Reply to ''Comment on 'Phase-invariant clock hypothesis for accelerating systems' ''

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A response is given to the work of Guangda *et al.* in which they criticize the paper ''Phase Invariant Clock Hypothesis.'' [S1050-2947(96)07305-2]

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The hypothesis dealing with the behavior of accelerating clocks that is most widely accepted (ACH) states that the rate of an accelerating clock is equal to that of a comoving unaccelerated clock. I have proposed a clock hypothesis [synchronized clock hypothesis (SCH)] that states that spatially separated clocks, which are synchronized in one inertial frame, maintain their synchronization in another inertial frame as long as their proper separation remains the same when they come to rest in any other inertial frame [1].

In Ref. [1], acceleration, in which the proper separation of the clocks is preserved, dominated the discussion. However, this is not necessary to satisfy the hypothesis. The clocks could, for instance, maintain their distance of separation in the initial inertial frame S. Then, upon reaching the inertial frame that the clocks find themselves in after the acceleration, S', they could achieve their proper separation by slow clock transport [2].

However, the intent of the paper was not to investigate the consequences of a variety of situations governed by the hypothesis but to concentrate on those effects that are most likely to be experimentally accessible. One such example is the redshift, which needs only to be calculated up to terms proportional to c^{-2} to satisfy this requirement. Additionally, the separation of the source and receiver was implicitly assumed to be no larger than the size of a normal laboratory and the acceleration was assumed to be within a few orders of magnitude of that given by gravity at the surface of the Earth.

These approximations may have been misunderstood in reading the paper. In frame S, for example, acceleration of spatially separated clocks, in which the proper length is maintained, requires that the acceleration a be different for the leading and the trailing clocks by a factor of $a/(1-aL/c^2)$, where L is the proper distance between the clocks [3]. In determining the redshift up to terms proportional to c^{-2}, the spatial dependence of the acceleration can therefore be neglected and the acceleration of both clocks appears to be the same in frame S.

The parameters have been chosen so that the product of the acceleration and the proper length is much less than c^2. This approximation then also eliminates the need to consider limitations on the proper length of an accelerating object [4] in the redshift calculation.

Let me now address the concerns, expressed by Guangda *et al.* [5] in the order that they appear in their Comment.

(1) Guangda *et al.* claim that I have made a mistake in deriving the frequency shift for spatially accelerating clocks using the SCH assumption. They claim that in going from one inertial frame to another ''the time for the leading clock Δt_l is greater than that for the trailing clock Δt_t.'' To what frame does this sentence refer: the proper frame of the clocks or the frame S? They justify this sentence by citing the work of Giannoni and Grøn [6]. Since Giannoni and Grøn's work is based on ACH, this frame must be the proper frame of the clocks and not S, the frame in which the calculation is to be carried out. Using this ACH result, Guangda *et al.* then proceed to calculate an average acceleration. It is not clear either in what frame this applies or exactly what this acceleration means.

The point of my paper is not to calculate the redshift using ACH, which has been done before [7], but it is to understand the behavior of spatially separated accelerating clocks using SCH.

The calculation in Eqs. (1) and (2) of my paper is done in frame S. Up to the order to which the calculation is carried out, the difference in the proper acceleration of the leading and trailing clocks is not significant, as discussed above. Therefore, both clocks experience constant acceleration. I state: ''These two clocks are now made to move at a new velocity $v+\Delta v$ in a time Δt, as seen in S.'' An observer in S then concludes that the acceleration of both clocks is $a=\Delta v/\Delta t$.

(2)–(4) Items 2, 3, and 4 are related. All involve the interaction of a clock (e.g., an atomic clock) with electromagnetic radiation. This is a separate issue from that of the behavior of a clock under acceleration. Neither ACH nor SCH deals with this interaction. ACH, for example, merely states that the rate of an accelerating clock is equal to that of a comoving unaccelerated clock. A supplemental assumption is needed here. I have chosen to assume that an accelerating atomic or nuclear clock interacts with electromagnetic radiation via the Doppler shift, where the velocity in this formula is given by the instantaneous velocity of the accelerating clock. Problems with this assumption have been discussed by Mashhoon [8]. Experimental evidence for this assumption, in the context of an accelerating mirror, have been given by Kowalski, Murray, and Head [9]. Therefore, my use of the Doppler relation is not ''obtained on the basis of ACH,'' as mentioned in objection (3) but is obtained on the basis of the assumption that the relativistic Doppler relation holds in the interaction of electromagnetic radiation with accelerating atomic or nuclear clocks.

SCH is incorporated into the Doppler shift via the frequencies that appear in the Doppler formula. For example, Eq. (3) of my paper is an expansion of the formula for the transverse Doppler shift. Equation (3) is not part of ACH because the frequency used in the transverse Doppler formula, $1/T^*$, which is given by my Eq. (2), is a consequence of SCH. Therefore objection (4) is unwarranted.

In the longitudinal Doppler example all calculations are carried out in frame S. An observer in this frame sees an electromagnetic wave train, emitted from a stationary source, pass by him with ν_{emitted} wave crests per second. He then uses the Doppler relation to determine the number of wave crests per second of this wave that pass by the trailing atom, which is now moving (to the approximation used, time dilation does not enter this calculation). Therefore the observer in S claims that the number of wave crests passing the trailing atom per second is given by $\nu_{\text{received}} = \nu_{\text{emitted}}(1+aL/c^2)$. This observer also concludes, using Eq. (2) of my paper, that this is the same frequency with which the trailing clock ticks. These two frequencies are those observed in frame S and not in different frames as claimed by Guangda *et al.* in objection (2).

Additionally, the authors misinterpret my discussion of the generalized form of SCH. The generalized hypothesis is: ''spatially separated clocks, synchronized in one inertial frame maintain their synchronization independent of the motion of the clocks as long as their proper separation remains the same when they come to rest with respect to each other in any *other* inertial frame.'' The intent was to have both clocks come to rest in a different inertial frame, (hence the word *other*) and therefore to have both clocks experience accelerated motion into this new frame. Such a stipulation is unnecessary if both clocks are constrained to maintain their proper separation. Nevertheless, in their interpretation Guangda *et al.* chose to have one clock of the pair of spatially separated clocks remain in the initial inertial frame while the other clock accelerates and returns to the initial inertial frame. They then conclude that SCH predicts that no twin paradox exists. Since this is not an example consistent with the assumptions of the generalized form of SCH their conclusion is inappropriate.

As discussed above, calculations are done only up to terms proportional to c^{-2}. The effect for terms proportional to c^{-4} is more subtle. The use of either the proper length or radar distance yields different answers using ACP [10]. My calculation was never intended to include terms proportional to c^{-4}. Yet Guangda *et al.* maintain in their calculations [see Eqs. (5)–(8)] terms proportional to c^{-4} without referring to the subtleties discussed by Landsberg and Bishop [10]. These issues remain even with SCH. By keeping terms proportional to c^{-4} they imply that their work provides new results for SCH. However, by ignoring the work of Landsberg and Bishop their results are at best incomplete in this regard.

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