szajkowski, Krause, and Bobkowski [13] and Koperski, Atkinson, and Krause [14]. They have given different molecular constants for the  $X$ ,  $A$ , and  $B$  states, which are slightly different from those of the former vdW experiments by Fuke, Saito, and Kaya [12] and Yamanouchi *et al.* [12]

We have reported previously the undulatory blue-wing spectra observed in absolute measurements of far-wing absorption spectra for the  $6s^1S_0 \rightarrow 6p^1P_1$  resonance lines of Yb and Ba broadened by rare gases [18–20]. These undulations can be interpreted as arising from free-free transitions [20]. In this study, we have extended our experimental approach to the Hg (253.7 nm)-Ar system to follow up the earlier line-shape studies cited above. Our absolute measurements of the blue-wing spectra will then be analyzed using the uniform semiclassical approach for the free-free transitions based on the formalism of Bieniek and Streeter [21]. The main purpose of this study is to understand the nature of the blue-wing satellites in the light of improved information on the relevant potentials. It will be shown that the present treatment for the free-free transition can provide excellent agreement with the experiment in both shape and intensity level upon introducing a small modification of the potentials given from the vdW experiments in the region of their repulsive branches. We will discuss in some detail the mechanism responsible for the appearance of the blue-wing satellites and reasons why the generalized line-shape theory based on the random-phase approximation fails to explain the satellite peaks.

## II. EXPERIMENT

We employed a classical double-beam absorption method using a Xe short-arc lamp as a continuum light source. The experimental setup and procedures were almost the same as those described in our previous paper [18]. Briefly, an absorption cell containing the Hg vapor and the Ar gas was placed in a test-beam section of the Mach-Zehnder interferometer. The cell, made of a quartz tube 100 cm long and 5 cm in diameter, with plane quartz windows at the ends, was operated at temperatures of 300–480 K. The perturber gas Ar is admixed in the cell at a pressure in the range 200–700 Torr. The light beam from a Xe short-arc lamp is divided into two: one passes through the absorption cell, while the other serves as a reference for the absorption. These two light beams and the interference beam, which is the superposition of these two beams, are alternately dispersed by a 50-cm Czerny-Turner spectrometer equipped with a grating of 1200 grooves/mm and are detected by a 1024-channel photodiode array. The overall resolution was about 0.1 nm. From the interference spectrum measured around the Hg resonance line, the number density of Hg atoms integrated over the line of sight, i.e., column density  $N_{\text{Hg}}l_a$ , is determined. The absorption spectrum, in terms of the optical depth, is given by

$$k(\nu)l_a = -\ln[I_T(\nu)/I_R(\nu)], \quad (2.1)$$

where  $I_T(\nu)$  and  $I_R(\nu)$  are the relative spectral intensities of the test beam and the reference beam, respectively.

## III. RESULTS

Absorption spectra of the Hg-Ar mixtures at 390 and 430 K for a variety of Ar pressures from 200 to 700 Torr are shown in Fig. 1 in terms of percentage absorption  $[1 - I_T(\nu)/I_R(\nu)] \times 100$  plotted against the wave-number shift  $\Delta$  from the Hg  $6s^1S_0 \rightarrow 6p^3P_1$  line at  $\nu_0 = 39412.3 \text{ cm}^{-1}$ . The Ar pressures are from 200 to 700 Torr at intervals of 100 Torr. The column density  $N_{\text{Hg}}l_a$  was observed to depend weakly on the Ar pressure even at a constant cell temperature.  $N_{\text{Hg}}l_a$  was typically  $1.69 \times 10^{18} \text{ cm}^{-2}$  at 390 K and 400 Torr, and  $13.9 \times 10^{18} \text{ cm}^{-2}$  at 430 K and 400 Torr. The spectra on the red wing show small undulations over a spectral range of about  $-100$  to  $-200 \text{ cm}^{-1}$ . The peak positions of these undulations coincides with those reported by Kielkopf and Miller [5], who have attributed them to the bound vibrational levels of the upper electronic state. We will not be concerned with these small red-wing undulations, but with the gross features averaged over the undulations. The spectra on the blue wing show the prominent satellites SI and SII, the undulations well known from the literature.

Assuming that the far-wing spectrum consists of a superposition of the Hg-Ar and Hg-Hg components, we have

$$k(\Delta) = \gamma_{\text{HgAr}}(\Delta)N_{\text{Hg}}n_{\text{Ar}} + \gamma_{\text{HgHg}}(\Delta)N_{\text{Hg}}^2 \quad (3.1)$$

and therefore

$$\frac{k(\Delta)l_a}{(N_{\text{Hg}}l_a)^2} = \gamma_{\text{HgAr}}(\Delta) \frac{n_{\text{Ar}}}{N_{\text{Hg}}l_a} + \frac{\gamma_{\text{HgHg}}(\Delta)}{l_a}, \quad (3.2)$$

where  $\gamma_{\text{HgAr}}(\Delta)$  and  $\gamma_{\text{HgHg}}(\Delta)$  are the reduced absorption coefficients (RACs) at a wave-number shift  $\Delta$  from the line

