Electromagnetic field quantization in absorbing dielectrics. II

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A quantization scheme for the electromagnetic field in absorbing dielectrics developed previously is extended to cover more complicated arrangements of dielectric media and to investigate various limiting cases of the general formalism. The limiting cases include media that have vanishing imaginary parts in their dielectric functions, because either the refractive index or the extinction coefficient vanishes. The further limit of a unit real dielectric function establishes the connection of the formalism with the well-known quantized field expressions in free space. Detailed calculations are presented for the quantization in the system of two different absorbing dielectrics in contact at a plane interface and for the cavity formed in the free space between two separated absorbing dielectrics. The forms of the field operators are determined for both systems, the canonical commutation relations are verified, and the spectra of the vacuum field fluctuations are calculated and illustrated. The calculations are restricted throughout to fields that propagate perpendicular to the dielectric interfaces. [S1050-2947(96)10706-X]

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I. INTRODUCTION

One of the main recent developments in quantum optics has been the study of processes, for example spontaneous emission, that take place inside, or adjacent to, material bodies. The need to interpret the growing body of experimental results has stimulated attempts to quantize the electromagnetic field in materials of increasingly general properties. Thus methods have recently been developed for the quantization of the electromagnetic field in dielectric media that simultaneously display the three properties of absorption, dispersion, and finite extent, in the special case where the sample has plane-parallel boundary surfaces [1,2]. The quantization scheme covers propagation perpendicular to the interfaces, and it is therefore applicable to a range of problems in which parallel light beams propagate through optical systems in a direction perpendicular to the surfaces of the components. It includes as special cases a range of earlier calculations that apply to dielectric media with only some of the three properties listed above (see [2] for a review of previous work). The formalism particularly generalizes earlier theories that apply to homogeneous absorbing dielectrics without any boundaries [3–6].

The presence of absorption has the effect of coupling the electromagnetic field to a reservoir, whose oscillators act as noise sources, and these are conveniently represented by Langevin forces that act on the fields (see [7] for a review). It has been shown that this representation leads to a straightforward quantization of the electromagnetic field, which has been applied to dielectrics with the spatial forms of an infinite medium, a semi-infinite medium with a flat surface, and a parallel-sided slab [1,2]. The aims of the present paper are, first, to make closer contact between this work and some of the previous calculations, and, second, to extend the results to more complicated systems.

In terms of the first aim, we show in Sec. II how the properties of the Langevin forces are related to the underlying noise spectrum of the microscopic oscillators of the particles that make up the dielectric medium. In Sec. III we show that the quantized field reduces to known forms when first the absorption and then the dispersion are removed, and we finally show that the well-known free-space result is obtained when the dielectric function is set equal to unity. The lossless medium is also considered in the case of frequencies within a stop band, for example a dielectric for frequencies between a transverse resonance and its associated longitudinal mode, or a plasma for frequencies lower than the plasma frequency, where the real part of the refractive index vanishes but its imaginary part, or extinction coefficient, is nonzero.

In terms of the second aim, in Secs. IV and V we consider the field quantization for two different lossy dielectrics in contact at a flat interface, and for an optical cavity formed from the region of free space between two parallel semi-infinite dielectric media. In both cases we derive expressions for the quantized field operators for propagation perpendicular to the interfaces, and we evaluate the vacuum field fluctuations in the different spatial regions. It is verified that the canonical commutation relation is satisfied in all spatial regions, and this forms a check on the validity of the quantization. The reduction of the general results to those for special cases derived previously is also confirmed. Particular attention is paid to the important practical case of an optical cavity formed from highly reflecting boundaries. The main conclusions are summarized in Sec. VI.

II. BASIC FORMALISM

A. Field equations

The formalism is the same as we used before [1,2] and it is only necessary to summarize here the basic equations and notation. In classical macroscopic electrodynamics, a linear nonmagnetic medium is described completely by the continuum approximation in which the medium is characterized by a dielectric function. A dielectric that shows dispersion must inevitably be lossy, in accordance with Kramers-

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Kronig relations, and the dielectric function must necessarily be complex. The complex refractive index $n(\omega)$ is related to the material dielectric function $\varepsilon(\omega)$ in the usual way,

$$\varepsilon(\omega)=\varepsilon_r(\omega)+i\varepsilon_i(\omega)=n^2(\omega)=[\eta(\omega)+i\kappa(\omega)]^2, \tag{2.1}$$

where the real and imaginary parts of $\varepsilon(\omega)$ are denoted by subscripts, and $\eta(\omega)$ and $\kappa(\omega)$ are the real refractive index and extinction coefficient, respectively.

Though these functions are defined for positive frequencies, nevertheless their negative-frequency values can be deduced from the crossing relations [8]

$$\varepsilon(-\omega)=\varepsilon^*(\omega), \quad n(-\omega)=n^*(\omega). \tag{2.2}$$

The form of $\varepsilon(\omega)$ in the upper half of the complex ω plane is restricted by causality considerations to have neither poles nor zeros, and it also conforms to the limit [8]

$$\varepsilon(\omega)\to 1 \quad \text{for} \quad \omega\to\infty \quad \text{in any manner.} \tag{2.3}$$

We consider linearly polarized electromagnetic waves that propagate parallel to the x axis with their transverse electric and magnetic vector operators $\hat{E}(x,t)$ and $\hat{B}(x,t)$ parallel to the y and z axes, respectively. The field operators satisfy Maxwell's equations in the forms

$$\frac{\partial\hat{E}(x,t)}{\partial x}=-\frac{\partial\hat{B}(x,t)}{\partial t} \tag{2.4}$$

and

$$-\frac{\partial\hat{B}(x,t)}{\partial x}=\mu_0\frac{\partial\hat{D}(x,t)}{\partial t}+\mu_0\hat{J}(x,t), \tag{2.5}$$

where $\hat{D}(x,t)$ is the electric displacement operator. In the absence of an externally applied current, the transverse operator $\hat{J}(x,t)$ plays the role of a Langevin force associated with the noise reservoir. The field operators are separated into positive- and negative-frequency components in the usual way,

$$\hat{E}(x,t)=\hat{E}^+(x,t)+\hat{E}^-(x,t), \tag{2.6}$$

and the frequency-space Fourier transform operators are defined according to

$$\hat{E}^+(x,t)=\frac{1}{\sqrt{2\pi}}\int_0^\infty d\omega\hat{E}^+(x,\omega)e^{-i\omega t}, \tag{2.7}$$

with similar separations and transforms for the magnetic induction, electric displacement, and noise current operators. The negative-frequency components are provided by the Hermitian conjugates of the positive-frequency operators.

The Fourier transform displacement operator for a system in which the dielectric function, denoted $\varepsilon(x,\omega)$, varies with both the frequency and the x coordinate, is determined by

$$\hat{D}^+(x,\omega)=\varepsilon_0\varepsilon(x,\omega)\hat{E}^+(x,\omega), \tag{2.8}$$

and the frequency-domain Maxwell's equations are obtained from (2.4) and (2.5) as

$$\frac{\partial\hat{E}^+(x,\omega)}{\partial x}=i\omega\hat{B}^+(x,\omega) \tag{2.9}$$

and

$$-\frac{\partial\hat{B}^+(x,\omega)}{\partial x}=-\frac{i\omega\varepsilon(x,\omega)}{c^2}\hat{E}^+(x,\omega)+\frac{1}{\varepsilon_0c^2}\hat{J}^+(x,\omega). \tag{2.10}$$

It is convenient to express the field operators in terms of a vector potential operator $\hat{A}(x,t)$, which has decompositions similar to (2.6) and (2.7), using the relations

$$\hat{E}^+(x,\omega)=i\omega\hat{A}^+(x,\omega), \qquad \hat{B}^+(x,\omega)=\frac{\partial\hat{A}^+(x,\omega)}{\partial x}. \tag{2.11}$$

The omission of the scalar potential is permitted by the one-dimensional nature of the system, where it is possible to choose a gauge in which only the transverse part of the vector potential participates in the quantization. The first Maxwell equation (2.9) is automatically satisfied when these expressions are substituted, while the second Maxwell equation (2.10) gives

$$\frac{\partial^2\hat{A}^+(x,\omega)}{\partial x^2}+\varepsilon(x,\omega)\frac{\omega^2}{c^2}\hat{A}^+(x,\omega)=-\frac{1}{\varepsilon_0c^2}\hat{J}^+(x,\omega). \tag{2.12}$$

The spatial dependence of the quantized vector potential is determined by this inhomogeneous Helmholtz differential equation.

The form of the generalized momentum operator that is conjugate to the vector potential in the quantized field theory is determined by the usual techniques of Lagrangian mechanics. The conjugate momentum takes different forms for different transverse gauges, but we particularly consider the Coulomb gauge where its form is $-\varepsilon_0\hat{E}(x,t)$ and the canonical commutation relation is

$$[\hat{A}(x,t),-\varepsilon_0\hat{E}(x',t)]=(i\hbar/S)\,\delta(x-x'), \tag{2.13}$$

where S is an area of quantization in the yz plane, perpendicular to the direction of propagation.

B. Langevin force commutator

The presence of fluctuations in the vacuum state of the electromagnetic field is apparent from the existence of the zero-point energy. What is not always appreciated is that a material medium, necessarily with complex dielectric function, also introduces a fluctuating noise current of the vacuum. The former demonstrates a characteristic property of the free-space electromagnetic field, while the latter reflects the same feature for the medium. This can be understood from the form of the relation

$$\hat{P}^+(x,\omega)=\varepsilon_0[\varepsilon(x,\omega)-1]\hat{E}^+_{\text{ext}}(x,\omega) \tag{2.14}$$

for the noise reservoir polarization operator $\hat{P}^{+}(x,\omega)$ induced by an externally applied electric field $\hat{E}^{+}_{\rm ext}(x,\omega)$. Thus if one regards the electric field as the *perturbing force*,, then the electromagnetic energy expression shows that the *responding quantity* is the polarization, and the function $\varepsilon_0[\varepsilon(x,\omega)-1]$ is the appropriate *generalized susceptibility*. The vacuum-state noise polarization correlation function can therefore be calculated by the help of the fluctuation-dissipation theorem and Kubo's formula [9] as

$$\langle 0|\hat{P}^{+}(x,\omega)\hat{P}^{-}(x',\omega')|0\rangle=\frac{2\hbar}{S}\varepsilon_0\varepsilon_i(x,\omega)\,\delta(x-x')\times\delta(\omega-\omega'),\tag{2.15}$$

where the factor S in the denominator results from the confinement of the quantization to a finite area in the yz plane.

