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Nonlocality of a single photon in cavity QED

Christopher C. Gerry
Department of Physics, Amherst College, Amherst, Massachusetts 01002

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We study a model of single-photon nonlocality in which two separated cavities are prepared in an entangled state containing only one photon. The nonlocality is then transferred to two atoms probing each cavity. When the atoms are subsequently analyzed by classical microwave fields and selective ionization, Bell's inequalities are found to be violated for a wide range of atomic velocities. [S1050-2947(96)02106-3]

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In recent years there has been a lively debate in the literature regarding the possibility that a single photon can exhibit nonlocal behavior [1–7]. Much of the debate seems to be concerned with whether or not only one photon is involved in the schemes proposed by Tan, Walls, and Collett [1] and Hardy [3]. Clearly, in their proposals involving beam splitters and nonlinear crystals many modes of the field are present as well as auxiliary photons responsible for parametric down conversion processes, etc. Peres [7] has shown that for ''single-photon'' nonlocality a two-particle state must be created by the detecting process. On the other hand, if one means an excitation of the quantized electromagnetic field which can be delocalized between two degenerate but spatially separated field modes as being a single photon then it would seem to make sense to talk about that photon being nonlocal. What does not seem to be in question is that nonlocal effects do indeed occur. In a broader sense it could be argued that any photon is nonlocal due to the lack of a position operator in quantum electrodynamics.

However, recently Freyberger [8] studied a simple example of nonlocality in the context of cavity QED. Two identical micromaser cavities are prepared in correlated fields containing a single excitation. Subsequently each cavity field interacts with a single ground-state atom upon which state selective measurements are performed. The nonlocality makes its appearance in the form involving no inequalities as recently discussed by Hardy [9]. A major drawback to the proposal of Freyberger is that the two ground-state atoms crossing the cavities must be precisely velocity selected in order to exactly replicate the field state onto the atomic state.

In this paper we consider how a two-cavity setup with a single photon excited nonlocally in the cavities may be used to produce an approximation to a maximally entangled Bell state of two probe atoms. Such a state cannot be used for demonstrating nonlocality without inequalities, but may of course be used to violate Bell's inequality. Essentially we show that the probe atoms can be analyzed with a combination of classical microwave fields (Ramsey zones) and ionization detectors from which follows violations of Bell's inequalities for a wide range of interaction times. What we propose here is similar in spirit to the technique proposed by Cirac and Zoller [10] to create two-atom entangled states where the atoms pass sequentially through the same cavity. In the present case the atoms pass through two separated cavities so that it is clear that the origin of the nonlocality is the single photon delocalized between the cavities. A distinct difference between the experiment proposed here and those of Refs. [1,3] is that in the latter case the photon immediately escapes the experimental area whereas in the former the photon is trapped in the two-cavity setup. Furthermore, it is clear that only one photon is involved since the preparing atom undergoes a single transition from the excited to ground state.

Our proposed experiment is pictured in Fig. 1. C_1 and C_2 are two identical micromaser cavities. The first step is to prepare the cavities which are assumed initially to be in vacuum states $|0_1\rangle|0\rangle_2$. Let $|e\rangle$ and $|g\rangle$ represent atomic Rydberg states with transition frequency resonant with the cavity field modes. The dynamics is then governed by the Jaynes-Cummings model interaction Hamiltonian [11]

$$H_I^i = \hbar\lambda(\sigma_+ a_i + a_i^\dagger \sigma_-), \quad i=1,2 \tag{1}$$

where $\sigma_\pm$ have their usual meanings and a_i and $a_i^\dagger$, $i=1,2$ are the annihilation and creation operators for cavities 1 and 2, respectively. This Hamiltonian effects the following transitions:

$$|e\rangle|n\rangle_i \rightarrow \cos(\lambda t\sqrt{n+1})|e\rangle|n\rangle_i - i\,\sin(\lambda t\sqrt{n+1})|g\rangle|n+1\rangle_i, \tag{2a}$$

$$|g\rangle|n\rangle_i \rightarrow \cos(\lambda t\sqrt{n})|g\rangle|n\rangle_i - i\,\sin(\lambda t\sqrt{n})|e\rangle|n-1\rangle_i. \tag{2b}$$