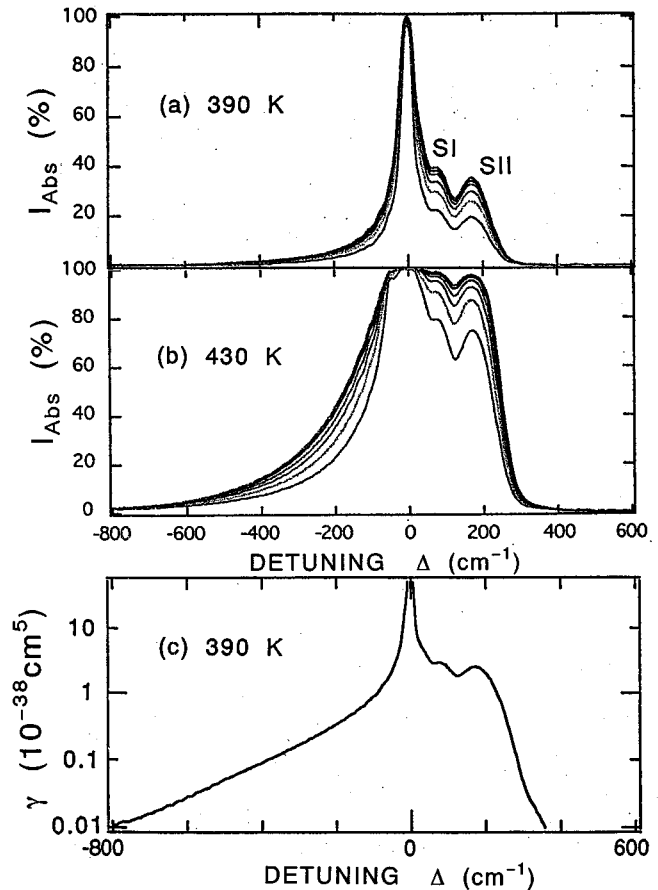


FIG. 1. (a) and (b) Absorption spectra, in terms of the percentage absorption  $(1 - I_T/I_R) \times 100$ , in the vicinity of the Hg  $6s^1S_0 - 6p^3P_1$  line at  $\lambda = 253.7 \text{ nm}$ , plotted versus the wave-number shift from the Hg line. The cell temperatures are (a) 390 and (b) 430 K. The Ar pressures are 200, 300, 400, 500, 600, and 700 Torr. (c) Reduced absorption coefficients of Hg-Ar at 430 K derived from the spectra in (b).

center for the Hg-Ar and the Hg-Hg systems, respectively. Thus we can obtain  $\gamma_{\text{HgAr}}$  as a gradient and  $\gamma_{\text{HgHg}}/l_a$  as an intercept via the straight-line fit of the  $kl_a/(N_{\text{Hg}}l_a)^2$  versus  $n_{\text{Ar}}/N_{\text{Hg}}l_a$  plot at each  $\Delta$  in the spectral range covered in this study. The resultant RAC spectrum  $\gamma_{\text{HgAr}}(\Delta)$  is plotted in Fig. 1(c). The RACs determined at different temperatures will be given in Sec. IV together with the calculated ones. The RACs showed no significant dependence on temperature within the present temperature variation from 390 to 430 K. The absolute RACs from the present measurements are in good agreement with those of Behmenburg and co-workers [3,4,7]. We focus here on the results of  $\gamma(\Delta)_{\text{HgAr}}$  for the Hg-Ar molecule. The results of  $\gamma_{\text{HgHg}}(\Delta)$  will be reported elsewhere.

## IV. DISCUSSION

### A. Red-wing spectrum

The present red-wing spectrum has been obtained at negative detunings smaller than  $-20 \text{ cm}^{-1}$ , where the spectrum can be assigned to the  $A^3O^+ \leftarrow X^1O^+$  transition. Fuke, Saito, and Kaya [11] and Yamanouchi *et al.* [12] have found that

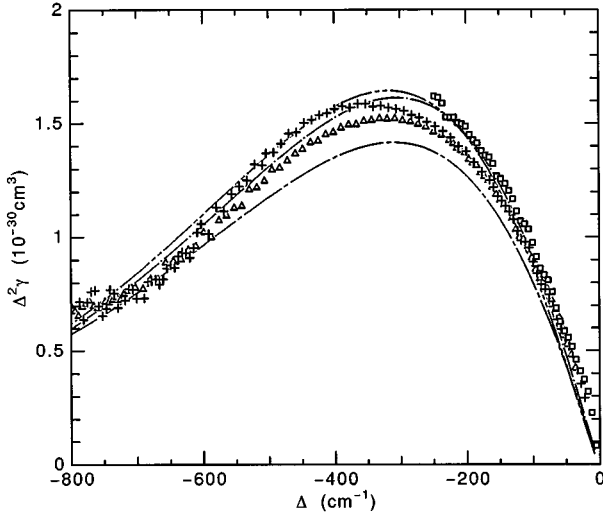


FIG. 2. Reduced absorption coefficient multiplied by the square of the wave-number shift from the Hg(253.7 nm) line center plotted against the wave-number shift for the red-wing  $A \leftarrow X$  transition of Hg-Ar. The experimental results at 390 K ( $\square$ ), 415 K ( $+$ ), and 430 K ( $\triangle$ ) are compared with the QST calculations for 415 K. The QST calculations are shown for the Morse potentials by Yamanouchi *et al.* Ref. [12] (-----), by Fuke, Saito, and Kaya [11] (-.-.-), and by Koperski, Atkinson, and Krause [14] (———).

