ARTICLES

Emergence of classicality via decoherence described by Lindblad operators

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(Received 13 June 1995)

Zurek, Habib, and Paz [Phys. Rev. Lett. **70**, 1187 (1993)] have characterized the set of states of maximal stability defined as the set of states having minimum entropy increase due to interaction with an environment and shown that coherent states are maximal for the particular environment model examined. To generalize these results, I consider entropy production within the Lindblad theory of open systems, treating environment effects perturbatively. I characterize the maximally predicitive states that emerge from several forms of effective dynamics, including decoherence from spatially correlated noise. Under a variety of conditions, coherent states emerge as the maximal states.

PACS number(s): 03.65.Bz, 05.40.+j

I. INTRODUCTION

Decoherence that results from a quantum system's interaction with an environment can provide a mechanism for characterizing the transition from quantum to classical behavior for a quantum open system [1,2] and has been an integral part of several programs addressing the emergence of classicality [3,4]. Zurek, Habib, and Paz (ZHP) have characterized the effectiveness of decoherence in terms of a predictability sieve and identified the maximally predictive states (defined as those with minimal entropy production) as the most classical [5]. ZHP considered an environment model consisting of an independent oscillator bath linearly coupled to the system of interest, which has been studied in the context of quantum Brownian motion [6]. ZHP demonstrated for the high-temperature limit of the environment that the coherent states of a harmonic oscillator are maximally predictive and that zero squeezing (corresponding to coherent states) is maximal for squeezed states considered at arbitrary environment temperatures.

The purpose of this paper is to extend the results of ZHP to additional environment models. All environmental models should be understood to be approximations in that there is, in prinicple, a more complete descripiton possible in terms of unitary evolution of a composite system containing the system of interest and external quantum-mechanical degrees of freedom. While some models have been derived from first-principle descriptions of such composite systems, others have been developed largely from phenomenological principles. The primary motivation for this paper is to use the phenomenology to get a better understanding of how interactions of a system with external quantum degrees of freedom lead to the emergence of classicality for that system. However, the general nature of the mathematical formulation employed in this paper does not restrict the results to this interpretation. The results can also readily be applied to models that have the same mathematical structure but are motivated by other constructs (for example, a modified quantum theory that contains intrinsic stochastic processes such as quantum mechanics with spontaneous localization [7]).

The effects of the environment are considered in the general framework of the Lindblad form for nonunitary evolution of a harmonic oscillator [8], correponding to the Markov limit. The Lindblad form of evolution can be written

$$L[\rho]=\frac{1}{i\hbar}[H,\rho]+\frac{1}{2\hbar}\sum_j\,[V_j\rho,V_j^\dagger]+[V_j,\rho V_j^\dagger] \tag{1.1}$$

and has proven to be a very useful starting point for theoretical studies of quantum open systems. It is the most general form for completely positive evolution (guaranteeing positive evolution for a quantum system quantum-mechanically entangled with another system) under a wide variety of conditions [8,9]. The Lindblad form of evolution does not suffer from nonphysical negative-entropy production for pure states, which occurs for some models in the literature. Strictly positive-entropy production for pure states can be established by examining the linear entropy

$$\varsigma(t)=\mathrm{Tr}[\rho(t)-\rho^2(t)]=1-\mathrm{Tr}[\rho^2(t)]. \tag{1.2}$$

For a pure state $\rho_0=|\psi\rangle\langle\psi|$ undergoing evolution generated by a Lindblad operator, the entropy production is

$$\begin{aligned}\dot{\varsigma}&=-2\ \mathrm{Tr}(\rho L[\rho])\\&=\frac{-2}{i\hbar}\ \mathrm{Tr}(\rho[H,\rho])-\frac{1}{\hbar}\ \mathrm{Tr}\left(\rho\sum_j\ [V_j\rho,V_j^\dagger]+[V_j,\rho V_j^\dagger]\right)\\&=0+\frac{2}{\hbar}(\langle\psi|V_j^\dagger V_j|\psi\rangle-\langle\psi|V_j|\psi\rangle\langle\psi|V_j^\dagger|\psi\rangle)\\&=\frac{2}{\hbar}|(V_j-\langle\psi|V_j|\psi\rangle)|\psi\rangle|^2\geqslant 0.\end{aligned} \tag{1.3}$$

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1050-2947/96/53(2)/655(6)/$06.00

For a mixed state there is the physically reasonable possibility of negative-entropy production that might occur, for example, if a "hot" system interacts with a cooler environment.