We denote the states of the preparing atom as $|e_p\rangle$ and

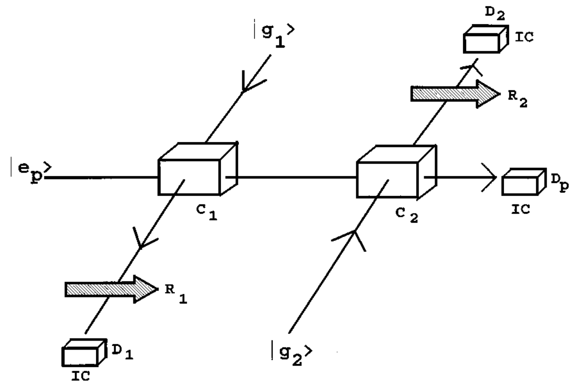

FIG. 1. Proposed experiment set to demonstrate single-photon nonlocality. The preparing atom in the excited state $|e_p\rangle$ enters the initially empty cavities C_1 and C_2. If the ionization detector D_p finds the atom in the ground state $|g_p\rangle$, a ground-state atom is then injected into each cavity. R_1 and R_2 are microwave Ramsey zones and D_1 and D_2 are ionization detection chambers.

$|g_p\rangle$. We assume that the atom is laser excited to state $|e_p\rangle$ so that the initial cavities-atom state is

$$|\psi(0)\rangle=|e_p\rangle|0\rangle_1|0\rangle_2. \tag{3}$$

After passage of the atom through the first cavity in time t_1 the state becomes

$$|\psi_1\rangle=[\cos(\lambda t_1)|e_p\rangle|0\rangle_1-i\ \sin(\lambda t_1)|g_p\rangle|1\rangle_1]|0\rangle_2 \tag{4}$$

and after passage through the second cavity in time t_2, the state is

$$\begin{aligned}|\psi_2\rangle= &\cos(\lambda t_1)\cos(\lambda t_2)|e_p\rangle|0\rangle_1|0\rangle_2\\ &-i[\cos(\lambda t_1)\sin(\lambda t_2)|0\rangle_1|1\rangle_2\\ &+\sin(\lambda t_1)|1\rangle_1|0\rangle_2]|g_p\rangle.\end{aligned} \tag{5}$$

If the atom is detected in the ground state the cavity field is projected into state

$$|\psi_F\rangle=N[\sin(\lambda t_1)|1\rangle_1|0\rangle_2+\cos(\lambda t_1)\sin(\lambda t_2)|0\rangle_1|1\rangle_2], \tag{6}$$

where

$$N=[\sin^2(\lambda t_1)+\cos^2(\lambda t_1)\sin^2(\lambda t_2)]^{-1/2} \tag{7}$$

is the normalization factor. Now suppose that the timings are arranged so that $\lambda t_1=\pi/4$ and $\lambda t_2=\pi/2$ or $\lambda t_2=3\pi/2$. This yields the field states

$$|\psi_{F\pm}\rangle=\frac{1}{\sqrt{2}}[|1\rangle_1|0\rangle_1\pm|0\rangle_1|1\rangle_2], \tag{8}$$

where the $+$ sign is for $\lambda t_2=\pi/2$ and the minus for $\lambda t_2=3\pi/2$. Either of these choices obviously leads to a maximally entangled state. In order to make the comparison with the usual spin-singlet states we imagine that the velocity is selected so that $\lambda t_2=3\pi/2$. The interaction time in the first cavity can be shortened by applying a constant electric field to Stark shift the atom out of resonances for just the right length of time [12].