The noise reservoir current density and polarization operators are related by

$$\hat{J}^{+}(x,\omega)=-i\omega\hat{P}^{+}(x,\omega),\tag{2.16}$$

and insertion into (2.15) gives

$$\langle 0|\hat{J}^{+}(x,\omega)\hat{J}^{-}(x',\omega')|0\rangle=\frac{2\hbar}{S}\omega^2\varepsilon_0\varepsilon_i(x,\omega)\,\delta(x-x')\times\delta(\omega-\omega').\tag{2.17}$$

Since

$$\langle 0|\hat{J}^{-}(x',\omega')\hat{J}^{+}(x,\omega)|0\rangle=0,\tag{2.18}$$

we find the vacuum expectation value of the noise-current commutator to be

$$\langle 0|[\hat{J}^{+}(x,\omega),\hat{J}^{-}(x',\omega')]|0\rangle=\frac{2\hbar}{S}\omega^2\varepsilon_0\varepsilon_i(x,\omega)\,\delta(x-x')\times\delta(\omega-\omega').\tag{2.19}$$

The correlation between polarization and current density can also be found by inserting (2.16) into (2.19), and we find that

$$\langle 0|[\hat{P}^{+}(x,\omega),\hat{J}^{-}(x',\omega')]|0\rangle=\frac{2\hbar}{S}i\omega\varepsilon_0\varepsilon_i(x,\omega)\,\delta(x-x')\times\delta(\omega-\omega').\tag{2.20}$$

The noise operators can be related to the spatial coordinates of the particles that make up the dielectric reservoir. Thus the polarization operator is defined in terms of the position operators for the charged particles by

$$\hat{P}^{+}(x,\omega)=-e\sum_{i=1}^{\nu}\hat{r}_i^{+}(x,\omega),\tag{2.21}$$

where ν is the number of charged particles per unit volume. Similarly, the current density operator is defined in terms of the charged particle momentum operators by

$$\hat{J}^{-}(x,\omega)=-\frac{e}{m}\sum_{i=1}^{\nu}\hat{p}_i^{-}(x,\omega),\tag{2.22}$$

where e and m are the charge and mass of the carriers, respectively. Here $\hat{p}_i$ are the usual canonical particle momentum operators, including the vector potential contributions. On inserting (2.21) and (2.22) into (2.20), we have that

$$\langle 0|[\hat{r}_i^{+}(x,\omega),\hat{p}_j^{-}(x',\omega')]|0\rangle=i\frac{2\hbar\omega}{S}\frac{m\varepsilon_0}{\nu e^2}\varepsilon_i(x,\omega)\times\delta_{ij}\delta(x-x')\,\delta(\omega-\omega').\tag{2.23}$$

As we see, an immediate outcome of the noise-current fluctuation is the expectation value of the commutation relation between the charged particle position and momentum operators. The noise-current fluctuation thus reflects the characteristic properties of the medium through an expression that incorporates its loss spectrum. The noise-current fluctuation can alternatively be considered as a consequence of the intrinsic quantum-mechanical natures of the position and momentum operators of the charged particles that constitute the dielectric reservoir. In this view, it is the commutation relations among the positions and momenta of the charged particles that are transmitted to the noise current and establish the noise-current fluctuations.

The field commutation relations are closely related to the commutation properties of the noise-current operator, or Langevin force, in (2.12). For noise that is uncorrelated at different frequencies, the latter commutation relations are taken in the forms

$$[\hat{J}^{+}(x,\omega),\hat{J}^{-}(x',\omega')]=\frac{2\varepsilon_0\hbar\omega^2}{S}\varepsilon_i(x,\omega)\,\delta(x-x')\times\delta(\omega-\omega'),$$
$$[\hat{J}^{+}(x,\omega),\hat{J}^{+}(x',\omega')]=[\hat{J}^{-}(x,\omega),\hat{J}^{-}(x',\omega')]=0,\tag{2.24}$$

consistent with (2.19). The noise current properties derived here agree with the results of the formalism presented in [7].

III. FIELD QUANTIZATION

A. Infinite homogeneous dielectric

The complete solution to the problem of quantization in a dispersive lossy dielectric can be obtained from (2.12), provided that the noise-current commutation relations (2.24) are satisfied. In the homogeneous case, in which the dielectric function has no spatial dependence, it is advantageous to convert the differential equation (2.12) into an algebraic equation. The wave-vector space Fourier transforms of $\hat{A}^{+}(x,\omega)$ and $\hat{J}^{+}(x,\omega)$ are defined by

$$\hat{A}^{+}(x,\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}dk\;\hat{A}^{+}(k,\omega)e^{ikx},\tag{3.1}$$
$$\hat{J}^{+}(x,\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}dk\;\hat{J}^{+}(k,\omega)e^{ikx},$$

and conversion of (2.12) gives

$$-k^2\hat{A}^+(k,\omega)+\varepsilon(\omega)\frac{\omega^2}{c^2}\hat{A}^+(k,\omega)=-\frac{1}{\varepsilon_0 c^2}\hat{J}^+(k,\omega). \tag{3.2}$$

Insertion of the solution for the vector potential operator into (3.1) gives

$$\hat{A}^+(x,\omega)=\frac{1}{\sqrt{2\pi}\varepsilon_0}\int_{-\infty}^{+\infty}dk\frac{1}{k^2c^2-\omega^2\varepsilon(\omega)}\hat{J}^+(k,\omega)\times\exp(ikx). \tag{3.3}$$

It is convenient to rewrite the above expression in terms of a boson-type operator as replacement for the noise-current operator [2]. We thus define a modified Langevin force operator in wave-vector space by

$$\hat{f}(k,\omega)=\hat{J}^+(k,\omega)\sqrt{S/2\varepsilon_0\hbar\omega^2\varepsilon_i(\omega)}, \tag{3.4}$$

whose commutation relation from (2.24) has the form

$$[\hat{f}(k,\omega),\hat{f}^\dagger(k',\omega')]=\delta(k-k')\delta(\omega-\omega'). \tag{3.5}$$

By the substitution of (3.4) into (3.3), the positive frequency part of the vector potential operator can be rewritten as

$$\hat{A}^+(x,\omega)=\left(\frac{\hbar\omega^2\varepsilon_i(\omega)}{\pi\varepsilon_0 S}\right)^{1/2}\int_{-\infty}^{+\infty}dk\frac{1}{k^2c^2-\omega^2\varepsilon(\omega)}\times\hat{f}(k,\omega)\exp(ikx), \tag{3.6}$$

and the form of the complete operator is

$$\hat{A}(x,t)=\left(\frac{\hbar}{2\pi^2\varepsilon_0 S}\right)^{1/2}\int_0^{+\infty}d\omega\int_{-\infty}^{+\infty}dk\times\left\{\frac{\omega\sqrt{\varepsilon_i(\omega)}}{k^2c^2-\omega^2\varepsilon(\omega)}\hat{f}(k,\omega)\exp[-i(\omega t-kx)]+\text{H.c.}\right\}. \tag{3.7}$$

It is not difficult to show that (3.7), with (3.5), is equivalent to expressions given previously [3,4,6].

The required commutation relation (2.13) between the vector potential operator and its conjugate momentum is easily checked. The one-dimensional vector potential derived above is valid for a three-dimensional system in which the noise-current density operator $\hat{J}(x,t)$ acts over an idealized surface which is perpendicular to the direction of propagation. Evidently, if $\hat{J}(x,t)$ is produced by a flat dipole layer, the longitudinal part of the electric field is identically zero, or equivalently, the scalar potential is a constant. The transverse electric field derived from the vector potential (3.7) with the use of (2.11) is then

$$\hat{E}(x,t)=i\left(\frac{\hbar}{2\pi^2\varepsilon_0 S}\right)^{1/2}\int_0^{+\infty}d\omega\int_{-\infty}^{+\infty}dk\left\{\frac{\omega^2\sqrt{\varepsilon_i(\omega)}}{k^2c^2-\omega^2\varepsilon(\omega)}\times\hat{f}(k,\omega)\exp[-i(\omega t-kx)]-\text{H.c.}\right\}, \tag{3.8}$$

and the equal-time commutator (2.13) is

$$[\hat{A}(x,t),-\varepsilon_0\hat{E}(x',t)]=\frac{i\hbar}{2\pi^2 S}\int_{-\infty}^{+\infty}dk\,\exp[ik(x-x')]\times\int_{-\infty}^{+\infty}d\omega\frac{\omega^3\varepsilon_i(\omega)}{|k^2c^2-\omega^2\varepsilon(\omega)|^2}, \tag{3.9}$$

where the commutation relation (3.5) has been used. The analyticity of $\varepsilon(\omega)$ in the upper half of the complex ω plane makes it possible to show that [4]

$$\int_{-\infty}^{+\infty}d\omega\frac{\omega^3\varepsilon_i(\omega)}{|k^2c^2-\omega^2\varepsilon(\omega)|^2}=\pi, \tag{3.10}$$

and insertion of this result into (3.9) then reproduces the canonical form (2.13).

It is worth emphasizing that by introducing an imaginary part for the dielectric function, which reflects the lossy nature of the medium, we lose the usual polariton dispersion relation in which a limited number of discrete frequencies ω are associated with each wave vector k [see Ref. [10] or Eq. (3.23) below]. The above formalism shows that for each k there is now an infinite continuous range of positive frequencies, and thus k and ω must be considered as independent real variables. A polariton dispersion relation can still be defined in this case, but it takes the form of a condition for the frequencies ω at which the field fluctuations with wave vector k have maximum values [11].

