Preparation of a four-atom Greenberger-Horne-Zeilinger state

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I propose a method to prepare four atoms in an entangled state of the type discussed by Greenberger, Horne, and Zeilinger [in *Bell's Theorem, Quantum Theory and Conceptions of the Universe*, edited by M. Kafatos (Kluwer Academic, Dordrecht, 1989), p. 107]. The method involves two micromaser cavities, each supporting two modes initially prepared in a nonlocalized two-photon state by an atom undergoing a two-photon transition. Subsequently, two atoms pass through each cavity, becoming entangled in a Greenberger-Horne-Zeilinger state leaving the cavities in the vacuum. [S1050-2947(96)04906-2]

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Recently, there has been considerable interest in the generation of multiparticle entangled states for the purpose of testing local hidden variable (LHV) theories against quantum mechanics [1]. Many of the tests carried out experimentally have involved two-particle entangled states of the form (in the language of spin-$\frac{1}{2}$ particles)

$$|\psi\rangle=\frac{1}{\sqrt{2}}[|+\rangle_1|-\rangle_2\pm|-\rangle_1|+\rangle_2], \tag{1}$$

where the subscripts 1 and 2 refer to the particles and $\pm$ refer to the spin being up or down [2]. This state violates Bell's [3] inequality in contrast to predictions of LHV theories. However, the contradictions between quantum mechanics and LHV theory are of a statistical nature. On the other hand, Greenberger, Horne, and Zeilinger [4] showed that much stronger refutations of local realism can be provided by entangled states involving three or more particles. For example, they investigated the four-particle spin state

$$|\psi\rangle=\frac{1}{\sqrt{2}}[|+\rangle_1|+\rangle_2|-\rangle_3|-\rangle_4\pm|-\rangle_1|-\rangle_2|+\rangle_3|+\rangle_4] \tag{2}$$

and showed that a contradiction with LHV theory can be obtained from a single set of measurements.

However, it is apparently not as easy to manufacture multiparticle Greenberger-Horne-Zeilinger (GHZ) states as it is for two-particle states of the form of Eq. (1). Some proposals have been made. For example, Reid and Munro [5] and Klyshko [6] have discussed realizations of a GHZ state involving three "particles." On the other hand, Wodkiewicz, Wang, and Eberly [7] have studied the perfect correlations of a three-particle entangled state in the context of cavity QED. However, the entanglement involves a single cavity supporting four modes interacting with a single three-level atom with interfering transition pathways. Cirac and Zoller [8] have proposed a much simpler three-particle GHZ state involving only two-level atoms. The state is prepared by injecting the atoms sequentially through a suitably prepared single-mode resonant cavity. The velocities of the atoms must be carefully selected, but a more imposing obstacle is that the cavity field must first be engineered into a superposition of photon number states $|0\rangle$ and $|3\rangle$, although schemes for such purposes have been proposed [9]. Previously, I [10] have discussed another cavity QED method for generating these states, but involving a dispersive atom-field interaction in which the cavity initially contains a coherent state. But this method requires a measurement of the cavity field in order to reduce the atomic states into the GHZ form.

In this paper I present another method for producing a GHZ state, this time involving four atoms and two micromaser cavities, each capable of supporting two modes of the quantized electromagnetic field. Cavity dissipation effects are assumed to be small during the atom-field interaction times.

The setup for the proposed method is pictured in Fig. 1. Cavities 1 and 2 are assumed to each support modes of frequencies ω_1 and ω_2, which are sufficiently different in a way to be discussed below. We let the modes of cavity 1 be labeled modes 1 and 2 and those of cavity 2 as modes 3 and 4. We further assume that initially the cavity fields are in the vacuum state, i.e., the initial field state is $|0_1 0_2 0_3 0_4\rangle$. Now an atom, which we call the preparation atom, capable of making a nondegenerate two-photon transition at exact resonance $\omega_1+\omega_2$ as indicated in Fig. 2 passes through both cavities. The atom-field dynamics can be described by the effective Hamiltonian of the Jaynes-Cummings type [11]

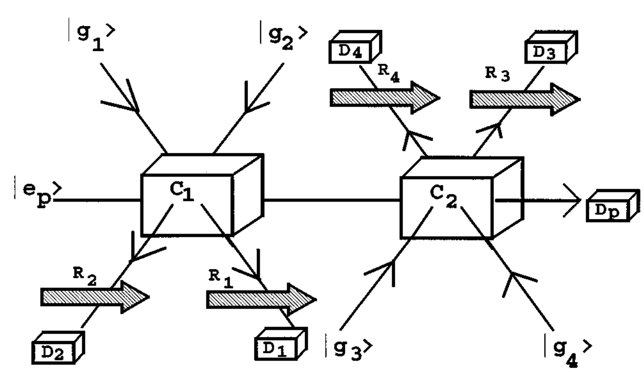

FIG. 1. Proposed experimental arrangement for generating a four-atom GHZ state. Cavities C_1 and C_2 each support modes at frequencies ω_1 and ω_2. The boxes labeled D are the ionization detectors and the R_s refer to the Ramsey zones used in analyzing the atomic states.

