Superluminal delays of coherent pulses in nondissipative media: A universal mechanism

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We identify the universal mechanism that is responsible for superluminal (faster-than-light) traversal times as well as the narrowing of wave packets transmitted through various nondissipative media. This mechanism is shown to be predominantly destructive interference between successive wave-packet components traversing all accessible causally retarded paths. It strongly depends on wave-packet coherence and width, and can cause superluminal traversal not only in evanescent-wave "tunneling" but also in allowed propagation.

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I. INTRODUCTION

As shown by a two-photon interference experiment [1], a photon that has tunneled as an evanescent wave packet through a dielectric-mirror "barrier" appears to have been delayed significantly less than its "twin" photon that has traversed the same distance in vacuum. Such a delay has been interpreted as signifying "superluminal" (faster-thanlight) barrier-traversal time. Similar "superluminal" time delay in tunneling through a dielectric mirror has now been measured in a classical two-pulse interference experiment [2]. The latter experiment has also revealed a remarkable feature, namely, that the temporal width of the transmitted wave packet is strongly narrowed down. These intriguing time-domain measurements add new insight to that offered by earlier spectral-domain observations of superluminal mean-phase delays, in frustrated total internal reflection [3] and waveguide transmission [4,5] of electromagnetic (EM) evanescent waves. Superluminal delays should also occur in tunneling of massive particles through potential barriers [6,7], which is analogous to EM evanescent wave-packet transmission [8].

It is always possible to trace numerically the evanescent wave-packet evolution and compare its features with different definitions of barrier traversal times [6,7,9] (see below). Nevertheless, the mechanism of superluminal time delays is still obscure [1] and regarded as a "poorly resolved mystery" [9]. A commonly invoked notion is that this mechanism is spectral reshaping (filtering) of the transmitted wave packet by dispersion. Indeed, such reshaping explains pulse narrowing and superluminal pulse traversal in absorbing [10] (or amplifying [11]) media, whose dispersion causes the faster spectral portion to be less absorbed (or more amplified) than the slower one. Analogous reshaping occurs in nonrelativistic electron wave packets which are dispersed in free space before hitting the barrier [9]. Yet why should spectral reshaping necessarily yield superluminal delays of EM pulses in nonabsorbing structures, after propagating in (dispersionless) vacuum? Is there a common mechanism for superluminal time delays and wave-packet narrowing, which applies to both EM pulses in dielectric structures and relativistic massive particles in potential barriers [4]? How is causality compatible with superluminal transmission, particularly in the single-photon case [1]?

We purport to show in this paper that the above questions can only be answered by a *universal description* of the temporal wave-packet transmission as *interference between its causally propagating consecutive components* [12]. Our description reveals the key role of phase coherence in tunneling, by demonstrating its dependence on the *coherence time* (phase randomization) of the wave packet. An important corrolary is that superluminal time delays can occur also in allowed propagation, namely, propagation which can only be described by *real wave vectors* (e.g., in Fabry-Pérot structures) and not only in evanescent-wave tunneling, where complex wave vectors can be employed (e.g., in photonic band-gap structures).

II. TRANSMISSION SPECTRUM AND MEAN TRAVERSAL TIME

Our general framework assumes a classical EM pulse with field amplitude $\psi_{in}(x,t)$ that is normally incident from $x \le 0$ onto a dielectric structure in 0 < x < L. The field amplitude $\psi_{tr}(x,t)$ transmitted through the structure is measured at x=L. It is related to the incident field amplitude at x=0 [13] by convolution with the impulse response $\sigma(t)$

$$\psi_{\rm tr}(L,t) = \int_0^\infty d\tau \ \sigma(\tau) \ \psi_{\rm in}(0,t-\tau), \tag{1}$$

where $\sigma(t)$ is the Fourier transform of the spectral transmission coefficient $\hat{\sigma}(\omega)$. The definitions of traversal times vary according to the wave-packet feature that is monitored [6,7,9]. We shall pick two of them and attempt to explain why they are superluminal, i.e., shorter than their free-space counterparts (similar explanation of other traversal times is deferred to Ref. [15]). (i) The mean traversal time is defined by

$$t_{\text{mean}} = \overline{\langle t \psi_{\text{tr}}^* \psi_{\text{tr}} \rangle} / \overline{\langle \psi_{\text{tr}}^* \psi_{\text{tr}} \rangle}, \qquad (2)$$

where the overbar denotes integration over all times and the angular brackets stand for an ensemble average (required for

