Comment on ''Phase-invariant clock hypothesis for accelerating systems''

Meng Guangda, Ma Guangwen, and Wu Youlin
Department of Physics, Zhengzhou University, Zhengzhou 450052, China

Zhang Guomin
Zhengzhou Institute of Surveying and Mapping, Zhengzhou 450052, China
(Received 16 March 1995)

In this paper the phase-invariant clock hypothesis for accelerating systems suggested by F. V. Kowalski [Phys. Rev. A **46**, 2261 (1992)] is investigated in detail. It is pointed out that in Kowalski's derivation of the most distinctive result of the hypothesis, predicting no frequency shift between two spatially separated clocks rigidly accelerating, there are a few points needing further discussion. In addition, it is shown that the generalized phase-invariant clock hypothesis denies the existence of the usual twin paradox.
[S1050-2947(96)07205-8]

PACS number(s): 03.30.+p, 04.20.Cv

In the current theory of relativity it is assumed that the acceleration of a clock has no influence on the rate of the clock, and that the rate of an accelerated clock is the same as that of an instantaneously co-moving unaccelerated clock [1]. This is usually referred to as the accepted clock hypothesis (ACH) or the accelerated clock principle. No observation causes one to suspect the correctness of this hypothesis. However, alternatives, which assume that the rate of a clock depends on its acceleration, have been proposed and investigated [2–5].

The phase-invariant clock hypothesis suggested by Kowalski [4] is very interesting. He assumes that spatially separated clocks synchronized in an inertial frame S are still synchronized when these clocks are accelerated rigidly into a new inertial frame S'. Kowalski has discussed the possibilities of experimental verification for this hypothesis. He concluded that ''It differs from the accepted clock hypothesis in predicting no frequency shift between two spatially separated clocks rigidly accelerating'' [4]. This confirmation may be regarded as the most distinctive result of the phase-invariant clock hypothesis. However, there are a few points in Kowalski's derivation that need further discussion.

(1) Let us point out that Eq. (1), $\Delta\nu \simeq La/Tc^2$, and Eq. (2), $\nu_{\text{trailing}} - \nu_{\text{leading}} \simeq La/Tc^2$, in Kowalski's paper should be corrected. In his derivation of these two equations the following condition is used: ''These two clocks are now made to move at a new velocity $v+\Delta v$ in a time Δt, as seen in S.'' However, the times that spatially separated clocks take for the same increase Δv of their velocities are different when they are accelerated rigidly; the time for the leading clock Δt_l is greater than that for the trailing clock ΔT_t [6,7]. Which one, then, should be used for the time Δt? It seems relatively reasonable to take $\Delta t = \frac{1}{2}(\Delta t_l + \Delta t_t)$. Thus Eqs. (1) and (2) of Kowalski's paper should be corrected as follows:

$$\Delta\nu \simeq \frac{\Delta N}{\Delta t} \simeq \frac{L\Delta v}{Tc^2\Delta t} \simeq \frac{L\overline{a}}{Tc^2}, \tag{1}$$

and

$$\nu_{\text{trailing}} - \nu_{\text{leading}} \simeq \frac{L\overline{a}}{Tc^2}, \tag{2}$$

where

$$\overline{a} \simeq \frac{\Delta v}{\Delta t} \simeq \frac{2\Delta v}{\Delta t_l + \Delta t_t}. \tag{3}$$

Let a_l and a denote the accelerations of the leading clock and the trailing clock, respectively. Thus we have

$$\overline{a} = \frac{2a_l a}{a_l + a}. \tag{4}$$

If calculations are carried out to second order in v/c, the equation $\nu_{\text{received}} \simeq \nu_{\text{emitted}}[1 + aL/c^2]$ in Kowalski's paper can be corrected as

$$\nu_{\text{received}} \simeq \nu_{\text{emitted}}\left[1 + \frac{v}{c} + \frac{v^2}{2c^2}\right]. \tag{5}$$

The last term $v^2/2c^2$ in Eq. (5) originates from relativistic effect; neglecting it, we get the same result as in the classical case.