I establish a systematic framework for evaluating the predictability of states using first-order perturbation theory in Sec. II; in Sec. III I apply this framework to a family of Lindblad generators $\{V_j\}$ linear in position and momentum whose general properties have been studied in the literature [9]. This family of Lindblad generators is of particular interest because it encompasses as special or limiting cases many environment models considered in the literature, such as the quantum optical master equation [10] and Dekker's model for quantum dissipation [11]. I consider models with environment spatial correlation effects [12–15] in Sec. IV. I comment on these results in Sec. V.

II. PERTURBATIVE APPROACH TO EVALUATING PREDICTABILITY

The predictability sieve was introduced by Zurek [3] as a means of characterizing those states that are most stable when considering not only the dynamics of the system, but also including the effects (such as decoherence) of interaction with the system's environment. The set of states having minimum (linear) entropy production are the best candidates for states corresponding to points of classical phase space. The effect of the environment on evolution is taken to be in the Markov regime and I assume that there is no further explicit time dependence on the Liouville generator for the isolated system. The form of the evolution is then stationary and we can write

$$\rho(t)=e^{Lt}[\rho]=J(t)[\rho]. \tag{2.1}$$

The generator L is assumed to be comprised of two parts: the evolution of the isolated system corresponding to L_0 with $J_0(t)=e^{L_0 t}$, and the effects of the environment ΔL. I consider the effects of the environment perturbatively in part because a wide variety of forms of evolution become tractable, and also because many of the effective evolution equations are derived using lowest-order approximations such as the weak-coupling limit [16]. From formal perturbation theory, the evolution of the Liouville generator can be written

$$\begin{aligned} J(t)&=J_0(t)+\int_0^t J_0(t-\tau)\Delta L J(\tau)d\tau \\ &\cong J_0(t)+\int_0^t J_0(t-\tau)\Delta L J_0(\tau)d\tau+O(\Delta L^2) \\ &=J_0(t)+J_1(t)+O(\Delta L^2), \end{aligned} \tag{2.2}$$

defining $J_1(t)$ as the first-order perturbation. For an initial state ρ_0, the entropy at time t becomes

$$\begin{aligned} \varsigma(t)&=1-\mathrm{Tr}\{J(t)[\rho_0]J(t)[\rho_0]\} \\ &\cong 1-\mathrm{Tr}\{J_0(t)[\rho_0]J_0(t)[\rho_0]\} \\ &\quad -2\,\mathrm{Tr}\{J_0(t)[\rho_0]J_1(t)[\rho_0]\}. \end{aligned} \tag{2.3}$$

If the isolated system has unitary evolution generated by a Hamiltonian H_0, then

$$\begin{aligned} L_0[\rho]&=\frac{1}{i\hbar}[H_0,\rho], \\ J_0[\rho]&=U(t)\rho U^\dagger(t), \\ U(t)&=e^{1/i\hbar H_0 t}. \end{aligned} \tag{2.4}$$

In this case

$$1-\mathrm{Tr}\{J_0(t)[\rho_0]J_0(t)[\rho_0]\}=1-\mathrm{Tr}[\rho_0\rho_0]=\varsigma(0). \tag{2.5}$$

The maximal states are those with minimal entropy production, that is, with minimized

$$\begin{aligned} \Delta\varsigma(t)&\equiv-2\,\mathrm{Tr}\{J_0(t)[\rho_0]J_1(t)[\rho_0]\} \\ &=-2\int_0^t(\mathrm{Tr}\{U(t)\rho_0U^\dagger(t)U(t-\tau)\Delta L[U(s)\rho_0U^\dagger(s)]U^\dagger(t-\tau)]\})d\tau \\ &=-2\int_0^t\mathrm{Tr}\{\rho_0U^\dagger(\tau)\Delta L[U(\tau)\rho_0U^\dagger(\tau)]U(\tau)\}d\tau, \end{aligned} \tag{2.6}$$

where the cyclic property of the trace and the unitarity of $U(t)$ have been used for the final simplification.