Morse functions are appropriate to describe the potential wells of the  $X$ ,  $A$ , and  $B$  states of the Hg-Ar molecule. We compare the observed red-wing spectra with the quasistatic line shapes calculated for the  $A \leftarrow X$  transition using the Morse potentials determined by the vibrationally resolved spectroscopy of the Hg-Ar molecule [11,12,14]. The calculation of the quasistatic wing is based on the classical quasistatic theory (QST) [22,23], which gives the RAC  $\gamma$  as a function of the detuning  $\Delta$  as

$$\gamma(\Delta) = \frac{(2\pi\alpha)^2 a_0 (f_a/3) R_\Delta^2}{\Delta V'_{A-X}(R_\Delta)} \exp\left[-\frac{V_X(R_\Delta)}{k_B T}\right], \quad (4.1)$$

where  $R_\Delta$  is the Hg-Ar distance that satisfies the classical Franck-Condon relation

$$\Delta = V_A(R_\Delta) - V_X(R_\Delta) - \nu_0, \quad (4.2)$$

$\alpha (= \frac{1}{137})$  is the fine-structure constant,  $a_0$  is the Bohr radius,  $f_a$  is the oscillator strength for the Hg atomic line (equal to 0.0255 [24]), and  $\Delta V'_{A-X}(R)$  is the first derivative of the difference potential  $V_A(R) - V_X(R)$  with respect to  $R$ .

The red-wing spectra observed at  $T=390, 415,$  and  $430$  K are plotted in Fig. 2 in terms of the RAC multiplied by the square of the wave-number shift from line center and compared with three QST curves calculated for  $T=415$  K using different potential curves. The observed spectra at these three different temperatures show small discrepancies with each other. However, these differences may not be significant, considering the present experimental accuracy. We regard the results as an indication of no significant temperature dependence within the present small temperature variation.

The three QST curves at Fig. 2 correspond to the Morse potentials from the spectroscopic constants of Fuke, Saito,

TABLE I. Summary of potential parameters.  $\beta_1^b$ ,  $\beta_1^c$ , and  $\beta_1^d$  are calculated assuming the Morse relation  $\beta_1 = 0.1218\omega_e(\mu/D_e)^{1/2}$ , where  $\beta_1$  is in units of  $\text{\AA}^{-1}$ ,  $\omega_e$  and  $D_e$  in  $\text{cm}^{-1}$ , and  $\mu$  in amu.

State	$D_e$ ( $\text{cm}^{-1}$ )	$R_e$ ( $\text{\AA}$ )	$\beta_1$ ( $\text{\AA}^{-1}$ )	$\beta_2$ ( $\text{\AA}^{-1}$ )
$X \ ^10^+$	143 <sup>a</sup> , 142 <sup>b</sup>	4.02 <sup>a</sup> , 4.01 <sup>b</sup>	1.40 <sup>a</sup> , 1.39 <sup>b</sup>	3.02 <sup>a</sup>
	143 <sup>c</sup> , 134 <sup>d</sup>	3.99 <sup>c</sup> , 3.99 <sup>d</sup>	1.38 <sup>c</sup> , 1.50 <sup>d</sup>	
$A \ ^30^+$	377 <sup>a</sup> , 369 <sup>b</sup>	3.36 <sup>a</sup> , 3.38 <sup>b</sup>	1.49 <sup>a</sup> , 1.45 <sup>b</sup>	
	377 <sup>c</sup> , 362 <sup>d</sup>	3.36 <sup>c</sup> , 3.38 <sup>d</sup>	1.49 <sup>c</sup> , 1.54 <sup>d</sup>	
$B \ ^31$	63 <sup>a</sup> , 67 <sup>b</sup>	4.62 <sup>a</sup> , 4.66 <sup>b</sup>	1.13 <sup>a</sup> , 1.04 <sup>b</sup>	2.22 <sup>a</sup>
	63 <sup>c</sup> , 67 <sup>d</sup>	4.70 <sup>c</sup> , 4.64 <sup>d</sup>	1.02 <sup>c</sup> , 1.09 <sup>d</sup>	

<sup>a</sup>Parameters used in this work.

<sup>b</sup>Reference [11].

<sup>c</sup>Reference [12].  $D_e$  is calculated as  $D_e = D_0 + \omega_e/2$  using  $\omega_e$  and  $D_0$  in Ref. [12].