For the case of $v \ll c$ we have [6,7]

$$a_l \simeq a\left(1 - \frac{La}{c^2} + \frac{L^2a^2}{c^4}\right). \tag{6}$$

Substituting Eq. (6) into Eq. (4), we get

$$\overline{a} \simeq a\left(1 - \frac{La}{2c^2} + \frac{3L^2a^2}{4c^4}\right). \tag{7}$$

Substituting Eq. (7) into Eq. (2), we obtain

$$\nu_{\text{trailing}} - \nu_{\text{leading}} \simeq \frac{1}{T}\frac{La}{c^2}\left[1 - \frac{La}{2c^2}\right]. \tag{8}$$

Taking into account the motion of the trailing clock, one can get the relation between a and v (v is the velocity of the trailing clock at the time when it receives the light wave train emitted by the leading clock):

$$\frac{La}{c^2} \simeq \frac{v}{c}\left(1+\frac{v}{2c}\right). \tag{9}$$

Substituting Eq. (9) into Eq. (8), we have

$$\nu_{\text{trailing}} - \nu_{\text{leading}} \simeq \frac{1}{T}\left(\frac{v}{c}\right). \tag{10}$$

From Eqs. (5) and (10), it can be seen that ν_{received} is different from ν_{trailing} if calculations are carried to second order in v/c.

(2) ν_{trailing} in Eq. (2) of Kowalski's paper is the frequency of the accelerated trailing clock determined by an observer in S, but ν_{received} is the frequency received by an observer moving with the trailing clock, which is the same as that received by a momentarily comoving unaccelerated observer; the two frequencies ν_{trailing} and ν_{received} are given with respect to different frames. Even if ν_{trailing} coincides with ν_{received}, the observer in frame S cannot conclude that ''. . . if the clocks are a Mössbauer source and receiver accelerating as described above then they will resonantly interact during the acceleration.''

(3) The conclusion that ''The frequency received by an observer moving with the trailing clock is the same as that of a momentarily comoving unaccelerated observer . . . '' is to be obtained on the basis of the ACH. It is not reasonable that Kowalski uses this conclusion when he makes calculations according to his hypothesis.

(4) Equation (3) of Kowalski's paper, $T_B \simeq T^*[1+v_B^2/2c^2]$ is based on ACH. This equation should not be used when calculations are carried out according to Kowalski's hypothesis.

In Kowalski's opinion his hypothesis can be generalized for arbitrary motion as follows: ''spatially separated clocks, synchronized in one inertial frame, maintain their synchronization in another inertial frame independent of the motion of the clocks as long as their proper separation remains the same when they come to rest with respect to each other in any other inertial frame,'' which is hereafter referred to as the generalized phase-invariant clock hypothesis. Now let us point out that the generalized phase-invariant clock hypothesis denies the existence of the usual twin paradox.

Because the separation L between two spatially separated clocks, the acceleration of the clocks, and the frame S' that they finally reach are arbitrary, we may consider the case in which in the proper separation $L=0$, one of the two clocks is not accelerated and the frame S'coincides with S. That is, at first the two clocks rest at the same point in an inertial frame S, then one of them takes a round trip with arbitrary acceleration. This case is just that of the usual twin paradox. According to Kowalski's generalized phase-invariant clock hypothesis, if the two clocks are synchronized before they separate then they are still synchronized after they rejoin. Therefore, this hypothesis denies the existence of the usual twin paradox. This is another distinctive result of Kowalski's clock hypothesis.

This work was supported by the National Natural Science Foundation of China.

[1] C. Møller, *The Theory of Relativity* (Oxford, New York, 1972), p. 47.
[2] J. E. Romain, Rev. Mod. Phys. **35**, 375 (1963).
[3] I. Khan, Int. J. Theor. Phys. **6**, 383 (1972).
[4] F. V. Kowalski, Phys. Rev. A **46**, 2261 (1992).
[5] S. R. Mainwaring and G. E. Stedman, Phys. Rev. A **47**, 3611 (1993).
[6] C. Giannoni and Ø. Grøn, Am. J. Phys. **47**, 431 (1979).
[7] C. Møller, *The Theory of Relativity* (Ref. [1]), p. 290.