Given the general form for a Lindblad operator, the contribution to environment interaction can be written as the perturbation

$$\Delta L[\rho]=\frac{1}{i\hbar}[\Delta H,\rho]+\frac{1}{2\hbar}\sum_j\ [V_j\rho,V_j^\dagger]+[V_j,\rho V_j^\dagger]. \tag{2.7}$$

ΔH contains any additional contributions to the effective system Hamiltonian arising from the interaction of the

$$\begin{aligned} \Delta\varsigma(t)=&-2\int_0^t\mathrm{Tr}\left(\rho_0U^\dagger(\tau)\frac{1}{i\hbar}[\Delta H,U(\tau)\rho_0U^\dagger(\tau)]U(\tau)\right)d\tau \\ &-\frac{1}{\hbar}\int_0^t\mathrm{Tr}\left(\rho_0U^\dagger(\tau)\sum_j\ \{[V_jU(\tau)\rho_0U^\dagger(\tau),V_j^\dagger]\right. \\ &\left.+[V_j,U(\tau)\rho_0U^\dagger(\tau)V_j^\dagger]\}U(\tau)\right)d\tau. \end{aligned} \tag{2.8}$$

The first term on the right-hand side of this equation is identically zero, from the cyclic property of the trace. The remainder can be written in a simpler form using the cyclic property of the trace, the unitarity of $U(t)$, and the identification

$$V_j(\tau)\equiv U^\dagger(\tau)V_jU(\tau) \tag{2.9}$$

to yield

$$\Delta\varsigma(t)=\frac{1}{\hbar}\int_0^t \sum_j \mathrm{Tr}[\rho_0\{V_j^\dagger(\tau)V_j(\tau),\rho_0\} - 2\rho_0V_j(\tau)\rho_0V_j^\dagger(\tau)]d\tau. \tag{2.10}$$

If ρ_0 is a pure state, then it is also a projection, with

$$\rho_0=P=|\psi\rangle\langle\psi|,$$

$$P^2=P,$$

$$POP=P\langle\psi|O|\psi\rangle=P\langle O\rangle \tag{2.11}$$

for an arbitrary operator O. Thus, for pure states, the entropy production is given by

$$\Delta\varsigma(t)=\frac{2}{\hbar}\int_0^t \sum_j [\langle V_j^\dagger(\tau)V_j(\tau)\rangle-\langle V_j(\tau)\rangle\langle V_j^\dagger(\tau)\rangle]d\tau=\frac{2}{\hbar}\int_0^t \sum_j \langle[V_j(\tau)-\langle V_j(\tau)\rangle]^\dagger[\langle V_j(\tau)\rangle-\langle V_j(\tau)\rangle]\rangle d\tau. \tag{2.12}$$

Minimization of this final quantity can then be used to determine the maximal states.

III. LINDBLAD GENERATORS LINEAR IN POSITION AND MOMENTUM

I will now apply the results of Sec. II to Lindblad operators that have $\{V_j\}$ linear in position and momentum. This family of generators have been studied extensively in the literature [9] and include as special or limiting cases the quantum optical master equation [10] and Dekker's phenomenological master equation [11]. In terms of Eq. (2.7), the operators are given by