<sup>d</sup>Reference [14].

and Kaya [11], Yamanouchi *et al.* [12], and Koperski *et al.* [14]. The Morse parameters deduced from their spectroscopic constants are listed in Table I. The Morse potentials by Fuke, Saito, and Kaya and Yamanouchi *et al.* give almost similar  $\gamma$ 's while those by Koperski, Atkinson, and Krause give  $\gamma$ 's a little smaller. All of these QST curves are in good agreement with the present experimental results in both shape and magnitude, as seen in Fig. 2. Note that the plot of  $\Delta^2\gamma$  accentuates a small difference in  $\gamma$  at larger absolute detunings. It is therefore concluded that the QST values of  $\gamma$  calculated from the Morse potentials by Fuke, Saito, and Kaya and Yamanouchi *et al.* agree remarkably well with the experimental data for red-wing detunings larger than  $80 \text{ cm}^{-1}$ . Discrepancies in  $\gamma$  between theory and experiment grow larger rather for  $|\Delta|$  smaller than  $80 \text{ cm}^{-1}$ . This may be attributed, as has been discussed by Segal and Harris [15], to the fact that the Morse potential converges to zero too quickly for a correct description of the long-range van der Waals interaction.

As will be discussed later, we modify the ground- $X$ -state potential in order to make a reasonable simulation for the blue-wing  $B \leftarrow X$  line-shape data. We assume an extended Morse function for the  $X \ ^10^+$  state potential

$$V(R) = a \left[ e^{-\beta_2 \hat{R}} - \frac{\beta_2}{\beta_1} e^{-\beta_1 \hat{R}} \right], \quad (4.3)$$

where  $\hat{R} = R - R_e$  and the parameter  $a$  is related to the well depth  $D_e$  as  $D_e = -a(1 - \beta_2/\beta_1)$ . While keeping  $D_e$  and  $R_e$  at the values given by Yamanouchi *et al.*, we determined  $\beta_1$  and  $\beta_2$  so that we obtained a quasistatic line shape in good agreement with the observed red-wing spectrum. The  $X$ -state potential thus determined is quite similar to the Morse potential of Yamanouchi *et al.* in the attractive branch, but has a steeper dependence on  $R$  in the repulsive branch (see Fig. 3). The potential parameters are summarized in Table I. The QST curve of  $\Delta^2\gamma$  calculated from the modified  $X$ -state potential and the  $A$ -state Morse potential by Yamanouchi *et al.*, though it is not shown in Fig. 2 to avoid complexity, lies between the curves corresponding to the Morse potentials of Fuke, Saito, and Kaya and Koperski, Atkinson, and Krause.

### B. Blue-wing spectrum

The blue-wing spectrum at detunings larger than  $20 \text{ cm}^{-1}$  corresponds to the  $B \leftarrow X$  transition. If the Morse potentials of Yamanouchi *et al.* are extended to the range above the dissociation limit, the difference potential  $\Delta V_{B-X}(R) = V_B(R) - V_X(R) - \nu_0$  gives a maximum  $\Delta_M \sim 600 \text{ cm}^{-1}$  at  $R \sim 3.1 \text{ \AA}$ . This maximum appears because the repulsive branch of  $V_X$  is steeper than that of  $V_B$ . Though the prominent satellite observed in the blue wing is obviously due to the existence of such a maximum in  $\Delta V_{B-X}$ , its height  $\Delta_M \sim 600 \text{ cm}^{-1}$  is too high considering that the peak of the outermost satellite SII appears at  $\Delta \sim 170 \text{ cm}^{-1}$  (see Fig. 1). Therefore, the repulsive branch of  $V_X$  must have a steeper dependence on  $R$  and/or that of  $V_B$  must depend on  $R$  less than expected from the Morse potentials of Yamanouchi *et al.* This is the reason why we have assumed the extended Morse function for  $V_X$ . In order to modify the repulsive part of  $V_B$ , we use an extended Morse form and further introduce a function  $f(R)$  [25]

$$V_B(R) = a \left[ e^{-\beta_2 \hat{R}} - \frac{\beta_2}{\beta_1} e^{-\beta_1 \hat{R}} \right] f(R), \quad (4.4)$$

with

$$f(R) = 1 - b_1 \exp \left[ - \left( \frac{R - b_2}{b_3} \right)^2 \right]. \quad (4.5)$$

The  $b$  parameters in  $f(R)$  are chosen so that the factor  $f(R)$  modifies only the repulsive part of  $V_B$  and changes neither the well depth nor the attractive part of  $V_B$ . The maximum  $\Delta_M$  of  $\Delta V_{B-X}$  is sensitive to the  $b$  parameters in  $f(R)$ . The values of the  $b$  parameters finally chosen are  $b_1 = 0.43$ ,  $b_2 = 2.96 \text{ \AA}$ , and  $b_3 = 0.635 \text{ \AA}$ . The other potential parameters for the  $B$  state employed to simulate the blue-wing spectrum are listed in Table I. Figure 3 shows the potential-energy curves we used to calculate the red- and blue-wing spectra. The resultant difference potential  $\Delta V_{B-X}$  is also shown in Fig. 3.  $\Delta V_{B-X}$  has a maximum  $\Delta_M \approx 220 \text{ cm}^{-1}$  at  $R \approx 3.6 \text{ \AA}$ .