The forms of vector potential operator given in (3.3) and (3.6) are useful for displaying the underlying polariton nature of the dielectric excitations, and they are used in Sec. III B to derive the quantization in a lossless dielectric. However, for the dielectric systems with boundaries treated later in the paper, the magnitude of the wave vector varies from one region of space to another. Since the boundary conditions are imposed on the monochromatic field operators at the interfaces, it is the frequency and spatial coordinates that are the fundamental variables. The integrand in (3.3) has simple poles at $k=\pm\omega n(\omega)/c$, and, with the noise operator Fourier transformed as in (3.1), it is straightforward to perform the contour integration. The complete vector potential operator (3.7) is accordingly replaced by

$$\hat{A}^+(x,t)=\int_0^{+\infty}d\omega\left(\frac{\hbar\eta(\omega)}{4\pi\varepsilon_0 c\omega\varepsilon(\omega)S}\right)^{1/2}\times\{\hat{c}_R(x,\omega)+\hat{c}_L(x,\omega)\}e^{-i\omega t}, \tag{3.11}$$

where the rightwards- and leftwards-traveling polariton annihilation operators are defined by

$$\hat{c}_R(x,\omega)=i\left(\frac{2\omega\kappa(\omega)}{c}\right)^{1/2}\int_{-\infty}^{x}dx'\times\exp\left(\frac{i\omega n(\omega)}{c}(x-x')\right)\hat{f}(x',\omega), \tag{3.12}$$

$$\hat{c}_L(x,\omega)=i\left(\frac{2\omega\kappa(\omega)}{c}\right)^{1/2}\int_x^{+\infty}dx'\times\exp\left(-\frac{i\omega n(\omega)}{c}(x-x')\right)\hat{f}(x',\omega),$$

and the transformed Langevin force operator has the simple boson commutation relation

$$[\hat{f}(x,\omega),\hat{f}^\dagger(x',\omega')]=\delta(x-x')\,\delta(\omega-\omega'). \quad (3.13)$$

The commutation relations of the operators $\hat{c}(x,\omega)$ at different positions are easily obtained from (3.12) as

$$[\hat{c}_R(x,\omega),\hat{c}_R^\dagger(x',\omega')]=[\hat{c}_L(x',\omega'),\hat{c}_L^\dagger(x,\omega)]=\delta(\omega-\omega')\exp\left\{\frac{i\omega\eta(\omega)}{c}(x-x')-\frac{\omega\kappa(\omega)}{c}|x-x'|\right\} \quad (3.14)$$

and

$$[\hat{c}_R(x,\omega),\hat{c}_L^\dagger(x',\omega')]=[\hat{c}_L(x',\omega'),\hat{c}_R^\dagger(x,\omega)]=\delta(\omega-\omega')\,\theta(x-x')\frac{2\kappa(\omega)}{\eta(\omega)}\times\sin\left(\frac{\omega\eta(\omega)}{c}(x-x')\right)\times\exp\left(-\frac{\omega\kappa(\omega)}{c}|x-x'|\right). \quad (3.15)$$

The form of vector potential operator given in (3.11) agrees with previous work [1,2], where it was derived by a Green's-function formalism, using the standard relation

$$\hat{A}^+(x,\omega)=S\int_{-\infty}^{+\infty}dx'\,G(x,x',\omega)\hat{J}^+(x',\omega). \quad (3.16)$$

The Green's function is determined from the Helmholtz equation (2.12) by solution of

$$\left(\frac{\partial^2}{\partial x^2}+\varepsilon(x,\omega)\frac{\omega^2}{c^2}\right)G(x,x',\omega)=-\frac{1}{\varepsilon_0 c^2 S}\,\delta(x-x'). \quad (3.17)$$

The Green's-function formalism is used in subsequent sections to treat dielectric systems with boundaries.

B. Lossless dielectric

The introduction of loss, which gives a complex part to the dielectric function, establishes the noise-current fluctuation and provides the quantization of the field. The imaginary part of the dielectric function plays a key role in this approach, as if the whole formalism would collapse in its absence. However, despite the requirement that a dispersive dielectric must inevitably be lossy, the quantization of the electromagnetic field in a nondissipative medium is a well-known problem in the literature. According to causality considerations, this problem must be considered in principle as a limiting case of the lossy dispersive formalism.

In order to calculate the limit, it is convenient to start from (3.7), and define

$$C_j\hat{c}_j(k)e^{-i\omega_j t}=\lim_{\varepsilon_i(\omega)\to 0}\int_0^{+\infty}d\omega\,\frac{\omega\sqrt{\varepsilon_i(\omega)}}{k^2c^2-\omega^2\varepsilon(\omega)}\times\hat{f}(k,\omega)e^{-i\omega t}, \quad (3.18)$$

where C_j is a constant of proportionality to be determined, ω_j is the frequency of the polariton in branch j, which is assumed to possess a well-defined relation with the wave vector in the lossless dielectric, and $\hat{c}_j(k)$ is the polariton annihilation operator in the dispersive medium. The constant C_j is determined by calculating the commutator of the polariton creation and annihilation operators,

$$|C_j|^2[\hat{c}_j(k),\hat{c}_j^\dagger(k')]=\delta(k-k')\lim_{\varepsilon_i(\omega)\to 0}\int_0^{+\infty}\frac{d\omega}{\omega^2}\times\frac{\varepsilon_i(\omega)}{[\varepsilon_r(\omega)-k^2c^2/\omega^2]^2+\varepsilon_i^2(\omega)}, \quad (3.19)$$

where (3.5) has been used. We assume initially that the vanishing of the imaginary part of the dielectric function is associated with a zero extinction coefficient, so that $\varepsilon_r(\omega)=\eta^2(\omega)$ from (2.1). Taking advantage of a standard representation of the Dirac δ function, we have

$$|C_j|^2[\hat{c}_j(k),\hat{c}_j^\dagger(k')]=\delta(k-k')\int\frac{d\omega}{\omega^2}\,\delta[\eta^2(\omega)-(kc/\omega)^2]. \quad (3.20)$$

The δ function is developed using the identity [12]

$$\delta(f(\omega))=\sum_j\frac{\delta(\omega-\omega_j)}{|f'(\omega_j)|}, \quad (3.21)$$

where ω_j are the roots of the δ-function argument, and the prime denotes differentiation with respect to this argument.

The simplest example of a dielectric function is provided by the Lorentz model of a material with a single resonance [13]. In the limit of zero damping, the dielectric function takes the form

$$\varepsilon(\omega)=\frac{\omega_L^2-\omega^2}{\omega_T^2-\omega^2}+i\pi\frac{\omega_L^2-\omega_T^2}{2\omega_T}\{\delta(\omega-\omega_T)+\delta(\omega+\omega_T)\}, \quad (3.22)$$

where ω_T is the transverse resonance frequency and ω_L is the associated longitudinal frequency. This function conforms to the crossing relation in (2.2), and it is real in the positive frequency range except at the single frequency ω_T. The argument of the δ function in (3.20) thus give the usual polariton dispersion relation [13].

$$k^2=\frac{\omega^2\varepsilon_r(\omega)}{c^2}=\frac{\omega^2\eta^2(\omega)}{c^2}=\frac{\omega^2}{c^2}\frac{\omega_L^2-\omega^2}{\omega_T^2-\omega^2}. \quad (3.23)$$

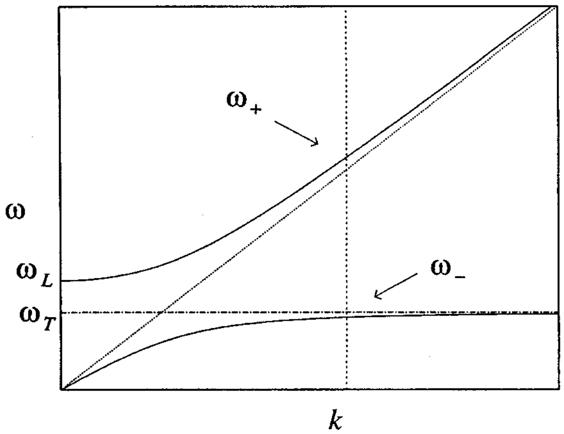

FIG. 1. Polariton dispersion curves, showing the two frequencies ω_+ and ω_- for each value of the wave vector k. The stop band lies between the frequencies ω_T and ω_L.

There are two allowed frequencies for each value of k, and the label j in (3.20) and (3.21) takes the two values $\pm$, corresponding to the two branches of the polariton dispersion curve, which we label by ω_- and ω_+ as in Fig. 1. The limiting procedure in (3.18) has therefore restored the usual polariton dispersion relation, which is a distinguishing feature of the lossless dielectric. Note that there are no propagating excitations for frequencies within the stop band that extends from ω_T to ω_L.

It is straightforward to show that

$$\frac{\partial}{\partial\omega}\left(\eta^2(\omega)-\frac{k^2c^2}{\omega^2}\right)_{\omega=\omega_\pm}=\frac{2c\,\eta(\omega_\pm)}{\omega_\pm v_g(\omega_\pm)},\qquad(3.24)$$

where $v_g(\omega)=\partial\omega/\partial k$ is the usual group velocity. Thus with the assumption of a single transverse resonance, (3.20) can be rewritten as

$$|C_\pm|^2[\hat{c}_\pm(k),\hat{c}_\pm^\dagger(k')]=\pi\delta(k-k')\frac{v_g(\omega_\pm)}{2\omega_\pm\eta^2(\omega_\pm)v_p(\omega_\pm)},\qquad(3.25)$$

where $v_p(\omega)=\omega/k$ is the phase velocity. The polariton creation and destruction operators are assumed to satisfy the independent boson commutation relations

$$[\hat{c}_+(k),\hat{c}_+^\dagger(k')]=[\hat{c}_-(k),\hat{c}_-^\dagger(k')]=\delta(k-k')$$
$$(3.26)$$
$$[\hat{c}_+(k),\hat{c}_-^\dagger(k')]=[\hat{c}_-(k),\hat{c}_+^\dagger(k')]=0,$$

and the constants $C_\pm$ are thus determined from (3.25) as

$$C_\pm=\left(\frac{\pi v_g(\omega_\pm)}{2\omega_\pm\eta^2(\omega_\pm)v_p(\omega_\pm)}\right)^{1/2}.\qquad(3.27)$$

The occurrence of the two sets of dressed operators results from the one-to-two relation between the wave vector and frequency in the dispersive medium, which appears as the two branches of the dispersion curve separated by the stop band between ω_T and ω_L in Fig. 1. On substitution of these results into (3.7), the quantized vector potential operator in a nondissipative dispersive medium can be written as

$$\hat{A}(x,t)=\left(\frac{\hbar}{4\pi\varepsilon_0 S}\right)^{1/2}\int_{-\infty}^{+\infty}dk\times\sum_{j=\pm}\left(\frac{v_g(\omega_j)}{\omega_j\eta^2(\omega_j)v_p(\omega_j)}\right)^{1/2}\times\{\hat{c}_j(k)\exp[-i(\omega t-kx)]+\text{H.c.}\}.\qquad(3.28)$$

This expression agrees with the results of previous work [10] based on the diagonalization of a Hamiltonian for the coupled system of dielectric and electromagnetic field, apart from a phase factor, which can be adjusted in the definition of the constants of proportionality in (3.27).