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FIG. 1. A Gaussian pulse transmission through an empty layer $(n_2=1)$ between two dielectric slabs with $n_1=4$. Inset: Successively transmitted terms corresponding to 2j-fold reflections in the empty layer. Thin curves: successively transmitted Gaussians numbered by $j=0,1,2,\ldots$ with phase delays $\bar{\omega}\tau_j=(2j+1)\pi$. Dotted curve: their coherent sum ψ_{tr} for narrow $\psi_{in} (\Delta_t=0.03 \ L/c)$. Thick curve: ψ_{tr} for a wide $\psi_{in} (\Delta_t=1.5 \ L/c)$ exhibits $t_{mean} \approx t_{peak} < L/c$ (arrow). Dashed curve: the square root of the ensemble-averaged intensity when $\tau_c=0.5 \ L/c$.

fluctuating fields). This traversal time coincides with the "center-of-gravity" arrival time [9] in the case where the propagation outside the barrier is not dispersive. We note that t_{mean} can be defined for any shape of the incident pulse. (ii) The peak traversal time, t_{peak} , can be defined if $|\psi_{\text{tr}}|$ is a smooth single-peaked function, as $\partial |\psi_{\text{tr}}(t)|^2 / \partial t|_{t=t_{\text{peak}}} = 0$. Only in the asymptotic limit of a spectrally narrow incident pulse, where the stationary phase approximation is valid, do these two traversal times coincide with the so-called "phase time" [6,7] $t_{\text{phase}} = \partial \phi / \partial \omega$, where $\phi(\omega)$ is the phase of the spectral transmission function $\hat{\sigma}(\omega)$.

III. SUPERLUMINAL TRAVERSAL TIMES IN LAYERED STRUCTURES

We first show how superluminal effects occur in the simple case of a single dielectric layer: the region 0 < x < L is filled with a (nondispersive) medium of refractive index n_2 and embedded in an infinite medium of refractive index n_1 . The transmission coefficient $\hat{\sigma}(\omega)$ can be expanded in a series

$$\hat{\sigma}(\omega) = \sum_{j=0}^{\infty} c_j e^{i\omega\tau_j}, \qquad (3)$$

where each term represents a causal path, corresponding to 2j boundary reflections: j round trips through L followed by transmission (Fig. 1, inset). Here $\tau_j = (2j+1)n_2L/c$ is the jth path traversal time and $c_j = (1-\lambda^2)\lambda^{2j}$ are determined by the reflection amplitudes $\lambda \equiv (n_1 - n_2)/(n_1 + n_2)$ at the boundaries x=0 and x=L. Equation (3) also describes the transmission of a Fabry-Pérot etalon (two mirrors separated by a dielectric medium) when λ is replaced by the (complex)

reflection coefficient of a mirror [14]. By Fouriertransforming the expansion (3) we obtain the impulse response

$$\sigma(t) = \sum_{j=0}^{\infty} c_j \delta(t - \tau_j) \tag{4}$$

consisting of successive impulses with causal propagation times and decreasing amplitudes.

Suppose that the incident pulse $\psi_{in}(0,t)$ is of the form $\psi_{in}(0,t) = g(t) \exp[-i\xi(t)]$, where g(t) is a normalized Gaussian of temporal width Δ_t and the phase $\xi(t)$ corresponds to an oscillation with frequency $\overline{\omega}$ and phase-coherence time τ_c such that $\langle e^{i\xi(t)}e^{-i\xi(t')}\rangle = e^{i\tilde{\omega}(t-t')}e^{-|t-t'|/\tau_c}$. Our main tool will be the following autocorrelation function:

$$\Gamma(\tau_j - \tau_k) \equiv \operatorname{Re} \langle \psi_{in}^*(t - \tau_j) \psi_{in}(t - \tau_k) \rangle$$

= cos[$\bar{\omega}(\tau_j - \tau_k)$]exp $\left[-\frac{(\tau_j - \tau_k)^2}{8\Delta_t^2} - \frac{|\tau_j - \tau_k|}{\tau_c} \right].$ (5)

 $\Gamma(\tau_j - \tau_k)$ consists of the cosine of the relative phase between the paths *i* and *j*, weighted by an exponential term, measuring the amount of overlap and phase correlation between wave packets traversing the two paths.