We have calculated the blue-wing line shape assuming that the free-free transitions between the  $B^3 1$  and  $X^1 0^+$  states are responsible for this continuum. The calculations of the RAC at temperature  $T$  were performed according to [26]

$$\begin{aligned} \gamma_T(\Delta) &= 8 \pi^4 \alpha^2 (2 f_a/3) \frac{hc}{Q_T} \\ &\times \int_0^\infty \left[ \sum_l (2l+1) |\langle \Psi_B | \Psi_X \rangle|^2 \right] \exp \left( - \frac{E}{k_B T} \right) dE, \end{aligned} \quad (4.6)$$

where  $Q_T$  is the translational partition function,  $l$  the angular-momentum quantum number of the relative nuclear motion,  $\langle \Psi_B | \Psi_X \rangle$  the overlap integral of the nuclear translational wave functions, and  $E$  the initial kinetic energy. We have used the uniform-semiclassical formula by Bieniek and Streeter [21] to calculate the free-free Franck-Condon overlap integrals.

When the difference potential  $\Delta V_{B-X}$  has a maximum at  $R = R_0$  as shown in Fig. 3, two Condon points  $R_1$  and  $R_2$  ( $R_1 < R_0 < R_2$ ) can contribute to the transition for a given

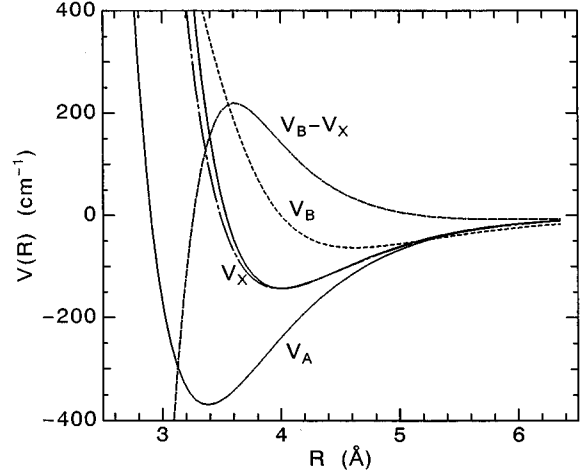


FIG. 3. Hg-Ar potentials used in this work. The potentials of the excited states  $V_A$  and  $V_B$  are displaced by their asymptotic value  $\nu_0$ .  $V_A$  is the  $A$  state Morse potential with the parameters from Ref. [12].  $V_B$  is the modified  $B$  state potential introduced in this study. Two curves are shown for the  $X$ -state potential: One is the modified  $X$  state potential of an extended Morse form (solid line) and the other is the Morse potential by Yamanouchi *et al.* [12]. The latter is plotted by a chain line for comparison. The difference potential  $\Delta V_{B-X} = V_B - V_X - \nu_0$  is also shown, where  $V_B$  and  $V_X$  are the modified potentials introduced in this study.

absorption wavenumber. Let  $l_{nM}$  ( $n=1$  or  $2$ ) be the maximum angular momentum that allows the collision pair to approach within the Condon point  $R_n$

$$l_{nM} = \text{Int} \left\{ k_E R_n \left[ 1 - \frac{V_X(R_n)}{E} \right]^{1/2} - \frac{1}{2} \right\}, \quad (4.7)$$

where  $\text{Int}$  denotes integer part of and  $k_E = \sqrt{2\mu E}/\hbar$  is the initial kinetic wave number. Then the summation over  $l$  in Eq. (4.6) must be divided into two parts

$$\sum_l (2l+1) |\langle \Psi_B | \Psi_X \rangle|^2 = q_{12} + q_2, \quad (4.8)$$

with

$$q_{12} = \sum_{l=0}^{l_{1M}} (2l+1) |T_1 + T_2|^2 \quad (4.9)$$

and

$$q_2 = \sum_{l=l_{1M}+1}^{l_{2M}} (2l+1) |T_2|^2. \quad (4.10)$$

The  $q_{12}$  term includes the contributions of both Condon points while the  $q_2$  term contains a contribution only from the outer Condon point  $R_2$ . According to the uniform-semiclassical expression by Bieniek [21,27],

$$\begin{aligned} T_n &= \pi \left( \frac{\hbar^2}{2\mu |\Delta V''(R_n)| k_n^2} \right)^{1/3} [\cos \beta_n \text{Ai}(-z_n) \\ &- (-1)^n \sin \beta_n \text{Bi}(-z_n)], \end{aligned} \quad (4.11)$$

where  $\text{Ai}(-z)$  and  $\text{Bi}(-z)$  are the regular and irregular homogeneous Airy functions respectively.  $\beta_n$  and  $z_n$  are related to the phase-shift difference  $\Delta\phi_n$  evaluated at the Condon point  $R_n$

$$\beta_n = \Delta\phi_n + \frac{2}{3}|z_n|^{3/2}, \quad (4.12)$$

$$z_n = \pm \left[ \frac{(\Delta\phi_n'')^3}{2(\Delta\phi_n''')^2} \right]^{2/3}, \quad (4.13)$$

$$\Delta\phi_n = \int_{R_B'}^{R_n} k_B(R) dR - \int_{R_X'}^{R_n} k_X(R) dR. \quad (4.14)$$