The quantized field in a lossless dielectric can also be obtained by taking an appropriate limit in the alternative formalism expressed in (3.11). Thus with $\kappa(\omega)$ set equal to zero, the commutation relations in (3.14) simplify while those in (3.15) vanish, and the polariton operators can be expressed in terms of a pair of independent boson operators $\hat{c}_R(\omega)$ and $\hat{c}_L(\omega)$ according to

$$\lim_{\varepsilon_i(\omega)\to 0}\hat{c}_R(x,\omega)=\hat{c}_R(\omega)\exp(i\omega\eta(\omega)/c),$$
$$(3.29)$$
$$\lim_{\varepsilon_i(\omega)\to 0}\hat{c}_L(x,\omega)=\hat{c}_L(\omega)\exp(-i\omega\eta(\omega)/c).$$

The substitution of (3.29) into (3.11) yields

$$\hat{A}(x,t)=\int d\omega\left(\frac{\hbar}{4\pi\varepsilon_0 c\,\omega\,\eta(\omega)S}\right)^{1/2}\times\left\{\hat{c}_R(\omega)\exp\left[-i\omega\left(t-\frac{\eta(\omega)x}{c}\right)\right]+\hat{c}_L(\omega)\exp\left[-i\omega\left(t+\frac{\eta(\omega)x}{c}\right)\right]+\text{H.c.}\right\},$$
$$(3.30)$$

in agreement with previous results [10,14].

The limit $\varepsilon_i(\omega)\to 0$ of a lossless dielectric is assumed above to be equivalent to the limit $\kappa(\omega)\to 0$ of a vanishing extinction coefficient, and this is valid for most frequencies ω. However, there is also the possibility that the limit is equivalent to a vanishing refractive index, $\eta(\omega)\to 0$. For example, the refractive index obtained from (3.23) is defined only for the frequency ranges $\omega<\omega_T$ and $\omega_L<\omega$, and the integration in (3.30) is restricted to these ranges. For the remaining frequencies within the stop band, $\omega_T<\omega<\omega_L$, it is the refractive index $\eta(\omega)$ that vanishes and the extinction coefficient $\kappa(\omega)$ is nonzero, so that the appropriate limits are

$$\eta(\omega)\to 0,$$
$$\varepsilon_i(\omega)\to 0,$$
$$\sqrt{\varepsilon_r(\omega)}\to\pm i\kappa(\omega).\qquad(3.31)$$

The traveling-wave nature in the dielectric is thus replaced by a decay behavior. The quantized field obtained from (3.7) is zero for these frequencies within the stop band, in accordance with the vanishing field correlation function and power spectrum found in (3.31) and (3.34) of Ref. [2] for frequencies where $\eta(\omega)=0$. A similar behavior occurs in the three-dimensional theory of an absorbing dielectric [15–17], and it leads to a vanishing spontaneous emission rate at frequencies within the stop band.

A one-component plasma is a special case of the single-resonance dielectric medium obtained by taking $\omega_T\to 0$ and $\omega_L\to\omega_P$, where ω_P is the plasma frequency. The real part of the plasma dielectric function is accordingly obtained from (3.22) as

$$\varepsilon_r(\omega)=1-\frac{\omega_P^2}{\omega^2}. \tag{3.32}$$

The plasma dielectric function is positive when $\omega>\omega_P$, and in this domain of frequencies the quantized field expression is the same as (3.30), provided that the ω integration covers the frequency range from ω_P to infinity. The dielectric function is negative when $\omega<\omega_P$, and the discussion of the previous paragraph applies in this frequency range.

The expression for the quantized field in homogeneous free space can also be obtained from the above general formalism, provided that a suitable limit is employed. The appropriate limit for this case takes the form

$$C\hat{a}(k)e^{-i\omega_k t}=\lim_{\substack{\varepsilon_i(\omega)\to 0\\ \varepsilon_r(\omega)\to 1}}\times\int_0^{+\infty}d\omega\,\frac{\omega\sqrt{\varepsilon_i(\omega)}}{k^2c^2-\omega^2\varepsilon(\omega)}\hat{f}(k,\omega)e^{-i\omega t}, \tag{3.33}$$

where $\hat{a}(k)$ is the photon annihilation operator. The constant C is determined by calculating the commutator of this annihilation operator with its creation counterpart, and we find

$$C=(\pi/2c|k|)^{1/2}. \tag{3.34}$$

Therefore, by substitution into (3.7), we obtain the vector potential operator

$$\hat{A}(x,t)=\int_{-\infty}^{+\infty}dk\left(\frac{\hbar}{4\pi\varepsilon_0 c|k|S}\right)^{1/2}\{\hat{a}(k)\exp[-i(\omega t-kx)]+\text{H.c.}\}, \tag{3.35}$$

where $\omega=c|k|$. This reproduces the well-known form of the free-space operator [6]. The free-space limit of the form of vector potential operator given in (3.30) is readily found to be

$$\hat{A}^+(x,t)=\int_0^{+\infty}d\omega\left(\frac{\hbar}{4\pi\varepsilon_0 c\omega S}\right)^{1/2}\left\{\hat{a}_R(\omega)\exp\left(\frac{i\omega x}{c}\right)+\hat{a}_L(\omega)\exp\left(-\frac{i\omega x}{c}\right)\right\}e^{-i\omega t}, \tag{3.36}$$

where the rightwards- and leftwards-traveling operators now refer to photons in free space.

IV. TWO ADJACENT SEMI-INFINITE LOSSY DISPERSIVE MEDIA

A. Field quantization

In this section we consider the field quantization in a space that is occupied half and half by two different absorbing media with different dielectric functions. The complete dielectric function is therefore

$$\varepsilon(x,\omega)=\begin{cases}\varepsilon_1(\omega)=n_1^2(\omega)=[\eta_1(\omega)+i\kappa_1(\omega)]^2, & x>0\\ \varepsilon_2(\omega)=n_2^2(\omega)=[\eta_2(\omega)+i\kappa_2(\omega)]^2, & x<0.\end{cases} \tag{4.1}$$

Hereafter in this section we label all quantities with subscript indices 1 and 2, corresponding to the media with dielectric functions $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$. The inhomogeneous nature of the problem requires the imposition of boundary conditions on the spatial mode functions in the two media. The vector potential is determined then by (3.16), with the Green's function obtained by the solution of (3.17), subject to the dielectric function (4.1). Though the calculation is rather lengthy, it is straightforward, and the final expressions for the coordinate-space Green's functions are

$$\begin{aligned}G(x,x',\omega)&=\frac{i}{2\varepsilon_0 c\omega n_1(\omega)S}\left\{R_L(\omega)\exp\left(\frac{i\omega n_1(\omega)(x+x')}{c}\right)+\exp\left(\frac{i\omega n_1(\omega)|x-x'|}{c}\right)\right\} \quad x'>0\\ &=\frac{i}{2\varepsilon_0 c\omega n_2(\omega)S}T_R(\omega)\exp\left(\frac{i\omega[n_1(\omega)x-n_2(\omega)x']}{c}\right) \quad x'<0\end{aligned} \tag{4.2}$$

for the positive-x domain, and

$$\begin{aligned}G(x,x',\omega)&=\frac{i}{2\varepsilon_0 c\omega n_2(\omega)S}\left\{R_R(\omega)\exp\left(-\frac{i\omega n_2(\omega)(x+x')}{c}\right)+\exp\left(\frac{i\omega n_2(\omega)|x-x'|}{c}\right)\right\} \quad x'<0\\ &=\frac{i}{2\varepsilon_0 c\omega n_1(\omega)S}T_L(\omega)\exp\left(-\frac{i\omega[n_2(\omega)x-n_1(\omega)x']}{c}\right) \quad x'>0\end{aligned} \tag{4.3}$$

for the negative-x domain, where $T(\omega)$ and $R(\omega)$ are the usual amplitude transmission and reflection coefficients, respectively, and the subscript indices R and L refer to the light incident on the interface from the right or left. These coefficients are given by

$$R_L(\omega)=-R_R(\omega)=\frac{n_1(\omega)-n_2(\omega)}{n_1(\omega)+n_2(\omega)} \tag{4.4}$$

and

$$\frac{T_L(\omega)}{n_1(\omega)}=\frac{T_R(\omega)}{n_2(\omega)}=\frac{2}{n_1(\omega)+n_2(\omega)}. \tag{4.5}$$

The two kinds of terms in the large brackets of the Green's functions (4.2) and (4.3) for the domains $x,x'>0$ and $x,x'<0$, respectively, are typical of the interface system. Thus the first terms correspond to communication between points x and x' via reflection in the interface, while the second terms correspond to direct communication between the two points. The reflection terms tend to zero when the observation point is far enough from the interface, while the direct term reproduces the form for an infinite homogeneous dielectric.