$$\Delta H=\frac{\mu}{2}\{x,p\},\quad V_j=a_jp+b_jx. \tag{3.1}$$

With the identifications

$$D_{qq}=\frac{\hbar}{2}\sum_j |a_j|^2,$$

$$D_{pp}=\frac{\hbar}{2}\sum_j |b_j|^2,$$

$$D_{pq}=\frac{\hbar}{2}\sum_j -\mathrm{Re}[a_jb_j^*],$$

$$\lambda=\sum_j \mathrm{Im}[a_jb_j^*], \tag{3.2}$$

the perturbation on the system evolution becomes

$$\Delta L[\rho]=\frac{1}{i\hbar}\left[\frac{\mu}{2}\{x,p\},\rho\right]-\frac{D_{qq}}{\hbar^2}[p,[p,\rho]]-\frac{D_{pp}}{\hbar^2}[x,[x,\rho]] +\frac{D_{pq}}{\hbar^2}([x,[p,\rho]]+[p,[x,\rho]]) +\frac{i\lambda}{2\hbar}([x,\{p,\rho\}]-[p,\{x,\rho\}]). \tag{3.3}$$

The particular choice of parameters D_{qq}, D_{pp}, D_{pq}, λ, and μ [Eq. (3.3)] determines the details of the evolution (i.e., evolution corresponding to the quantum optical master equation, Dekker's master equation, etc.). In many cases one can obtain an evolution equation from the literature simply by selecting the appropriate diffusion constants in Eq. (3.3), subject to the constraints imposed by Eq. (3.2) (the coefficients are not completely independent). In other cases, terms might be omitted from the evolution equations as an approximation under conditions that will, of course, depend upon the specific density operators. The results of such approximations can be determined by setting corresponding constants in Eq. (3.3) to zero [although this would be in apparent conflict with the conditions of Eq. (3.2)]. Anomalous behavior (such as negative-entropy production for pure states) arises only under conditions where the underlying approximations are not valid. A more complete discussion of the comparisons of the Lindblad form with other evolutionary models can be found in Ref. [9]. It has been determined that only when

$$D_{pq}=\frac{\hbar}{2}\sum_j -\mathrm{Re}[a_jb_j^*]=0 \tag{3.4}$$

will the system relax into a thermal equilibrium state [9], so I adopt this condition for the remainder of the paper.

For the simple harmonic oscillator, the operator equations of motion are easily solved by

$$x(\tau)=U^{\dagger}(\tau)xU(\tau)$$
$$=x\cos(\omega\tau)+\frac{p}{m\omega}\sin(\omega\tau),$$
$$p(\tau)g=U^{\dagger}(\tau)pU(\tau)$$
$$=p\cos(\omega\tau)-m\omega x\sin(\omega\tau) \tag{3.5}$$

to yield

$$V_j(\tau)=a_j[p\cos(\omega\tau)-m\omega x\sin(\omega\tau)]$$
$$+b_j\left(x\cos(\omega\tau)+\frac{p}{m\omega}\sin(\omega\tau)\right). \tag{3.6}$$

Substituting Eq. (3.6) into Eq. (2.12) and evaluating the elementary trigonometric integrals over τ,

$$\Delta\varsigma(t)=f_1(t)\left(\frac{1}{2m}\langle p^2-\langle p\rangle^2\rangle+\frac{m\omega^2}{2}\langle x^2-\langle x\rangle^2\rangle\right)-2\lambda t$$
$$+f_2(t)\left(\frac{1}{2m}\langle p^2-\langle p\rangle^2\rangle-\frac{m\omega^2}{2}\langle x^2-\langle x\rangle^2\rangle\right)+f_3(t)$$
$$\times\left(\frac{\omega}{2}\langle\{x,p\}\rangle-\omega\langle x\rangle\langle p\rangle\right), \tag{3.7}$$

where

$$f_1(t)=\frac{4mt}{\hbar^2}\left(D_{qq}+\frac{D_{pp}}{(m\omega)^2}\right),$$
$$f_2(t)=\frac{2m}{\hbar^2\omega}\sin(2\omega t)\left(D_{qq}-\frac{D_{pp}}{(m\omega)^2}\right),$$
$$f_3(t)=-\frac{4m}{\hbar^2\omega}\sin^2(\omega t)(D_{qq}-\frac{D_{pp}}{(m\omega)^2}). \tag{3.8}$$