$R_X'$  and  $R_B'$  are the classical turning points on the  $X$  and  $B$  states, respectively, and  $k(R)$  is the semiclassical local wave number

$$k_X(R) = k_E \left[ 1 - \frac{V_X(R)}{E} - \left( \frac{l + \frac{1}{2}}{k_E R} \right)^2 \right]^{1/2}, \quad (4.15)$$

$$k_B(R) = k_E \left[ 1 - \frac{V_B(R) - (\nu_0 + \Delta)}{E} - \left( \frac{l + \frac{1}{2}}{k_E R} \right)^2 \right]^{1/2}. \quad (4.16)$$

The above two local wave numbers have the same value at the Condon point

$$k_X(R_n) = k_B(R_n) = k_n. \quad (4.17)$$

This is the semiclassical equivalent to the classical Franck-Condon relation in Eq. (4.2) under the  $Q$ -branch approximation [28]. For the derivatives of the phase difference in Eq. (4.13), the following approximations are used [8]:

$$|\Delta\phi_n''| = \frac{\mu}{\hbar^2 k_n} |\Delta V_n'|, \quad |\Delta\phi_n'''| = \frac{\mu}{\hbar^2 k_n} |\Delta V_n''|, \quad (4.18)$$

where  $\Delta V_n' = \Delta V'(R_n)$  and  $\Delta V_n'' = \Delta V''(R_n)$ .

We have examined another uniform-semiclassical expression for the  $q_{12}$  term. This expression includes only the second derivative of the phase difference  $\Delta\phi_n$  [21]

$$q_{12} = \sum_{l=0}^{l_{1M}} (2l+1)(U_a^2 + U_b^2), \quad (4.19)$$

$$U_a = (u_1 + u_2) \zeta^{1/4} \cos\left[\frac{1}{2}(\Delta\phi_1 + \Delta\phi_2)\right] \text{Ai}(-\zeta), \quad (4.20)$$

$$U_b = (u_1 - u_2) \zeta^{1/4} \sin\left[\frac{1}{2}(\Delta\phi_1 + \Delta\phi_2)\right] \text{Bi}(-\zeta), \quad (4.21)$$

$$u_n = \frac{\pi}{\sqrt{2}} k_n^{-1} |\Delta\phi_n''|^{-1/2} = \frac{\pi\hbar}{(2\mu k_n |\Delta V_n'|)^{1/2}}, \quad (4.22)$$

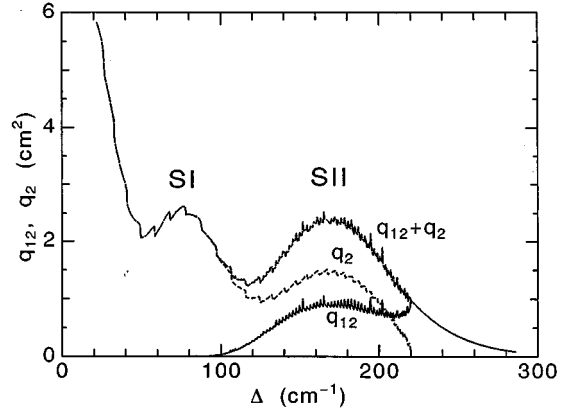


FIG. 4. Dependence of the partial Franck-Condon factors, the  $q_1$  and  $q_2$  terms, on the wave-number shift  $\Delta$  for the Hg-Ar  $B \leftarrow X$  transition. The initial kinetic energy is  $345 \text{ cm}^{-1}$ .

where  $\zeta = (\mu/\hbar^2 k_0)^{2/3} |\Delta V_0''|^{-1/3} (\Delta_M - \Delta)$  for  $\Delta \geq \Delta_M$ ,  $\zeta = \left[ \frac{3}{4} \Delta\phi_{12} \right]^{2/3}$  for  $\Delta < \Delta_M$ ,  $k_0 = k_X(R_0)$ ,  $\Delta V_0'' = \Delta V''(R_0)$ , and  $\Delta\phi_{12} = \Delta\phi_2 - \Delta\phi_1$ .