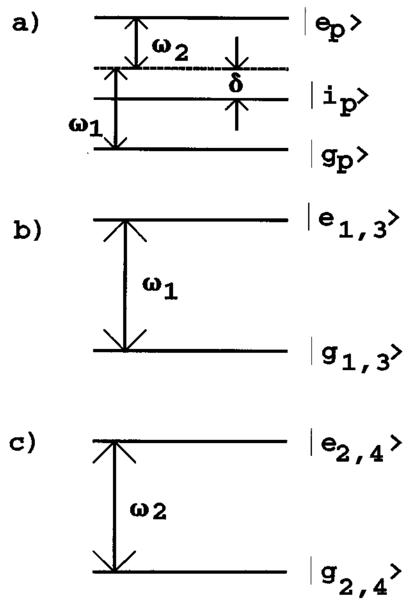

FIG. 2. (a) Energy levels of the preparing atom undergoing the two-photon transition. δ is the detuning with an intermediate state. (b) Atoms 1 and 3 are resonant at frequency ω_1. (c) Atoms 2 and 4 are resonant at frequency ω_2.

$$H^{1,2}_{I,\mathrm{eff}}=\hbar[\beta_1\sigma^p_{gg}a^+_{1,3}a_{1,3}+\beta_2\sigma^p_{ee}a^+_{2,4}a_{2,4} +\lambda_p(a_{1,3}a_{2,4}\sigma^p_{eg}+a^+_{1,3}a^+_{2,4}\sigma^p_{ge})], \tag{3}$$

where $a_{1,3}$, $a_{2,4}$, etc., are the field operators assuming modes 1 and 3 are of frequency ω_1 and 2 and 4 of ω_2, $\sigma^p_{eg}=|e_p\rangle\langle g_p|$, $\sigma^p_{ge}=|g_p\rangle\langle e_p|$, $\sigma^p_{ee}=|e_p\rangle\langle e_p|$, $\sigma^p_{gg}=|g_p\rangle\langle g_p|$, and

$$\beta_1=\frac{\lambda_1^2}{\delta},\quad \beta_2=\frac{\lambda_2^2}{\delta},\quad \lambda_p=\frac{\lambda_1\lambda_2}{\delta}. \tag{4}$$

Here λ_1 and λ_2 are the dipole moments for transitions to the intermediate state of Fig. 2 from $|e_p\rangle$ and $|g_p\rangle$ and δ is the indicated detuning. The first two terms of Eq. (3) contain the intensity-dependent Stark shifts. Our strategy will be to assume that a high-velocity atom passes through the cavities in a time t short enough so that the time evolution may be approximated by

$$|\psi(t)\rangle\simeq|\psi(0)\rangle-iH_{I,\mathrm{eff}}\frac{t}{\hbar}|\psi(0)\rangle. \tag{5}$$

Suppose $|\psi(0)\rangle=|e_p\rangle|0_10_20_30_4$. Then after passage through the first cavity the state vector is

$$|\psi_1\rangle\simeq|e_p\rangle|0_10_20_30_4\rangle-i\lambda_pt|g_p\rangle|1_11_20_10_3\rangle, \tag{6}$$

where we assume that $\lambda_pt\ll1$, t being the transit time across the cavity. Note that the Stark shift terms make no contribution to order λ_pt owing to the initial vacuum states of the cavities. After passing through the second cavity in the same time t, we have

$$|\psi_2\rangle\simeq|e_p\rangle|0_10_20_30_4\rangle-i\lambda_pt|g_p\rangle[|1_11_20_30_4\rangle +|0_10_21_31_4\rangle]. \tag{7}$$

If, after leaving the second cavity, the atom is detected in the ground state $|g_p\rangle$, the cavity fields are projected into the state

$$|\psi_F\rangle=\frac{1}{\sqrt{2}}[|1_11_20_30_4\rangle+0_10_21_31_4\rangle]. \tag{8}$$

Thus we obtain a GHZ state in which two correlated photons are delocalized between two cavities. As said above, the atomic velocity v_p should be very fast so that $\lambda_pt\ll1$ or $v_p\gg\lambda_pL$, where L is the length of the cavity. Beyond that, however, the velocity need not be precisely determined; all that is required is that the atom be detected in state $|g_p\rangle$ in order to generate Eq. (8). Thus the use of fast atoms reduces the need for precise timing and eliminates the effects of the Stark shifts.