Equation (3.7) is the expectation of a c number times the harmonic-oscillator Hamiltonian plus a second c-number constant after squeezeing and translation by $\langle x\rangle$ in position and $\langle p\rangle$ in momentum. The state that minimizes Eq. (3.7) will be the corresponding squeezed and translated ground state, which is simply a coherent squeezed state [17]. In terms of the harmonic-oscillator creation and annihilation operators $a^{\dagger}$ and a, the squeeze operator is given by

$$S(\zeta)=e^{1/2(\zeta^{*}a^2-\zeta a^{\dagger 2})}, \tag{3.9}$$

and Glauber's displacement operator is given by

$$D(\alpha)=e^{(\alpha a^{\dagger}-\alpha a)}. \tag{3.10}$$

The appropriate selection of the parameter α for the displacement operator

$$\mathrm{Im}(\alpha)=\left(\frac{\hbar}{2m\omega}\right)^{1/2}\langle p\rangle,\quad \mathrm{Re}(\alpha)=\left(\frac{\hbar m\omega}{2}\right)^{1/2}\langle x\rangle \tag{3.11}$$

provides the necessary translation (in phase space). For simplicity, I take $\langle x\rangle=0$ and $\langle p\rangle=0$ for the rest of this section. The effect of squeezing on the anihillation operator can be written

$$S^{\dagger}(\zeta)aS(\zeta)=\mu a+\nu a^{\dagger}, \tag{3.12}$$

where

$$\zeta=se^{i\theta},\quad \mu=\cosh(s),\quad \nu=\sinh(s)e^{i\theta}. \tag{3.13}$$

The parameter s determines the amount of squeezing ($s=0$ corresponding to no squeezing) and the parameter θ determines the orientation of the squeeze axis. The squeezed harmonic-oscillator Hamiltonian (scaled by a constant factor A) is given by

$$S^{\dagger}(\zeta)A\left(\frac{p^2}{2m}+\frac{m\omega^2}{2}x^2\right)S(\zeta)$$
$$=A\cosh(2s)\left(\frac{m\omega^2}{2}x^2+\frac{p^2}{2m}\right)+A\sinh(2s)\cos(\theta)$$
$$\times\left(\frac{m\omega^2}{2}x^2-\frac{p^2}{2m}\right)+A\sinh(2s)\sin(\theta)\frac{\omega}{2}\{x,p\}. \tag{3.14}$$

Equating the expectation of the last expression with Eq. (3.7) (and letting $\langle x\rangle=0=\langle p\rangle$) and equating the corresponding constants produces the conditions

$$A\cosh(2s)=f_1(t)=\frac{4mt}{\hbar^2}\left(D_{qq}+\frac{D_{pp}}{(m\omega)^2}\right),$$
$$A\sinh(2s)\cos(\theta)=f_2(t)=\frac{2m}{\hbar^2\omega}\sin(2\omega t)$$
$$\times\left(D_{qq}-\frac{D_{pp}}{(m\omega)^2}\right),$$
$$A\sinh(2s)\sin(\theta)=f_3(t)=-\frac{4m}{\hbar^2\omega}\sin^2(\omega t)$$
$$\times\left(D_{qq}-\frac{D_{pp}}{(m\omega)^2}\right). \tag{3.15}$$

The most advantageous orientation for squeezing is determined by

$$\tan\theta=\frac{f_3(t)}{f_2(t)}=-\tan\omega t. \tag{3.16}$$

The amount of squeezing is determined by

$$\tanh^2 2s=\frac{f_2^2(t)+f_3^2(t)}{f_1^2(t)}. \tag{3.17}$$

Since f_1 is linear in t while f_1 and f_1 oscillate, $\tanh^2 2s$ (and hence s) tends towards zero. The long time behavior of the amount of squeezing required to minimize entropy production is zero. Reintroducing the translations in position and momentum ignored in comparing Eq. (3.7) to Eq. (3.14) will not affect this result. For times on the order of the dynamical time scale of the system (more than a few cycles), coherent states are the maximal states, just as ZHP found for their environment model.