The time-integrated and ensemble-averaged transmitted intensity $I_{tr} = \overline{\langle \psi_{tr}^* \psi_{tr} \rangle}$ is expressed in term of this autocorrelation function as

$$I_{\rm tr} = \sum_j c_j^2 + \sum_{j \neq k} c_j c_k \Gamma(\tau_j - \tau_k).$$
(6)

If the pulse is either very narrow $(\Delta_t \ll \tau_0)$ or incoherent $(\tau_c \ll \tau_0)$, then the wave packets traveling along different paths $\tau_i \neq \tau_k$ are no longer correlated, due to phase randomization or lack of overlap. The second (coherent) term in (6) is then washed out, and the transmission becomes frequency independent. In the opposite limit, when the incident pulse is wide, smooth, and coherent (transform-limited, such that $\Delta_t, \tau_c \gg \tau_0$, strong interference takes place between the different overlapping wave packets and the second term plays a major role. When this term is large and negative, this means that the interference is strong and predominantly destructive, due to the negativity of the $\Gamma(\tau_i - \tau_k)$ terms with the largest $c_i c_k$ weights. In the single-layer example, the transmission of a wide, transform-limited pulse becomes minimal when $\bar{\omega}(\tau_{i+1}-\tau_i)$ is an odd multiple of π , yielding $I_{\rm tr} = [2n_1n_2/(n_1^2 + n_2^2)]^2.$

The mean traversal time [Eq. (2)] through such a structure is

$$t_{\text{mean}} = \frac{1}{I_{\text{tr}}} \left\{ \sum_{j} c_{j}^{2} \tau_{j} + \frac{1}{2} \sum_{j \neq k} c_{j} c_{k} (\tau_{j} + \tau_{k}) \Gamma(\tau_{j} - \tau_{k}) \right\}.$$
(7)

The first (diagonal) term in (7), which is predominant for an incoherent or temporally narrow incident pulse, is always larger than the shortest causal arrival time $\tau_0 = n_2 L/c$, as expected for τ_j weighted with positive probabilities $(c_j^2/\Sigma_j c_j^2)$. For a wide, coherent pulse with carrier frequency $\bar{\omega}$ such that the transmitted intensity [Eq. (6)] becomes minimal due to strong, predominantly destructive interference between the paths, the second (coherent) term in Eq. (7) can become sufficiently negative to cause $t_{\text{mean}} < \tau_0$. In the present example the minimal value of t_{mean} is $[2n_1n_2^2/(n_1^2+n_2^2)]L/c$, which is less than L/c if $n_2=1$, i.e., superluminal, although we deal with allowed propagation.

The width of the transmitted pulse is given by $(\Delta t)^2 = I_{\text{tr}}^{-1} \overline{\langle (t - t_{\text{mean}})^2 \psi_{\text{tr}}^* \psi_{\text{tr}} \rangle}$. It can be shown that

$$(\Delta t)^{2} = \Delta_{t}^{2} + \frac{1}{4I_{\text{tr}}} \sum_{j,k} c_{j}c_{k}(\tau_{j} + \tau_{k})^{2}\Gamma(\tau_{j} - \tau_{k}) - t_{\text{mean}}^{2}.$$
 (8)

In the limit of total incoherence, we find that $(\Delta t)^2$ is just the sum of the squared widths of the incident wave packet Δ_t^2 and the impulse response $\sum_j \bar{c}_j^2 \tau_j^2 - (\sum_j \bar{c}_j^2 \tau_j)^2$ where $\bar{c}_j^2 = c_j^2 / \sum_j c_j^2$. In the opposite limit of strong interference between coherent and wide wave packets, $\Gamma(\tau_j - \tau_k) \sim e^{i\omega(\tau_j - \tau_k)}$. It then follows from Eqs. (5)–(7) that Eq. (8) becomes

$$(\Delta t)^2 = \Delta_t^2 - \frac{1}{4} \left. \frac{\partial^2}{\partial \omega^2} \left[\ln(I_{\rm tr}(\omega)) \right] \right|_{\bar{\omega}}.$$
 (9)

If $\bar{\omega}$ lies in a dip of the spectral transmission curve $I_{tr}(\omega)$, then $\partial^2 [\ln I_{tr}] / \partial \omega^2 > 0$ and the pulse will be narrowed. The temporal narrowing effect will be most salient when $\Delta_t \sim (\partial^2 / \partial \omega^2) [\ln(I_{tr}(\omega))]$, provided Δ_t is large enough to allow overlap of successive wave packets. This effect is seen to be sensitive to coherence: the phase incoherence τ_c^{-1} , which contributes only to the total spectral width of ψ_{in} and ψ_{tr} [due to the fluctuating phase $\xi(t)$], exponentially diminishing the narrowing in (8).