However, it is not clear how one could probe the cavity fields directly to obtain the relevant measurements needed to demonstrate nonlocality. One solution is to replicate this state onto four atoms where two atoms pass through each cavity as indicated in Fig. 1. We assume that atoms 1 and 3 have pairs of levels resonant with frequency ω_1 and that atoms 2 and 4 are likewise resonant with frequency ω_2. The practical issues regarding how this could be realized are discussed below.

Now we assume that all four atoms are initially in their ground states so that the new initial state is $|\psi_F\rangle|g_1g_2g_3g_4\rangle$. The dynamics for each atom-field state is governed by the single photon Jaynes-Cummings model [12] for which

$$|g\rangle|0\rangle\rightarrow|g\rangle|0\rangle,$$
$$|g\rangle|1\rangle\rightarrow\cos(\lambda t)|g\rangle|1\rangle-i\sin(\lambda t)|e\rangle|0\rangle. \tag{9}$$

Thus, if λ_1 and λ_2 are the coupling constants associated with the modes of frequencies 1 and 2 and if t_i, $i=1,\ldots,4$ are the interaction times of atoms 1–4, then with $\lambda_1t_{1,3}=\lambda_2t_{2,4}=\pi/2$ the final state of the atom-field system is

$$|\psi\rangle=\frac{1}{\sqrt{2}}[|e_1e_2g_3g_4\rangle+|g_1g_2e_3e_4\rangle]|0_10_20_30_4\rangle, \tag{10}$$

the atomic part of which is obviously a GHZ state of the form of Eq. (1). The cavities are left in the vacuum state so that cavity dissipation effects can be ignored after the passage of the atoms. The atomic state can be analyzed to test for nonlocality by the use of classical microwave fields (Ramsey zones) followed by selective ionization.

It is necessary to address the practical matters regarding the experimental realization of the proposed scheme. First we examine a possible scheme to generate the nonlocal two-photon state of Eq. (8). Previously, Gou [11] has suggested that a two-photon transition, as used here to prepare the cavities could be realized with the Rb Rydberg states $63P_{1/2}$ and $62P_{1/2}$ as the states $|e_p\rangle$ and $|g_p\rangle$, respectively, and $61D_{3/2}$ for the intermediate state $|i_p\rangle$. The transition frequencies for $|e_p\rangle\leftrightarrow|i_p\rangle$ and $|i_p\rangle\leftrightarrow|g_p\rangle$ are, respectively, 21.111 and 9.591 GHz. These are millimeter waves. The atomic dipole moments for these transitions are on the order of 3000 a.u. On the other hand, a rectangular cavity of dimensions 2.20, 2.20, and 3.40 cm can be constructed to support the frequencies 9.6 and 21.12 GHz in the TE_{110} and TE_{222} modes, respectively. Thus the detuning δ between the states $|e_p\rangle$, $|g_p\rangle$, and $|i_p\rangle$, as indicated in Fig. 2(a), is $\delta = 0.009$ GHz. With these choices the two-photon transition rate between $|e_p\rangle$ and $|g_p\rangle$ can be large, as required. Thus a two-photon state of the form of Eq. (8) can be prepared. In order to probe the cavity it is necessary to employ atoms presumably of two different species, where one species has a pair of levels resonant at frequency ω_1 and the other a pair resonant at frequency ω_2. Furthermore, we require that for a given species there are no adjacent levels resonant at the other cavity frequency such that no competition between the modes can occur. This may be difficult to realize in practice, but could possibly be achieved by using a single atomic species that can be brought into resonance with ω_1 or ω_2 by Stark shifting with an applied electric field. Stark shifting could be applied sequentially, first tuning a set of atoms, one in each cavity, at ω_1 followed by a second set tuned at ω_2, all prior to selective ionization of any of the atoms.

Thus we have shown how a four-particle GHZ state of the form of Eq. (1) can be produced in the context of cavity QED. The advantages of using atoms in tests of nonlocality are twofold: the directions of the atomic beams are easily controlled and the detection of the atomic state is nearly 100% efficient. Furthermore, our procedure does not require a carefully engineered superposition state of the form $|0\rangle - |n\rangle$, which requires many atoms passing through a cavity. However, our method does require careful velocity selection for the atoms in order to achieve the precise interaction times indicated above. But this is also true of the Cirac-Zoller method [8] and of the two-cavity test of nonlocality in the form of Hardy [13] as recently discussed by Freyberger [14]. Even if the ideal state of Eq. (8) cannot be produced, it should be possible to demonstrate nonlocality since, as has been shown by Mermin [15], the GHZ-type state produces much stronger violations of Bell-type inequalities than does the traditional two-particle state.

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