IV. CORRELATION EFFECTS IN ENVIRONMENT NOISE

In this section I wish to consider environment models that include the effects of finite correlation lengths in the environment [12–15]. From a strictly phenomenological point of view, quantum mechanics with spontaneous localization can be included by virtue of the effective form of the dynamics, although this is actually a fundamental modification of quantum mechanics [7]. Many of these models can be written in the form

$$\frac{\partial\rho(x,x';t)}{\partial t}=(\text{Hamiltonian})+(\text{dissipation})+\cdots -g(x,x')\rho(x,x';t), \tag{4.1}$$

where the decoherence term, upon which we shall focus, can be expressed in terms of the correlations of a classical fluctuating potential $V(x,t)$ with

$$g(x;y)=\frac{1}{\hbar^2}[c(x;x)+c(y;y)-2c(x;y)], \tag{4.2}$$

where

$$\langle V(x,t)V(y,\tau)\rangle_{\text{av}}=c(x;y)\,\delta(t-\tau). \tag{4.3}$$

For simplicity, I consider only a homogeneous and isotropic environment for which $c(x;y)=c(x-y)$ and $g(x;y)=g(x-y)$. I will also restrict my attention to weak dissipation and consider only the evolution due to the unperturbed Hamiltonian and the (spatially correlated) noise term.

The Lindblad form can be used to represent the noise term with

$$\{V_j\}=a(k)e^{ikx} \tag{4.4}$$

and replacing the discrete sum over j in Eq. (2.7) with an integral over k. The evolution in this case can be written

$$\frac{\partial\rho(x,x';t)}{\partial t}=(\text{Hamiltonian})-\frac{1}{\hbar}\int|a(k)|^2\times(1-e^{ik(x-x')})\rho(x,x';t), \tag{4.5}$$

so that $|a(k)|^2$ and $c(r)$ are Fourier transform pairs

$$c(r)=\frac{\hbar}{2}\int|a(k)|^2e^{ik(r)}dk. \tag{4.6}$$

Thus a noise term with short correlation length scales will have a narrow $c(r)$ and a broad $|a(k)|^2$ while a long correlation length scale implies a narrow $|a(k)|^2$. Inserting Eq. (4.4) into Eq. (2.12) yields

$$\Delta\varsigma(t)=\frac{1}{\hbar}\int_0^t\int|a(k)|_2(1-|\langle e^{ikx(\tau)}\rangle|_2)dk\,d\tau, \tag{4.7}$$

where $x(\tau)$ is given by Eq. (3.5) and $e^{ikx(s)}$ can immediately be recognized as Glauber's displacement operator, with a translation in momentum of $\hbar k\cos(\omega\tau)$ and a translation in position of $-(\hbar k)/(m\omega)\sin(\omega\tau)$. In terms of the translated state

$$|\psi;k,\tau\rangle\equiv e^{ikx(\tau)}|\psi\rangle, \tag{4.8}$$

the entropy production is given by

$$\Delta\varsigma(t)=\frac{1}{\hbar}\int_0^t\int|a(k)|^2(1-|\langle\psi|\psi;k,\tau\rangle|^2)dk\,d\tau. \tag{4.9}$$

While it is not possible to find general solutions for the minimization of Eq. (4.9) for arbitrary environment correlations, it is possible to extract important limiting cases. Restricting attention to times of several oscillator periods or more, entropy minimization requires the *maximization* of $|\langle\psi|\psi;k,\tau\rangle|^2$ for typical values of k, on the order of δk [the spread of $|a(k)|^2$], resulting in typical translations of $\hbar\delta k$ in momentum and $(\hbar\delta k)/(m\omega)$ in position. The maximization of the square of the inner product of any two normalized vectors occurs when the vectors are identical (up to a phase). Thus the maximal states and the translated maximal states will be approximately equal for typical translations, requiring that the width of $|a(k)|^2$ be much less than the width of the maximal states

$$\delta k\ll\frac{m\omega}{\hbar}\Delta x,\quad \delta k\ll\frac{1}{\hbar}\Delta p. \tag{4.10}$$