It is convenient to retain form (3.11) for the positive-frequency part of the complete quantized vector potential. This is now written as

$$\hat{A}_\alpha^+(x,t)=\int_0^{+\infty}d\omega\left(\frac{\hbar\,\eta_\alpha(\omega)}{4\pi\varepsilon_0 c\,\omega\varepsilon_\alpha(\omega)S}\right)^{1/2}\times\{\hat{c}_{\alpha R}(x,\omega)+\hat{c}_{\alpha L}(x,\omega)\}e^{-i\omega t}, \tag{4.6}$$

where $\alpha=1$ and 2 refers to quantities in media 1 and 2. The complete expressions for the operators $\hat{c}_{1L}(x,\omega)$ and $\hat{c}_{1R}(x,\omega)$ in the positive-x domain are determined by the insertion of (4.2) into (3.16) as

$$\hat{c}_{1L}(x,\omega)=i\left(\frac{2\omega\kappa_1(\omega)}{c}\right)^{1/2}\int_x^{+\infty}dx'\times\exp\left(-\frac{i\omega n_1(\omega)}{c}(x-x')\right)\hat{f}(x',\omega) \tag{4.7}$$

and

$$\hat{c}_{1R}(x,\omega)=\left\{i\left(\frac{2\omega\,\eta_2(\omega)\kappa_2(\omega)}{c\,\eta_1(\omega)}\right)^{1/2}\frac{n_1(\omega)}{n_2(\omega)}T_R(\omega)\int_{-\infty}^0 dx'\exp\left(-\frac{i\omega n_2(\omega)x'}{c}\right)\hat{f}(x',\omega)+i\left(\frac{2\omega\kappa_1(\omega)}{c}\right)^{1/2}\times\left[R_L(\omega)\int_0^{+\infty}dx'\exp\left(\frac{i\omega n_1(\omega)x'}{c}\right)\hat{f}(x',\omega)+\int_0^x dx'\exp\left(-\frac{i\omega n_1(\omega)x'}{c}\right)\hat{f}(x',\omega)\right]\right\}\exp\left(\frac{i\omega n_1(\omega)x}{c}\right), \tag{4.8}$$

where the modified Langevin force operators obey the commutation relation (3.13) for all x'. The contribution in $\hat{c}_{1L}(x,\omega)$, which represents leftwards-traveling noise in dielectric 1 is unaffected by the presence of the interface at $x=0$, and its form in the same as (3.12). The contribution in $\hat{c}_{1R}(x,\omega)$, which represents rightwards-traveling noise in dielectric 1, has transmitted noise from medium 2 into medium 1, reflected noise from medium 1, and rightwards-traveling noise generated within medium 1. The later expression reduces to the form (3.12) obtained for the infinite homogeneous dielectric when $\omega\kappa_1(\omega)x/c\gg 1$.

A similar calculation can be performed to obtain the vector potential operator (4.6) in the domain $x<0$, by the substitution of (4.3) into the (3.16). The complete expressions for the operators $\hat{c}_{2R}(x,\omega)$ and $\hat{c}_{2L}(x,\omega)$ are the same as (4.7) and (4.8) with the changes $1\leftrightarrow 2$, $L\leftrightarrow R$, $x\to -x$, and $x'\to -x'$ [except in $\hat{f}(x',\omega)$]. As expected, $\hat{c}_{2R}(x,\omega)$ is unaffected by the presence of the interface, while there are additional terms in $\hat{c}_{2L}(x,\omega)$ that result from the presence of the interface at $x=0$. The notation for these rightwards- and leftwards-traveling operators is illustrated in Fig. 2.

The evaluation of the canonical commutation relation (2.13) is more complicated in this system as the field operators have more complicated structures compared to the infinite homogeneous case. Though there are two regions to consider, on taking symmetry considerations into account it is sufficient to check the required equal-time commutation relation in the domain $x>0$. The commutator of the operators $\hat{c}_{1R}(x,\omega)$ and $\hat{c}_{1R}^\dagger(x,\omega)$ is obtained from (4.8), on employing (3.13), with the result

$$[\hat{c}_{1R}(x,\omega),\hat{c}_{1R}^\dagger(x',\omega')]=\delta(\omega-\omega')\left\{\exp\left(\frac{i\omega\,\eta_1(\omega)(x-x')-\omega\kappa_1(\omega)|x-x'|}{c}\right)-i\frac{\kappa_1(\omega)}{\eta_1(\omega)}\left[R_L(\omega)\exp\left(\frac{i\omega n_1(\omega)(x+x')}{c}\right)-\text{c.c.}\right]\right\}. \tag{4.9}$$

The corresponding relation for $\hat{c}_{1L}(x,\omega)$ is the same as in (3.15). The commutation relations between the leftwards and rightwards annihilation and creation operators are

$$[\hat{c}_{1R}(x,\omega),\hat{c}_{1L}^{\dagger}(x',\omega')]=[\hat{c}_{1L}(x',\omega'),\hat{c}_{1R}^{\dagger}(x,\omega)]^{*}$$
$$=\delta(\omega-\omega')\left\{R_L(\omega)\exp\left(\frac{i\omega n_1(\omega)(x+x')}{c}\right)+\theta(x-x')\frac{2\kappa_1(\omega)}{\eta_1(\omega)}\sin\left(\frac{\omega\eta_1(\omega)(x-x')}{c}\right)\right.$$
$$\left.\times\exp\left(-\frac{\omega\kappa_1(\omega)(x+x')}{c}\right)\right\}. \tag{4.10}$$

The canonical commutation relation (2.13) can now be evaluated with use of expression (4.6) for the vector potential operator, together with the commutators given above. It is straightforward to show that

$$[\hat{A}(x,t),-\varepsilon_0\hat{E}(x',t)]=\frac{i\hbar}{2\pi cS}\int_{-\infty}^{+\infty}d\omega\left\{R_L(\omega)\right.$$
$$\times\frac{\exp[i\omega n_1(\omega)(x+x')/c]}{n_1(\omega)}$$
$$\left.+\frac{\exp[i\omega n_1(\omega)|x-x'|/c]}{n_1(\omega)}\right\}$$
$$=\frac{2i\varepsilon_0\hbar}{\pi}\int_0^{\infty}d\omega\,\omega\;\mathrm{Im}G(x,x',\omega), \tag{4.11}$$

where (2.2) and (4.2) have been used, and the general relation between the canonical commutator and the Green's function is discussed in the Appendix. The integrals can be evaluated by the same method as used previously, and indeed the second term in the large bracket of (4.11) reproduces the desired result for the canonical commutator. For the first term in the large bracket, we use the property that $R_L(\omega)$ defined in (4.4) is itself a linear-response function, which determines the reflected field produced by an incident field. Thus causality requires that the function can have no poles in the upper half of the complex ω plane [18]. Furthermore, in view of (2.1) and (2.3), $R_L(\omega)$ has the property that

$$R_L(\omega)\to 0 \quad \text{for} \quad \omega\to\infty. \tag{4.12}$$

Thus, when the contribution of the first term in the large brackets is evaluated by conversion to polar coordinates, the integral vanishes on the semicircle at infinity, and this term is zero. The canonical commutator thus has the expected form (2.13). When one position coordinate lies in medium 1 and the other in medium 2, a similar calculation shows that the canonical commutator vanishes.

The formalism of the present section can be developed, on employing an appropriate limit to the general form (3.11) of the vector potential operator and the destruction operators (4.7) and (4.8), to derive the quantized field in a space where the half $x<0$ is occupied by free space and the half $x>0$ by a lossy dielectric medium. The leftwards-traveling annihilation operator in domain $x<0$ can be considered in this case as the sum of two terms—the complementary function and the particular integral parts—determined by the usual boundary conditions and the appropriate Green's function. This agrees with previous results [2].

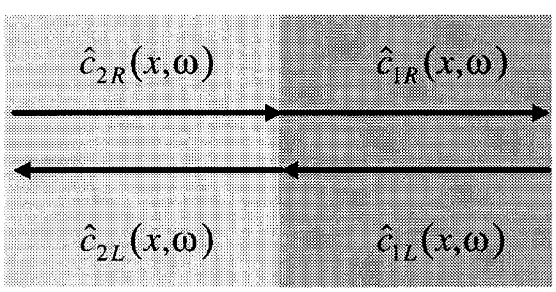

FIG. 2. Representation of the notation for the annihilation operators used in the definition of the vector potential operator for two adjacent dielectrics.

B. Vacuum field fluctuations

The contribution to the electromagnetic field fluctuations from the modes that propagate perpendicular to the interface is obtained with the use of the associated Green's functions in the domains $x,x'>0$ and $x,x'<0$. The value of the correlation function at a common spatial position determines the power spectrum $S(x,\omega)$ of the field fluctuations according to

$$\langle 0|\hat{E}(x,\omega)\hat{E}(x,\omega')|0\rangle=S(x,\omega)\delta(\omega-\omega'). \tag{4.13}$$

The power spectrum needed for substitution in (4.13) is determined by the fluctuation-dissipation theorem as

$$S(x,\omega)=2\hbar\omega^2\mathrm{Im}G(x,x,\omega), \tag{4.14}$$

and insertion of the Green's functions from (4.2) and (4.3) gives

$$S(x,\omega)=\begin{cases}\dfrac{\hbar\omega}{\varepsilon_0 cS}\mathrm{Re}\left\{\dfrac{1}{n_2(\omega)}\left[R_R(\omega)\exp\left(\dfrac{-2i\omega n_2(\omega)x}{c}\right)+1\right]\right\}, & x<0\\ \dfrac{\hbar\omega}{\varepsilon_0 cS}\mathrm{Re}\left\{\dfrac{1}{n_1(\omega)}\left[R_L(\omega)\exp\left(\dfrac{+2i\omega n_1(\omega)x}{c}\right)+1\right]\right\}, & x>0.\end{cases}\tag{4.15}$$

Alternatively, the same expressions for the power spectra can be obtained without use of the fluctuation-dissipation theorem by substitution of the electric-field operators given by (2.11) and (4.6) into (4.13). The result for an infinite homogeneous medium is reproduced by (4.15) far away from the dielectric interface at $x=0$, where $\omega\kappa_\alpha(\omega)|x|/c \gg 1$, with $\alpha=1$ and 2. The continuity of the electric field through the interface requires the fluctuations to be continuous at $x=0$, where both expressions in (4.15) give

$$S(0,\omega)=\frac{2\hbar\omega}{\varepsilon_0 cS}\,\frac{\eta_1(\omega)+\eta_2(\omega)}{|n_1(\omega)+n_2(\omega)|^2}.\tag{4.16}$$

This result is in agreement with the infinite homogeneous case [2] when $n_1(\omega)=n_2(\omega)$.

The spatial variation of the fluctuations is shown in Fig. 3 for representative values of the refractive indices and extinction coefficients. The magnitude of the fluctuation is normalized to its free-space value. The oscillations far from the interface decay to the values for the homogeneous dielectrics, and the influence of the interface is evanescent. It should be emphasized that the fluctuations illustrated are only those associated with the modes that propagate perpendicular to the interface.