Figure 1 allows more insight into the case of wide and coherent ψ_{in} , for which $\psi_{tr}(L,t)$ (thick curve) consists of overlapping, destructively interfering Gaussians (thin curves). Amplitude suppression of the first transmitted wave packet $|c_0\psi_{in}(t-\tau_0)|$ by the next one is stronger throughout its back half than throughout its forward half, since the envelopes $|c_j\psi_{in}(t-\tau_j)|$ with $j \ge 1$ are maximal at $t > \tau_0$. Consequently, the forward tail, which has extended through the structure already at t<0, becomes the peak of ψ_{tr} , corresponding to superluminal t_{peak} and t_{mean} . Destruction of the back half of ψ_{in} by interference also makes the transmitted pulse narrower, because it consists mostly of the forward tail of ψ_{in} . By comparison, an incoherent (fluctuating) input Gaussian of the same envelope results in a broad, intense $\langle |\psi_{tr}|^2 \rangle$ with subluminal t_{peak} .

The foregoing results render the essence of superluminal effects in the transmission through any layered structure, since the impulse response is then a discrete sum of δ functions as in (4). Consider specifically the structure used in Refs. [1] and [2], which contains N periods, each consisting



FIG. 2. Transmission through five double-layer periods of length $2\ell'$ ($n_1=1.5$, $n_2=1$). Inset: successively transmitted terms due to interlayer reflections. Impulse response spikes are shown at $t = \tau_{j1} + \tau_{j2}$. Solid curve: transmitted Gaussian envelope with carrier frequency $\bar{\omega}$ at the center of the band gap and $\Delta_t = 1.5 L/c$ exhibits $t_{\text{peak}} \approx 0.6 L/c$.

of two layers with refractive indices n_1 and n_2 , and thicknesses ℓ_1, ℓ_2 . We have obtained the transmission coefficient $\hat{\sigma}_N(\omega)$ of N such periods by a recurrence relation from the single-period transmission and reflection coefficients. Its Fourier transform can be expanded, as seen from Fig. 2 (inset), in the form

$$\sigma_N(t) = \sum_{j_1, j_2=0}^{\infty} C_{j_1, j_2}^{(N)} \delta(t - (\tau_{j_1} + \tau_{j_2})).$$
(10)

Here each coefficient $C_{j_1,j_2}^{(N)}$ is the sum of the amplitudes of all possible paths traversing the n_1 and n_2 layers $N+2j_1$ and $N+2j_2$ times, respectively, with causal delay times $\tau_{ji}=(N+2j_i)n_i\ell_i/2c$ (i=1,2). Equations (6)–(8) and the foregoing discussion of the single layer (Fig. 1) fully apply to the multilayered barrier, on substituting $c_j \rightarrow C_{j_1,j_2}^{(N)}$ and $\tau_j \rightarrow \tau_{j1} + \tau_{j2}$. We find that (Fig. 2) an evanescent wave is a sum of propagating transmitted waves whose leading terms interfere destructively at band-gap frequencies. Correspondingly, there is constructive interference in the leading reflected waves (the Bragg reflection condition).

IV. SUPERLUMINAL TRANSMISSION TIMES IN DISPERSIVE MEDIA

The foregoing results can be extended to nondissipative dispersive media which are characterized by a continuous impulse response, e.g., a dielectric medium with continuously varying refractive index or an optical waveguide [4,9]. The sums over paths $\sum_{j,k} c_j c_k \Gamma(\tau_j - \tau_k)$ in Eqs. (6)–(8) should be replaced by the integrals $\int d\tau \int d\tau' \sigma(\tau) \sigma(\tau') \Gamma(\tau - \tau')$. Reduced intensity, superluminal time delay, and temporal narrowing of pulses transmitted through such media follow, as in layered structures, from destructive interference between components of ψ_{tr} at different delay times $\tau \neq \tau'$, corresponding to predominantly

negative contributions of $\sigma(\tau)\sigma(\tau')\Gamma(\tau-\tau')$. As an example, consider an infinite waveguide, with cutoff frequency ω_c . Its impulse response for transmission from x=0 to x=L has been found by us to have the causal form [15]

$$\sigma(t) = \delta(t - L/c) - \omega_c(L/c) [J_1(\omega_c s)/s] \theta(t - L/c), \qquad (11)$$

where J_1 is the first-order Bessel function, θ is the Heaviside step function, and $s = \sqrt{t^2 - (L/c)^2}$. If ψ_{in} is coherent and temporally wide $(\Delta_t, \tau_c \ge L/c)$ we can divide the integrals over $\sigma(\tau)\sigma(\tau')\Gamma(\tau-\tau')$ into intervals that exhibit strong cancellations in the expressions for t_{mean} , I_{tr} , and $(\Delta t)^2$ [the continuous limits of Eqs. (6), (7), and (8), respectively] provided that $\bar{\omega} < \omega_c$. The superluminal delays observed in waveguide transmission below cutoff [4] are obtainable from this description. It is important that massive relativistic particles in potential barriers obeying the Klein-Gordon equation fit the same description, since their energy-momentum dispersion is the same as in light in a waveguide [8].