Since $|a(k)|^2$ and $c(r)$ are Fourier transform pairs, this last condition also implies that the noise spatial correlation function $c(r)$ is wide (compared to the maximal states). In this long correlation length scale limit, $g(x,x')$ in Eq. (4.1) is quadratic [13,14], corresponding to the low dissipation limit studied by ZHP and to the results of Sec. III with $b_j=0$ for all j. The coherent states are then the maximal states if they are consistent with the condition expressed in Eq. (4.10) using coherent state values for Δx and Δp,

$$\delta k\ll\sqrt{\frac{m\omega}{2\hbar}}, \tag{4.11}$$

which corresponds to a environment corellatation length much larger than the width of a coherent state $\Delta x=\sqrt{\hbar/2m\omega}$.

If the environment correlation length is shorter than the width of the coherent state, then the approximation described above is not valid. It is useful to examine the entropy production in the position representation and absorbing the time dependence into the (Schrödinger picture) state vectors, where

$$\begin{aligned}\langle\psi|e_{ikx(\tau)}|\psi\rangle&=\langle\psi|U_\dagger(\tau)e^{ikx}U(\tau)|\psi\rangle\\&=\langle\psi(\tau)|e^{ikx}|\psi(\tau)\rangle\\&=\int dx\,e^{ikx}|\psi(x,\tau)|^2\\&=\int dx\,e^{ikx}P(x,\tau).\end{aligned} \tag{4.12}$$

Entropy production then becomes

$$\Delta\varsigma(t)=\frac{2}{\hbar^2}\left(c(0)t-\int d\tau dx dx' c(x-x')P(x,\tau)P(x',\tau)\right), \tag{4.13}$$

using the Fourier transform relation expressed in Eq. (4.6). For narrow $c(r)$ this expression becomes independent of the function P. To see this, consider an example where the spatial correlation of the environment is given by

$$c(r)=\lambda e^{(r/\sigma)^2}. \tag{4.14}$$

In the limit $\sigma\rightarrow$ zero

$$\int dx dx' \lambda e^{[(x-x')/\sigma]^2}P(x,\tau)P(x',\tau)$$
$$\cong \lambda\sigma\sqrt{\pi}\int P^2(x,\tau)dx\rightarrow 0, \tag{4.15}$$

so that in the short correlation length limit

$$\Delta\varsigma(t)=\frac{2}{\hbar^2}c(0)t. \tag{4.16}$$

In this regime, all states produce the same entropy; there are no maximal states. However, the decay rate of the off-diagonal terms given by the decoherence term in Eq. (4.1) in the low short correlation length regime is generally at a maximum

$$\frac{\partial\rho(x,x';t)}{\partial t}=\cdots-\frac{2c(0)}{\hbar^2}\rho(x,x';t), \tag{4.17}$$

and if decoherence is to be effective, the decay time must be comparable to dynamical time scales. There will necessarily be a significant increase in the entropy for all pure states, so that all states will be rapidly ‘‘mixed’’ by the noise.

V. COMMENTS AND CONCLUSIONS

I have established an approximation scheme for determining maximal states (as defined by ZHP) and applied it to two families of Lindblad operators. For Lindblad generators linear in position and momentum, squeezed states emerge as the maximal states for intermediate times compared to the dynamical time scales. The amount of squeezing decreases with time, so that coherent states are maximal for large time scales. Large time scales are the most relevant, since an object's classicality should be an enduring property, not a transient one. These results can be applied to a wide variety of models in the literature by recognizing that those models are either particular cases or limiting cases of this Lindblad form.

For an environment with finite spatial correlation, coherent states emerge as maximal when the environment has long correlation length, but all states rapidly become mixed states when the environment length scale is long. Thus environment correlation effects will not be important in establishing the nature of maximal states and the character of quasiclassical states. However, correlation effects can still be important when considering quantum interference between two such states. One important result that emerges from these calculations is that coherent states are a robust choice for the maximal states.

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