Equation (4.19) gives almost the same numerical results as Eq. (4.9). We have used Eq. (4.19) for the present analysis. Figure 4 shows the  $\Delta$  dependence of the  $q_{12}$  and  $q_2$  terms for an initial kinetic energy  $E = 345 \text{ cm}^{-1}$ . This value of  $E$  corresponds to the mean velocity for  $T = 390 \text{ K}$ , i.e., one of the temperatures at which our measurements have been made. As seen in the figure, the  $q_{12}$  term is important at the outermost SII satellite region including the antistatic wing beyond  $\Delta_M$ . In most of the regions of  $\Delta$  smaller than  $\Delta_M$ , the  $q_2$  term dominates over the  $q_{12}$  term. Moreover, the  $q_2$  term shows undulatory structure; at least two peaks can be clearly seen. The main satellite SII is the additive superposition of the  $q_{12}$  and  $q_2$  terms, while the satellite SI at about  $80 \text{ cm}^{-1}$  comes from the  $q_2$  term. For  $\Delta$ 's smaller than  $100 \text{ cm}^{-1}$ , the  $q_{12}$  term vanishes. This is because the potential height at the inner Condon point  $V_X(R_1)$  is higher than the initial kinetic energy and therefore the Hg-Ar pair cannot reach  $R_1$ . In these regions, the summation over  $l$  in the  $q_2$  term extends from 0 to  $l_{2M}$ .

The oscillation in  $q_2$  comes from the fact that the phase difference  $\Delta\phi_2$  in Eq. (4.14) has a maximum at some  $l$ , say  $l^*$ . The main contribution to  $q_2$  comes from the stationary phase difference  $\Delta\phi_2(l^*)$ . If the asymptotic form of the Airy functions are used in Eq. (4.11), we have

$$T_2 \approx \left( \frac{\pi\hbar^2}{2\mu |\Delta V_2'| k_2} \right)^{1/2} \sin \left[ -\Delta\phi_2 + \frac{\pi}{4} \right]. \quad (4.23)$$

Equation (4.23) is a good approximation for the uniform-semiclassical expression in Eq. (4.11) for most of the range of  $\Delta$ ,  $0 < \Delta \leq 0.9\Delta_M$ , except the region close to  $\Delta_M$ .  $\Delta\phi_2$  in Eq. (4.14) may be approximated as

$$\Delta\phi_2(l) \approx \frac{1}{\hbar v} \int_{b_l}^{R_2} \frac{\Delta V(R) - \Delta}{\sqrt{R^2 - b_l^2}} R dR, \quad (4.24)$$

where  $b_l = (l + \frac{1}{2})/k_E$  is the impact parameter corresponding to the angular momentum  $l$ . Equation (4.24) implies that if

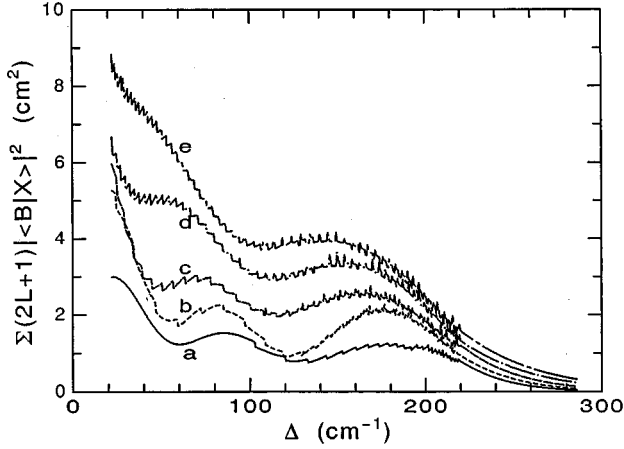


FIG. 5.  $\Delta$  dependence of the free-free Franck-Condon factor  $q_{12}+q_2$  for various initial kinetic energies: a,  $100 \text{ cm}^{-1}$ ; b,  $260 \text{ cm}^{-1}$ ; c,  $500 \text{ cm}^{-1}$ ; d,  $1000 \text{ cm}^{-1}$ ; and e,  $1500 \text{ cm}^{-1}$ .

$\Delta V(R)$  has a maximum within  $R_2$ , it is possible for  $\Delta\phi_2$  to have a maximum as a function of  $l$ . This is exactly the case for the blue-wing satellite, because  $\Delta V_{B-X}(R)$  has a maximum  $\Delta_M$  at  $R=R_0$  ( $R_0 < R_2$ ). One may employ the stationary-phase approximation instead of the random-phase approximation by replacing the summation by the integration over  $l$  in Eq. (4.10) to obtain a smoothly varying term and an oscillatory term

$$q_2 \approx \bar{q}_2 - \frac{\pi^{3/2} \hbar^2 (l^* + \frac{1}{2})}{2\mu |\Delta V_2'| [k_2 |d^2 \Delta \phi_2 / dl^2|^{1/2}]_{l^*}} \cos[2\Delta \phi_2(l^*) - \pi/4]. \quad (4.25)$$

Since  $\Delta\phi_2(l^*)$  depends on  $\Delta$ ,  $q_2$  oscillates as a function of  $\Delta$ . This type of oscillation mechanism is quite parallel to the one that has been employed to interpret the oscillations in the integrated cross sections of inelastic atomic collisions arising from curve crossing [29,30].