V. CAUSALITY AND SIGNAL TRANSMISSION

Had the peak of the transmitted wave packet carried any new information, its arrival after a superluminal time delay t_{peak} would have violated causality. However, for an *analytic* input ψ_{in} in Eq. (1) the interfering forward tails of $\psi_{in}(t-\tau_i)$ which are present at x=L at t<0 already contain all the information on the rest of the pulse to follow. New information can only be transmitted by a nonanalytic (abrupt) disturbance of ψ_{in} , which would be causally delayed. This can be experimentally demonstrated by switching off the incident Gaussian at its peak (t=0) much faster than $\bar{\omega}^{-1}$ (for $\bar{\omega}$ in the microwave range, this is achievable by a subpicosecond optical pulse that drastically changes the transmissivity of a dielectric medium). Such abrupt switching off corresponds to $\psi_{in}(x=0,t) = \theta(-t)g(t)$, where $\theta(t)$ is the Heaviside step function [16]. The transmission of this ψ_{in} through any layered structure, e.g., a single layer [Eq. (4) yields by Eq. (1)

$$\psi_{\rm tr}(t) = \sum_j c_j \theta(\tau_j - t) g(t - \tau_j).$$
(12)

The interference is unaffected by this switching off at $t < \tau_0$. Hence, the forward half of ψ_{tr} looks the same as for an unchopped Gaussian ψ_{in} and may exhibit a superluminal t_{peak} . The true character of ψ_{in} is revealed only at $t = \tau_0 = L/c$, when the first transmitted Gaussian, $\psi_{in}(t - \tau_0)$, vanishes, causing ψ_{tr} (thick curve in Fig. 3) to drop below $|c_1\psi_{in}(t - \tau_1)|$. This demonstrates a fundamental point: The steplike decrease at successive τ_j transmits the switching-off information in a causal fashion, whereas superluminal features, such as t_{peak} , carry no information.

VI. PHOTON DETECTION AND SUPERLUMINAL EFFECTS

The discussion thus far has been classical, but it can easily be rendered in quantal terms, appropriate for two-photon interference [1] or one-photon detection. We must replace



FIG. 3. Same as Fig. 1 for an incident Gaussian pulse that is chopped at t=0 [Eq. (12)].

 ψ_{in} by a field operator $\Psi_{in}^{(+)}(x,t) = \sum_{\kappa} a_{\kappa} e^{i[\kappa x - \omega(\kappa)t]}$ where a_{κ} are the annihilation operators of the modes κ of the free field. In the single-photon case the incident field state is $|1\rangle = \sum_{\kappa} g_{\kappa} a_{\kappa}^{\dagger} |0\rangle$, where g_{κ} is a Gaussian function of $\omega(\kappa)$ centered around $\bar{\omega}$ and $|0\rangle$ is the vacuum state. In this case $\Gamma(\tau_i - \tau_k)$ in Eq. (5) must be evaluated using $\langle 1 | \Psi^{(-)}(x,t) \Psi^{(+)}(x',t') | 1 \rangle = \psi^*_{in}(x,t) \psi_{in}(x',t'),$ where ψ_{in} is the classical wave packet. Equations (6)–(8) are then valid, if $I_{\rm tr}$ is interpreted as the detection rate or probability of a transmitted photon. A transmitted photon is likely to be detected at "superluminal" times when its I_{tr} is peaked at the forward tail of the classical $|\psi_{in}(L,t)|^2$, which was already present at the detector even at t < 0. By contrast, a similar photon that has propagated the same distance through vacuum is characterized by the *peak* of $|\psi_{in}(L,t)|^2$, which arrives later (at t = L/c).

VII. CONCLUSIONS

Our theory has demonstrated that the universal mechanism of predominantly destructive interference between accessible causal paths [12] is responsible for transmission attenuation, superluminal delay times, and wave-packet narrowing. Two other characteristics of evanescent waves, namely, exponential attenuation and traversal-length independence of t_{mean} or t_{peak} for opaque barriers [2], can also be explained in terms of this universal mechanism [15]. This theory overcomes the limitations of previous approaches, since it applies to arbitrary pulse shapes, widths, and coherence times, and *explicitly* reveals the causal nature of their transmission. The understanding provided by this theory may open new perspectives in the design of the velocity, intensity, and shape of transmitted pulses, by manipulating the phase delays along the accessible paths in the medium.

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