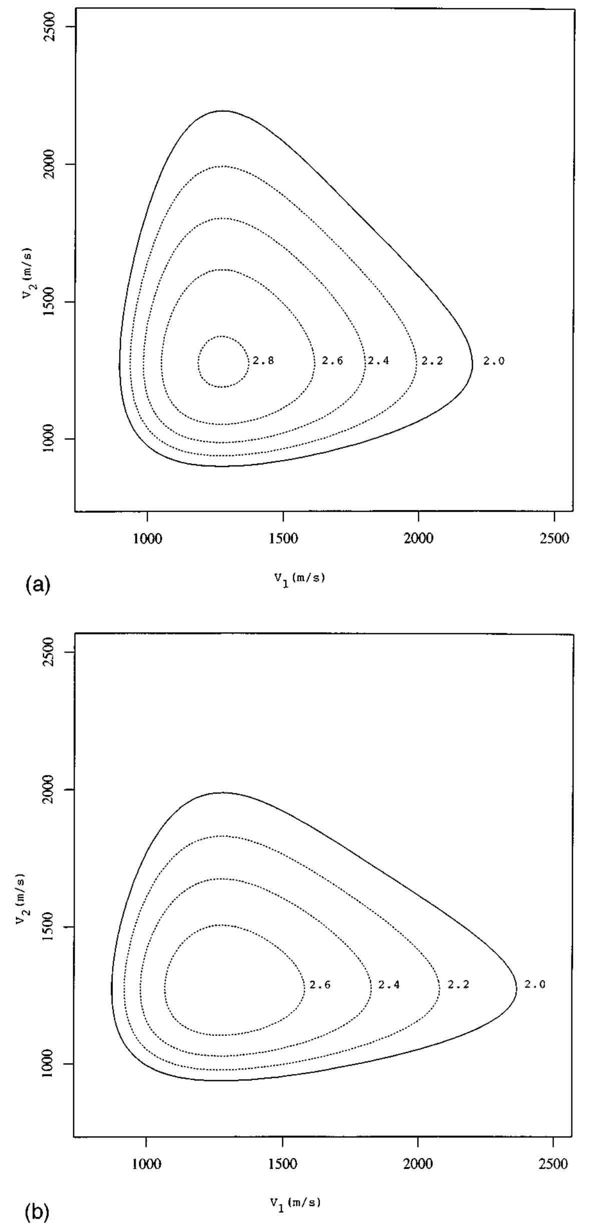

FIG. 2. Contour plots of $|S|$ versus v_1 and v_2 for different v_p. The solid lines indicate the contour $|S|=2$. In (a) $\lambda t_1=\pi/4$, $\lambda t_2=3\pi/2$, and in (b) $\lambda t_1=\pi/4$, $\lambda t_2=5\pi/4$. All velocities are chosen so that the point of maximum violation occurs for $\lambda\tau_1=\lambda\tau_2=\pi/2$.

However, for the moment we assume the effective interaction times t_1 and t_2 are arbitrary so that the field state has the form of Eq. (6) after the atom is detected in the ground state. We now assume that two ground-state atoms $|g_1\rangle$ and $|g_2\rangle$ are injected into cavities 1 and 2, respectively, as shown in Fig. 1. That is, the new initial state is $|\psi_F\rangle|g_1\rangle|g_2\rangle$. If τ_1 and τ_2 are the respective interaction times for these atoms in the cavities then using Eq. (2) we have

$$\begin{aligned}|\psi\rangle=N\{&\sin(\lambda t_1)[\cos(\lambda\tau_1)|g_1\rangle|g_2\rangle|1\rangle_1|0\rangle_2\\ &-i\ \sin(\lambda\tau_1)|e_1\rangle|g_2\rangle|0\rangle_1|0\rangle_1|0\rangle_2]\\ &+\cos(\lambda t_1)\sin(\lambda t_2)[\cos(\lambda\tau_2)|g_1\rangle|g_2\rangle|0\rangle_1|1\rangle_2\\ &-i\ \sin(\lambda\tau_2)|g_1\rangle|e_2\rangle|0_1\rangle|0\rangle_2]\}.\end{aligned} \tag{9}$$

In the special case where $\lambda t_1=\pi/4$, $\lambda t_2=3\pi/2$, $\lambda\tau_1=\lambda\tau_2=\pi/2$, the above reduces to

$$|\psi\rangle=\frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle-|g_1\rangle|e_2\rangle)|0\rangle_1|0\rangle_2, \tag{10}$$

where the atomic part is the usual spin-singlet state. Note that in this special case the photon disappears from the cavity.

We follow the argument of Cirac and Zoller [10] to determine the atomic velocities required to produce a state of the form of Eq. (9) close to the ideal spin singlet in Eq. (10). For the condition that $\lambda t_2=3\pi/2$, the velocity of the preparing atom should be $v_p^0=2\lambda L/3\pi$, where L is the length of the cavity. The effective interaction time in cavity 1 may be adjusted by applying an electric field in order to Stark shift the atom out of resonance for just the right amount of time to give an on-resonance time of $t_1=\pi/4\lambda$. On the other hand, the velocities of the probe atoms should be $v_i^0=2\lambda L/\pi$ ($i=1,2$) to make $\lambda\tau_i=\pi/2$. The assumption is made that the spread of velocities Δv_i is small such that $\lambda\tau_i\Delta v_i/v_i\ll 1$ or that $\Delta v_i/v_i\ll 2/\pi$. Now with $L=10^{-2}$ m, $\lambda\sim 2\times 10^5$ s^{-1} corresponding to a circular Rydberg atom transition from the states of principal quantum numbers 50 and 51, we require $v_{1,2}^0\simeq 1300$ m/s, $v_p^0=v_{1,2}^0/3\simeq 400$ m/s, and $\Delta v_i\ll v^i$. These numbers should be accessible in current or planned experiments.