The vacuum field fluctuation spectrum in a lossy dielectric at an interface with free space reduces to an expression derived in [2], and the spatial dependence is illustrated in Fig. 2 of this reference. However, here we consider the additional special case where, in addition to the assumption of free space for dielectric 2, we also assume that dielectric 1 is lossless and that the fluctuation frequency ω of interest lies

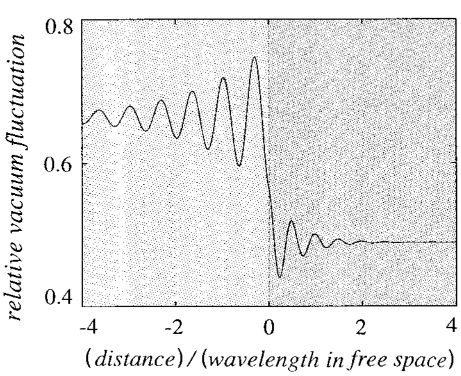

FIG. 3. Spatial variation of the spectrum $S(x,\omega)$ of the vacuum electric-field fluctuations, normalized to the free-space spectrum $\hbar\omega/\varepsilon_0 cS$, for the complex refractive indices $n_1=2.0+i0.3$ and $n_2=1.5+i0.1$.

within the stop band between ω_T and ω_L. The properties of the dielectric function in this case are specified by (3.31) and the power spectrum (4.15) takes the form

$$S(x,\omega)=\begin{cases}\dfrac{2\hbar\omega}{\varepsilon_0 cS}\,\dfrac{[\cos(\omega x/c)-\kappa(\omega)\sin(\omega x/c)]^2}{1+\kappa^2(\omega)}, & x<0\\ \dfrac{2\hbar\omega}{\varepsilon_0 cS}\,\dfrac{\exp[-2\omega\kappa(\omega)x/c]}{1+\kappa^2(\omega)}, & x>0.\end{cases}\tag{4.17}$$

The fluctuations therefore vanish deep inside the lossless dielectric for frequencies within the stop band, in accordance with the discussion following (3.31). The nonvanishing spectrum close to the interface is caused by the transmission of fluctuations generated in the free-space region at $x<0$. The fluctuations vanish everywhere in the dielectric in the limit $\kappa(\omega)\to\infty$, and they acquire a standing-wave form in the free-space region.

V. ONE-DIMENSIONAL CAVITY

A. Field quantization

The procedure of Sec. IV, for the field quantization in a space that is occupied half and half by two different lossy dielectric media, can also be used to quantize the electromagnetic field in a cavity. To take advantage of the symmetry of the problem, we define the dielectric function as

$$\varepsilon(x,\omega)=\begin{cases}\varepsilon(\omega)=n^2(\omega)=[\eta(\omega)+i\kappa(\omega)]^2, & |x|>l\\ 1, & -l<x<+l.\end{cases}\tag{5.1}$$

Though the two semi-infinite dielectric media on either side of the cavity are the same, it is convenient to label them by different indices to avoid any notational ambiguity. The notation for this geometry is illustrated in Fig. 4. The subscripts P and N refer to the dielectric media located on the positive and negative domains of x.

Since the leftwards field in the positive-x domain, and rightwards field in the negative-x domain are both incoming fields inside the dielectric media, there is no need for the complementary function parts in this system and the particular integral parts give the complete field expression [2]. The appropriate Green's function for the domain $x>l$ is

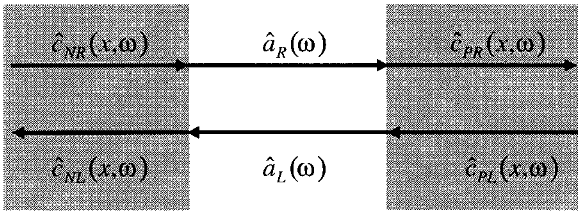

FIG. 4. Representation of the notation for the annihilation operators used in the definition of the vector potential operator for the cavity system.

$$G(x,x',\omega)=\frac{i}{2\varepsilon_0 c\omega n(\omega)S}\left\{R_C(\omega)\exp\left(\frac{i\omega n(\omega)(x+x')}{c}\right)+\exp\left(\frac{i\omega n(\omega)|x-x'|}{c}\right)\right\}\quad x'>l$$
$$=\frac{i}{2\varepsilon_0 c\omega n(\omega)S}T_C(\omega)\exp\left(\frac{i\omega n(\omega)(x-x')}{c}\right)\quad x'<-l, \tag{5.2}$$

where $T_C(\omega)$ and $R_C(\omega)$ are the transmission and reflection coefficients of the cavity. It is evident that the symmetrical nature of the cavity requires these coefficients to be the same for light incident from the left and right toward the cavity, and their explicit forms are

$$T_C(\omega)=\frac{4n(\omega)}{D_C(\omega)}\exp\left(-\frac{2i\omega[n(\omega)-1]l}{c}\right) \tag{5.3}$$

and

$$R_C(\omega)=\frac{n^2(\omega)-1}{D_C(\omega)}\exp\left(-\frac{2i\omega n(\omega)l}{c}\right)\left[1-\exp\left(\frac{4i\omega l}{c}\right)\right], \tag{5.4}$$

where the denominator $D_C(\omega)$ is

$$D_C(\omega)=[n(\omega)+1]^2-[n(\omega)-1]^2\exp(4i\omega l/c). \tag{5.5}$$

The Green's function (5.2) has the same structure as (4.2), but with the particular forms of the transmission and reflection coefficients associated with the geometry of the problem. In the limit $l\to 0$, (5.2) reproduces the infinite homogeneous Green's function rather than (4.2), because the two semi-infinite media involved in this problem are the same.

As in the semi-infinite case, it is convenient to retain the form (4.6) for the vector potential inside the material medium. The positive frequency part of the vector potential operator is determined by the substitution of (5.2) and (3.16). The contribution in $\hat{c}_{\rm PL}(x,\omega)$, which represents the leftwards-traveling annihilation operator in the dielectric located at the positive-x domain, has a form identical to (3.12),

$$\hat{c}_{\rm PL}(x,\omega)=i\left(\frac{2\omega\kappa(\omega)}{c}\right)^{1/2}\int_x^{+\infty}dx'\times\exp\left(-\frac{i\omega n(\omega)}{c}(x-x')\right)\hat{f}(x',\omega). \tag{5.6}$$

The contribution in $\hat{c}_{\rm PR}(x,\omega)$, which represents the rightwards-traveling annihilation operator in the dielectric has a structure similar to that of (4.8),

$$\hat{c}_{\rm PR}(x,\omega)=\left\{i\left(\frac{2\omega\kappa(\omega)}{c}\right)^{1/2}\left[T_C(\omega)\int_{-\infty}^{-l}dx'\times\exp\left(-\frac{i\omega n(\omega)x'}{c}\right)\hat{f}(x',\omega)+R_C(\omega)\times\int_{+l}^{+\infty}dx'\exp\left(\frac{i\omega n(\omega)x'}{c}\right)\hat{f}(x',\omega)+\int_{+l}^{x}dx'\exp\left(-\frac{i\omega n(\omega)x'}{c}\right)\hat{f}(x',\omega)\right]\right\}\times\exp\left(\frac{i\omega n(\omega)x}{c}\right). \tag{5.7}$$

The commutation relation of the operator $\hat{c}_{\rm PL}(x,\omega)$ with its creation counterpart is the same as in (3.14). The commutation relation of the operators $\hat{c}_{\rm PR}(x,\omega)$ and $\hat{c}^\dagger_{\rm PR}(x,\omega)$ is obtained from (5.7), by using (5.3) and (5.4), and the result is identical to (4.9) when the subscripts are removed from the complex refractive index and the reflection coefficient is replaced by its cavity counterpart (5.4). The commutation relation between the leftwards- and rightwards-traveling annihilation and creation operators is likewise identical to (4.10). The canonical commutation relation (2.13) can be calculated therefore with the use of the expression (4.6) for the vector potential operator. The result is clearly the same as (4.11), and on taking advantage of the nature of $R_C(\omega)$ as a linear-response function with the same analytic properties as $R_L(\omega)$, one finds the required equal-time canonical commutation relation. Very similar calculations apply to the dielectric located in the negative-x domain. The structure of this calculation is in fact the same as that given for medium 2 in Sec. IV, and there is no need to repeat the details.

In the domain $-l<x<+l$, the appropriate Green's function is

$$G(x,x',\omega)=\frac{i}{2\varepsilon_0 c\omega S}\left\{V_C(\omega)\exp\left(\pm\frac{i\omega[x-n(\omega)x']}{c}\right)+W_C(\omega)\exp\left(\mp\frac{i\omega[x+n(\omega)x']}{c}\right)\right\}, \tag{5.8}$$

where the upper and lower signs refer to the ranges of x' on the left and right of the cavity, respectively,

$$V_C(\omega)=\frac{2[n(\omega)+1]}{D_C(\omega)}\exp\left(-\frac{i\omega[n(\omega)-1]l}{c}\right),$$
$$W_C(\omega)=-\frac{2[n(\omega)-1]}{D_C(\omega)}\exp\left(-\frac{i\omega[n(\omega)-3]l}{c}\right), \tag{5.9}$$

and $D_C(\omega)$ is given by (5.5).