The upper limit of the integration over  $E$  in Eq. (4.6) can safely be taken to be  $6k_B T$  to calculate  $\gamma_T$  for a given temperature  $T$ . Figure 5 shows the total Franck-Condon factor  $q_{12}+q_2$  calculated for several different initial energies  $E$ . As  $E$  increases, the positions of the satellites shift towards lower  $\Delta$ . However, the shift is relatively small within most of the effective range of  $E$ , so that the oscillatory structure will not be smeared out by Boltzmann averaging over the energy  $E$ . We have carried out the integration over  $E$  in Eq. (4.6) to calculate the RAC,  $\gamma_T(\Delta)$ , for a given temperature  $T$ . The calculated  $\gamma_T(\Delta)$  for the blue wing of Hg-Ar at  $T=390 \text{ K}$  is compared with the experimental one in Fig. 6. Decomposition of the calculated  $\gamma_T(\Delta)$  into components corresponding to the  $q_{12}$  and  $q_2$  terms is also shown in Fig. 6. The calculated  $\gamma_T(\Delta)$  shows excellent agreement with the experiment in both shape and intensity level. The oscillations seen in  $q_2$  for a given initial energy  $E$  (Fig. 6) are not smeared out even after Boltzmann averaging. The positions of the satellites in  $\gamma_T(\Delta)$  at a temperature  $T$  are well represented by the peaks seen in Fig. 6 for  $q_{12}+q_2$ , at an energy  $E=4k_B T/\pi$  corresponding to a mean velocity  $\bar{v}=\sqrt{8k_B T/\pi\mu}$  at that temperature. The maximum  $\Delta_M$  of the difference potential  $\Delta V_{B-X}$  is

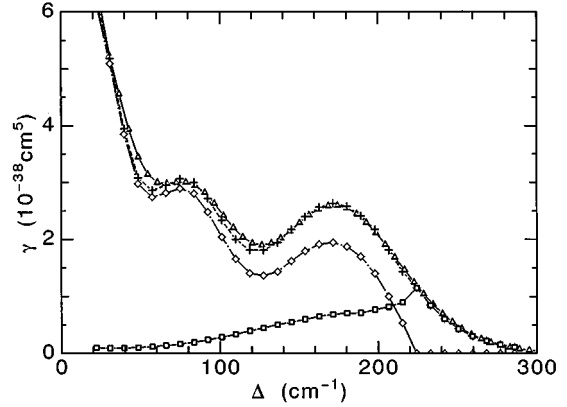


FIG. 6. Reduced absorption coefficients of Hg-Ar at 390 K versus the blue-wing shift of the wave number from the Hg(253.7 nm) line center:  $\Delta$ , from the experiment; +, from the calculation. The calculated spectrum is decomposed into components corresponding to the  $q_{12}$  term ( $\diamond$ ) and the  $q_2$  term ( $\square$ ).

located at a detuning wave number beyond the outermost satellite peak where the intensity is about 45% of the peak satellite intensity.

## V. CONCLUSION

We have employed the classical double-beam absorption-dispersion method to determine the absolute reduced absorption coefficient  $\gamma_T(\Delta)$  for the far wings of the Hg  $6s^1 S_0 \rightarrow 6p^3 P_1$  resonance line broadened by Ar at temperatures from 390 to 430 K. The resultant  $\gamma_T(\Delta)$ 's are in reasonable agreement with those by Behmenburg and co-workers [3,4,7]. The observed  $A \leftarrow X$  red-wing spectrum in a wave-number shift between 80 and  $800 \text{ cm}^{-1}$  from the line center agrees remarkably well in both shape and magnitude with the quasistatic line shape calculated using the Morse potentials determined from vibrationally resolved spectroscopy of the Hg-Ar molecule [11,12].

The repulsive parts of the potential-energy curves for the  $X$  and  $B$  states are determined from the analysis of the blue-wing spectrum using the uniform semiclassical treatment of Bieniek and Streeter [21] for the free-free Franck-Condon factors. Excellent agreement between the calculation and the experiment has been obtained in both shape and magnitude for the blue-wing  $B \leftarrow X$  spectrum. The blue-wing spectrum with satellites SI and SII is interpreted as free-free transitions in the  $B^3 1 \leftarrow X^1 0^+$  band of Hg-Ar. The satellite SII is the result of combined contributions from the inner and outer Condon-Points while the satellite SI results from the outer Condon point alone. In summing the contributions from all the angular-momentum components of the nuclear motion, the random-phase approximation employed in the generalized line-shape theories [8,10] is not valid for analyzing the satellite structures. The appearance of the satellites is due to the existence of a stationary phase-shift difference between the  $B$  and  $X$  state, i.e., a maximum of  $\Delta\phi(l)$  as a function of the nuclear angular momentum  $l$ , which in turn is due to the existence of a maximum in the difference potential  $V_B(R) - V_X(R)$ .

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