Nonlocality can be demonstrated by testing Bell's inequality. This can be done with measurements of the quantity [13]

$$S=E(a,b)-E(a,b')+E(a',b)+E(a',b'), \tag{11}$$

where

$$E(a,b)=\langle \boldsymbol{\sigma}_1\cdot\hat{\mathbf{a}}\ \boldsymbol{\sigma}_2\cdot\hat{\mathbf{b}}\rangle \tag{12}$$

and where $\boldsymbol{\sigma}_i=(\sigma_{ix},\sigma_{iy},\sigma_{iz})$ are the usual Pauli spin operators. According to local hidden variable theories one should always have $|S|\leqslant 2$ whereas in quantum mechanics one could have $|S|>2$ for some values of the angles $\angle(\hat{\mathbf{a}},\hat{\mathbf{b}})$, $\angle(\hat{\mathbf{a}},\hat{\mathbf{b}}')$, etc. These angles in the present case are controlled by the classical microwave fields applied to the atoms in Ramsey zones R_1 and R_2 of Fig. 1 equivalent to rotating a Stern-Gerlach magnet for a spin one-half particle, prior to selective ionization. From Eq. (9) we find that

$$E(a,b)=a_zb_zf+(a_xb_x+a_yb_y)h, \tag{13}$$

where

$$f=N^2[\sin^2(\lambda t_1)\cos^2(\lambda\tau_1)+\cos^2(\lambda t_1)\sin^2(\lambda t_2)\cos^2(\lambda\tau_2)-\sin^2(\lambda t_1)\sin^2(\lambda\tau_1)-\cos^2(\lambda t_1)\sin^2(\lambda t_2)\sin^2(\lambda\tau_2)] \tag{14}$$

and

$$h=2N^2\sin(\lambda t_1)\cos(\lambda t_1)\sin(\lambda t_2)\sin(\lambda\tau_1)\sin(\lambda\tau_2). \tag{15}$$

For the special case when $\lambda t_1=\pi/4$, $\lambda t_2=3\pi/2$, $\lambda\tau_1=\lambda\tau_2=\pi/2$ this reduces to

$$E(a,b)=-(a_xb_x+a_yb_y+a_zb_z)=-\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}. \tag{16}$$

Setting $a_y=b_y=0$, $a_x=\sin\alpha$, $a_z=\cos\alpha$, $b_x=\sin\beta$, $b_z=\cos\beta$, etc., and with $\alpha=0$, $\alpha'=\pi/2$, $\beta=\pi/4$, and $\beta'=3\pi/4$ we obtain from Eq. (11) $|S|=2\sqrt{2}$, a violation of Bell's inequality. With these choices of angles the more general case of arbitrary t_1, t_2, τ_1, and τ_2 yields

$$|S|=\sqrt{2}|f+h|. \tag{17}$$

In Fig. 2 we show contour plots of $|S|$ versus the velocities v_1 and v_2 of the ground-state atoms probing the cavities. The solid lines are for $|S|=2$ and the dashed contours represent $|S|=2.2$, 2.4, 2.6, and 2.8. At the peak $\tau_1=L/v_1$ and $\tau_2=L/v_2$ are such that $\lambda\tau_1=\lambda\tau_2=\pi/2$. Figure 2(a) is a contour plot for the case when the preparation times are such that $\lambda t_1=\pi/4$ and $\lambda t_2=3\pi/2$, which results in the cavities being prepared in the maximally entangled state

$$|\psi_{F-}\rangle=\frac{1}{\sqrt{2}}(|1\rangle_1|0\rangle_2-|0\rangle_1|1\rangle_2), \tag{18}$$

whereas in Fig. 2(b), the preparation times are changed to $\lambda t_1=\pi/4$ and $\lambda t_2=5\pi/4$. Although the cavities are no longer prepared ideally, we see that Bell's inequality can still be violated for a wide range of velocities of the probe atoms.

In summary, we have shown that two-atom entangled states may be generated from two atoms passing through separated cavities that have initially been prepared with a single photon delocalized between them. Since the entangled atomic states violate Bell's inequality and thus display nonlocal behavior, it is apparent that the origin of the nonlocality can be attributed only to the nonlocality of the single photon emitted by the preparing atom. We have shown that Bell's inequalities will be violated for a wide range of atomic velocities.

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