The positive-frequency part of the vector potential operator inside the cavity is determined by the substitution of (5.8) into (3.16). On using the conventional form (3.36) of the vector potential operator in free space and employing the modified Langevin force operator, the complete expressions for the operators $\hat{a}_R(\omega)$ and $\hat{a}_L(\omega)$ are

$$\hat{a}_R(\omega)=i\left(\frac{2\omega\eta(\omega)\kappa(\omega)}{c}\right)^{1/2}\left\{V_C(\omega)\int_{-\infty}^{-l}dx' \times\exp\left(-\frac{i\omega n(\omega)x'}{c}\right)\hat{f}(x',\omega) + W_C(\omega)\int_{+l}^{+\infty}dx'\exp\left(\frac{i\omega n(\omega)x'}{c}\right)\hat{f}(x',\omega)\right\} \tag{5.10}$$

and

$$\hat{a}_L(\omega)=i\left(\frac{2\omega\eta(\omega)\kappa(\omega)}{c}\right)^{1/2}\left\{W_C(\omega)\int_{-\infty}^{-l}dx' \times\exp\left(-\frac{i\omega n(\omega)x'}{c}\right)\hat{f}(x',\omega) + V_C(\omega)\int_{+l}^{+\infty}dx'\exp\left(\frac{i\omega n(\omega)x'}{c}\right)\hat{f}(x',\omega)\right\}. \tag{5.11}$$

It is not difficult, with the help of (5.9) to (5.11), to show that the rightwards- and leftwards-traveling photon operators possess the commutation relation

$$[\hat{a}_I(\omega),\hat{a}_I^\dagger(\omega')]=\delta(\omega-\omega')\left\{1+\left[\frac{[n(\omega)-1]^2}{D_C(\omega)}\exp\left(\frac{4i\omega l}{c}\right)+\text{c.c.}\right]\right\},\quad I=R,L, \tag{5.12}$$

and the commutation relations between the two operators are

$$[\hat{a}_R(\omega),\hat{a}_L^\dagger(\omega')]=[\hat{a}_L(\omega),\hat{a}_R^\dagger(\omega')] =\delta(\omega-\omega')\left\{-\left[\frac{[n^2(\omega)-1]}{D_C(\omega)} \times\exp\left(\frac{2i\omega l}{c}\right)+\text{c.c.}\right]\right\}. \tag{5.13}$$

The verification of the canonical commutation relation (2.13) inside the cavity may seem to be difficult compared to the calculation inside the dielectric media, but this is not in fact the case. The nature of $R_C(\omega)$ as a linear-response function ensures that the expression $D_C(\omega)$ has no zeros in the upper half of the complex ω plane, and therefore all expressions in which $D_C(\omega)$ is in the denominator can have no poles in the upper half of the complex ω plane. On changing the variable of integration from ω to $-\omega$ in those terms in which $D_C^*(\omega)$ is in the denominator, it is not difficult to show that the whole integral can be converted to the range $-\infty<\omega<+\infty$, with an integrand which has no poles in the upper half of the complex ω plane. The latter integral along the real ω axis is therefore the negative of the integral around the semicircle at infinity in the upper half-plane. The final result is obtained on taking (2.3) into account. It is straightforward to show that the only nonvanishing term in the integral is of the form

$$[\hat{A}(x,t),-\varepsilon_0\hat{E}(x',t)]=\frac{i\hbar}{2\pi cS}\int_{-\infty}^{+\infty}d\omega\exp\left(\frac{i\omega(x-x')}{c}\right) =\frac{i\hbar}{S}\delta(x-x'), \tag{5.14}$$

and that the canonical commutation relation is again verified.

Some results for the cavity system considered here have been obtained independently using similar methods [19], and using an explicit diagonalization of a Hamiltonian for the electromagnetic and dielectric polarization fields [20]. The diagonalization method generalizes earlier work on the infinite dielectric [3,4], but the forms of the results make a direct comparison of the quantized field expressions difficult. A different cavity model, consisting of absorbing dielectric mirrors of infinitesimal thickness, has also been treated [21]; the destruction and creation operator commutators were derived, and their consistency with the canonical commutation relation (2.13) was demonstrated.

B. Vacuum field fluctuations

To employ the fluctuation-dissipation theorem and Kubo's formula to calculate the vacuum field fluctuation in the cavity, one needs the explicit form of the Green's function when both the source and observation points x and x' are in each of the three domains illustrated in Fig. 4. Expression (5.2), together with the symmetrical configuration of the cavity, shows that the appropriate forms of the Green's functions in the dielectric media on either side are

$$G(x,x',\omega)=\frac{i}{2\varepsilon_0 c\omega n(\omega)S}\left\{R_C(\omega)\exp\left(\pm\frac{i\omega n(\omega)(x+x')}{c}\right)+\exp\left(\frac{i\omega n(\omega)|x-x'|}{c}\right)\right\}, \tag{5.15}$$

where the plus and minus signs refer to the media on the right and left of the cavity, respectively. The form of the Green's function, when both x and x' are inside the cavity, is

$$G(x,x',\omega)=\frac{i}{2\varepsilon_0 c\omega S}\left\{\frac{1-n^2(\omega)}{D_C(\omega)}\left[\exp\left(\frac{i\omega(2l+x+x')}{c}\right)+\exp\left(\frac{i\omega(2l-x-x')}{c}\right)\right] + \frac{[1-n(\omega)]^2}{D_C(\omega)}\left[\exp\left(\frac{i\omega(4l+x-x')}{c}\right)+\exp\left(\frac{i\omega(4l-x+x')}{c}\right)\right]+\exp\left(\frac{i\omega|x-x'|}{c}\right)\right\}. \tag{5.16}$$

The power spectrum obtained from (4.14) is rather complicated, on account of the complicated form of the Green's function, and we do not write down the general expressions.

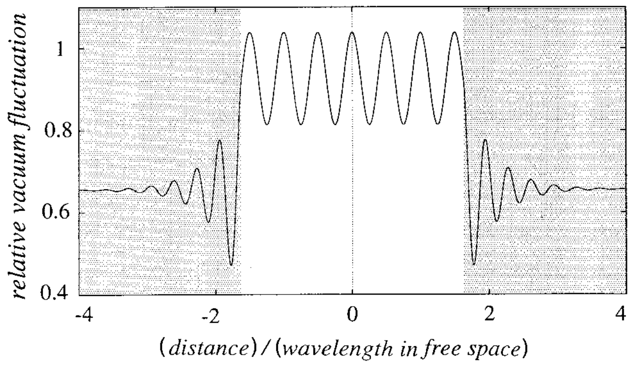

FIG. 5. Spatial variation of the spectrum $S(x,\omega)$ of vacuum electric-field fluctuations in the cavity system, normalized to their free-space value, for $2\omega l/c=6.5\pi$ and $n=1.5+i0.02$.

The typical form of spatial variation of the field fluctuations is shown in Fig. 5. The oscillations inside the cavity have the partial standing-wave character of the field excitation, while on either side of the cavity they show oscillatory decay to the value for the infinite homogeneous dielectric. Each half of this figure resembles the field fluctuation at the interface between an absorbing dielectric and free space, shown in Fig. 2 of Ref. [2]. However, the cavity geometry also displays quite a rich variety of distinctive spatial behaviors that occur for certain specific frequencies.

Consider first the transmission resonances of the cavity, which can be identified from expression (5.4). It is clear that the cavity reflection coefficient $R_C(\omega)$ vanishes at the frequencies $\omega=\omega_m$, where ω_m is defined by

$$2\omega_m l/c=m\pi, \quad m=0,1,2\ldots. \tag{5.17}$$

Therefore at these frequencies, it is clear from the form of (5.15) that the vacuum fluctuations on both sides of the cavity have no spatial-oscillatory character, and their values are equal to that for the infinite homogeneous dielectric. This behavior is demonstrated in Fig. 6 for $m=8$, and the parameter values given in the figure caption. ω_m are the resonant frequencies of a cavity with perfectly reflecting mirrors, for

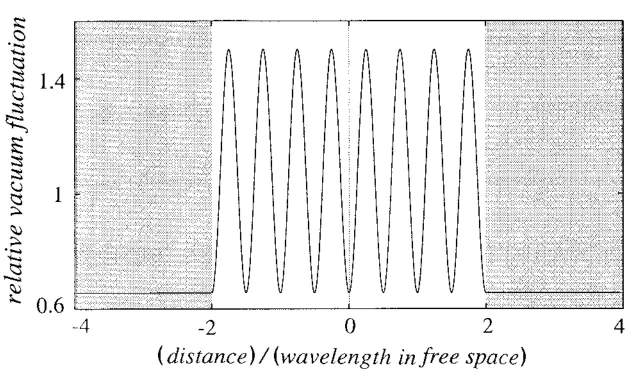

FIG. 6. Spatial variation of the spectrum $S(x,\omega)$ of vacuum electric-field fluctuations in the cavity system, normalized to their free-space value, for $2\omega l/c=8\pi$ and $n=1.5+i0.02$.

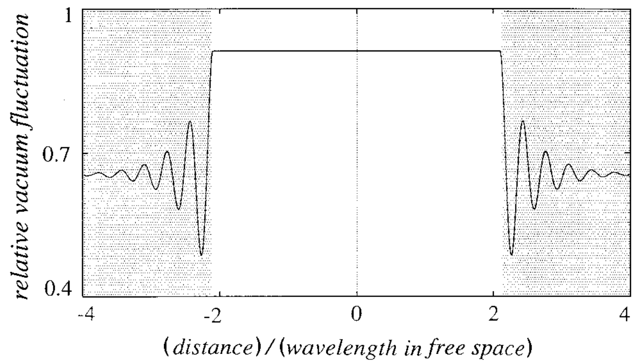

FIG. 7. Spatial variation of the spectrum $S(x,\omega)$ of vacuum electric-field fluctuations in the cavity system, normalized to their free-space value, for $2\omega l/c=8\pi+\tan^{-1}31=8.49\pi$ and $n=1.5+i0.02$.

which the cavity length exactly equals an integer number of half-wavelengths. They can be visualized as the frequencies for which there is no interference term between the rightwards- and leftwards-traveling field operators outside the cavity. A spatial variation similar to that shown in Fig. 6 occurs in the spontaneous radiative decay rate of an atom placed in the cavity when the emission is restricted to modes that propagate perpendicular to the mirrors [20].

There are analogous frequencies for which the vacuum fluctuations inside the cavity have no spatial-oscillatory character. These are obtained by setting the oscillatory terms in the power spectrum given by (4.14) and (5.16) equal to zero, and the detailed calculation shows that the required frequencies satisfy

$$\frac{2\omega l}{c}=m\pi+\tan^{-1}\left(\frac{|n(\omega)|^2-1}{2\kappa(\omega)}\right), \quad m=0,1,2,\ldots. \tag{5.18}$$

These are the frequencies for which there is no interference term between the rightwards- and leftwards-traveling field operators inside the cavity. For high-reflectivity dielectric materials with $\kappa(\omega)\gg\eta(\omega)$ and $\kappa(\omega)\gg1$, the inverse tangent in (5.18) tends to $\pi/2$, and the frequencies defined by this equation lie midway between the resonant frequencies ω_m defined by (5.17). The same values of the frequency occur in the limit of low-loss dielectric media with $\kappa(\omega)\ll|n(\omega)|^2-1$ [20]. Figure 7 shows the spatial behavior of the power spectrum for the quite low-loss parameter values given in the caption.

The properties of the system approach those of a high-finesse Fabry-Pérot cavity in the limit of high-reflectivity dielectric mirrors. A mirror reflection coefficient of the form $\xi e^{i\delta}$, for example with $\xi=0.99$ and $\delta=0.1$ [22], corresponds to the surface of a dielectric medium whose refractive index and extinction coefficient are $\eta=2$ and $\kappa=20$, respectively. In the limit of $\kappa(\omega)\gg\eta(\omega)$, it is evident from (5.15) that the Green's-function expression on either side of the cavity falls rapidly to zero, with very small vacuum fluctuations inside the material media. This corresponds to the quenching of the electromagnetic field in a conducting medium. In this limit, the Green's function inside the cavity given by (5.16) shows a series of very sharp peaks centered on frequencies ω'_m, slightly below the ω_m defined in (5.17). With the use of a suitable expansion in powers of $1/\kappa(\omega)$ around the peak frequencies, the power spectrum can be written as

$$S(x,\omega)=\frac{\hbar\omega\eta(\omega)}{\varepsilon_0 cS\kappa^2(\omega)}\left\{\sum_{m\ \mathrm{odd}}\frac{\cos^2\left(\frac{m\pi x}{2l}\right)}{\frac{(\omega-\omega'_m)^2l^2}{c^2}+\frac{\eta^2(\omega)}{\kappa^4(\omega)}}+\sum_{m\ \mathrm{even}}\frac{\sin^2\left(\frac{m\pi x}{2l}\right)}{\frac{(\omega-\omega'_m)^2l^2}{c^2}+\frac{\eta^2(\omega)}{\kappa^4(\omega)}}\right\}, \tag{5.19}$$

where

$$\omega'_m=\omega_m-\frac{c}{\kappa(\omega_m)l}. \tag{5.20}$$

The spectrum therefore consists of a series of equally spaced Lorentzians of maximum height

$$S(x_m,\omega_m)=\frac{\hbar\omega_m\kappa^2(\omega_m)}{\varepsilon_0 cS\eta(\omega_m)} \tag{5.21}$$

at the positions

$$x_m=\begin{cases}0,\pm\frac{2l}{m},\ldots,\pm\frac{(m-1)l}{m} & \text{for } m \text{ odd}\\ \pm\frac{l}{m},\pm\frac{3l}{m},\ldots,\pm\frac{(m-1)l}{m} & \text{for } m \text{ even}\end{cases} \tag{5.22}$$

of the antinodes in the cavity. The full widths of the Lorentzians at half maximum height are

$$\Delta\omega=\frac{2c\eta(\omega_m)}{l\kappa^2(\omega_m)}. \tag{5.23}$$

Note that these results are valid only to leading order in $1/\kappa(\omega)$, and that the correction of this order in the positions of the antinodes, obtained from (5.20), has been omitted. The variations of the field fluctuations for a cavity with $\eta=2$ and $\kappa=20$ are illustrated in Figs. 8 and 9. Figure 8 shows the spectrum after averaging over the spatial position x, with sharp Lorenzian contributions centered on the peak frequencies ω'_m given by (5.20). Figure 9 shows the fluctuation as a function of position for a fixed frequency that coincides with one of the ω'_m. It is seen that the field fluctuation vanishes on the walls of the cavity. These field fluctuations are clearly associated with the usual standing-wave mode of a high-finesse Fabry-Pérot cavity.

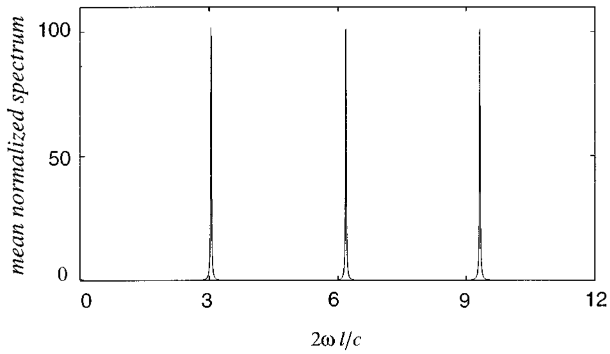

FIG. 8. Normalized spectrum of vacuum electric-field fluctuations averaged over position for the cavity system with $n=2+i20$.

VI. CONCLUSIONS

The results derived in this paper amplify and extend the Langevin noise current method for electromagnetic field quantization in absorbing dielectrics developed previously [1,2]. The first aim has been to explore the properties of the basic noise sources used in the quantization scheme, and to show how the quantized field operators conform with earlier derivations for special cases, particularly the results found in the absence of dielectric absorption. Thus the position-momentum commutator for the underlying charged particles that generate the quantum noise is determined in (2.23), which establishes the role of the imaginary part of the macroscopic dielectric function in controlling the magnitude of the noise currents. Alternative general forms (3.7), (3.11), and (3.16) of the vector potential operator in an infinite homogeneous dielectric medium derived from the Langevin force approach have been related and compared to previous results obtained by a microscopic canonical quantization scheme [3,4]. For the quantization in a lossless dielectric, we have used the simple example of a single-resonance dielectric function given by (3.22) in the limit of zero damping. It has been shown that the polariton form of field operator is recovered, in complete agreement with earlier work [5,10]. The lossless dielectric possesses an absolute stop band between its transverse and longitudinal frequencies, where the effects of the noise currents are completely quenched and the

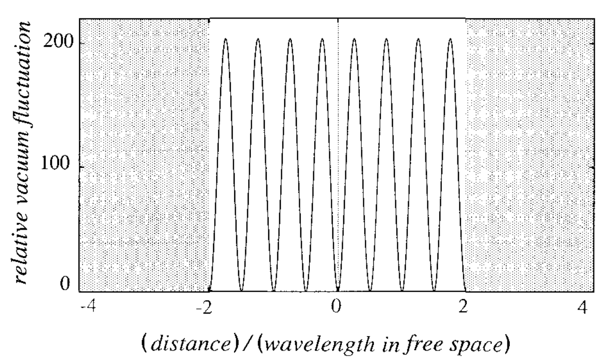

FIG. 9. Spatial variation of the normalized spectrum of vacuum electric-field fluctuations for $2\omega l/c=8\pi-0.1$ in the same cavity system as Fig. 8.

field operators vanish. The field-operator spectrum is therefore restricted to the frequency ranges that lie above and below the stop band.

The method of field quantization used here applies generally to one-dimensional propagation through a series of absorbing dielectric materials for which $\varepsilon(x,\omega)$ is a piecewise constant, with abrupt changes occurring at the interfaces between different media. The method was applied in Refs. [1,2] to the interface between free space and a semi-infinite dielectric and to a dielectric slab surrounded by free space. The second aim of the present paper has been to apply the method to further examples of dielectric configuration. The vector potential Green's functions and the field operators have accordingly been determined for the system of two different semi-infinite absorbing dielectric materials in contact, and for the free-space optical cavity formed from two separated semi-infinite dielectric samples. It has been verified that the canonical commutation relation is satisfied by conjugate field operators for both systems, and the operators have been used to determine the spectra and the spatial distributions of the vacuum field fluctuations. For two dielectrics in contact, the vacuum field fluctuations show characteristic spatial oscillations that have a maximum amplitude at the interface and decay to the values for the individual bulk dielectrics on either side. For the free-space cavity, the vacuum field fluctuations show a variety of behaviors for different choices of optical frequency. Thus in general there are spatial oscillations of a partial standing-wave nature inside the cavity, and oscillations that decay with distance in the dielectrics on either side. However, the oscillations in either the cavity or the dielectrics disappear for suitable choices of frequency that remove interference effects. In the limit of the high-finesse cavity formed from dielectrics with large values of the extinction coefficient, the vacuum field fluctuation spectrum has a succession of sharply peaked Lorenzian lines characteristic of a Fabry-Pérot cavity. In the extreme limit of a lossless dielectric where the refractive index vanishes but the extinction coefficient is nonzero for frequencies within a stop band, the intrinsic noise fields inside the dielectric are totally quenched, as discussed above, but vacuum field fluctuations can penetrate some way into the dielectric by transmission across interfaces with free space or with other dielectrics of nonzero refractive index.

The one-dimensional theory used here is adequate for the treatment of propagation of light along optical fibers or of plane-parallel beams through perpendicular optical components. Extensions to three dimensions, currently under way, are needed for the calculation of such quantities as spontaneous emission rates or Casimir forces, where there is no experimental control of the spatial modes of the electromagnetic field that participate in the process.

ACKNOWLEDGMENTS

We thank S. M. Barnett for helpful discussions, and C. H. Henry for the proof of the relation for the canonical commutator given in the Appendix. R.M. thanks the Iranian Ministry of Culture and Higher Education and University of Kerman for financial support.

APPENDIX

In this Appendix we outline the proof of the relation between the canonical commutator and the Green's function given in (4.11). The canonical commutator can be expressed in the form

$$[\hat{A}(x,t),-\varepsilon_0\hat{E}(x',t)]=\frac{i\varepsilon_0^2\hbar S}{\pi}\int_0^\infty d\omega\int_{-\infty}^\infty dx''\omega^3\varepsilon_i(x'',\omega)\times\{G(x,x'',\omega)G^*(x',x'',\omega)+G^*(x,x'',\omega)G(x',x'',\omega)\}, \quad \text{(A1)}$$

where (2.11), (2.24), and (3.16) have been used. It follows by manipulations of the Green's-function equations (3.17) for $G(x',x'',\omega)$ and its complex conjugate for $G^*(x,x'',\omega)$, similar to the procedure in Appendix B of [7], together with the symmetry of the Green's function under interchange of its first two arguments, that

$$\omega^2\int_{-\infty}^\infty dx''\varepsilon_i(x'',\omega)G(x,x'',\omega)G^*(x',x'',\omega)=\frac{1}{\varepsilon_0 S}\mathrm{Im}G(x,x',\omega). \quad \text{(A2)}$$

Substitution into (A1) now leads to the general relation for the canonical commutator given in (4.11).

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