Unbalanced homodyning for quantum state measurements

S. Wallentowitz and W. Vogel
*Arbeitsgruppe Quantenoptik, Fachbereich Physik, Universität Rostock,
Universitätsplatz 3, D-18051 Rostock, Germany*
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We propose the reconstruction of the quantum state of light from the homodyne photocounting statistics of a single, realistic photodetector. Contrary to the development of homodyning over the last decade, our approach is based on unbalanced detection with a weak local oscillator. Representing the quantum state in terms of s-parametrized quasiprobability distributions, the method even allows one to determine the Wigner function provided the quantum efficiency of the detector is sufficiently large. We show that perturbing effects due to classical noise of the local oscillator are small. [S1050-2947(96)07006-0]

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I. INTRODUCTION

One of the most fundamental problems of optical measurements is the reconstruction of the full information on the quantum state of a given light field. Its solution is the prerequisite for many fundamental experiments in quantum optics. The maximum amount of information that can be obtained from the statistics of the photoelectric counts of a photodetector is the photon number distribution [1–3]. Its direct measurement, however, would require an idealized photodetector with a quantum efficiency of unity. In more realistic cases, the determination of the photon statistics requires a reconstruction procedure to eliminate the effects of nonideal detection [4–6].

For getting insight into phase-sensitive properties of light, it is convenient to use homodyne detection schemes [7,8]. Usually the signal field is superimposed by a strong local oscillator before it is measured. However, even small classical fluctuations of the strong local oscillator can alter the measured data significantly. To solve this problem balanced homodyne detection has been proposed, where the difference statistics of the events in two channels is recorded [9,10]. This method has been applied in squeezed-light experiments [11,12].

Recently it has been demonstrated experimentally that balanced homodyning can be used to reconstruct the quantum state of light via optical homodyne tomography [13,14]. A four-port homodyne detection scheme is used to measure the statistics of difference events in the two output channels of the device for various values of the phase difference between local oscillator and signal field. The measured distributions have been used to reconstruct the Wigner function and the density matrix by inverse Radon transform, which effectively corresponds to various integrations of the measured data [15]. Related work has been performed to determine the quantum state of a molecular vibration [16]. In both kinds of experiments the data include the effects of imperfect detection with quantum efficiencies significantly smaller than one, so that ''smoothed'' Wigner functions are reconstructed. A deconvolution procedure is required to eliminate these perturbations. This would include the multiplication of the data with exponentially rising functions [17–20], so that sampling noise may become crucial. To our knowledge, a general solution of this problem does not exist yet. A recent proposal of a tomographic method for determining the quantum mechanical state of a trapped ion is free of such problems [21]. This method is based on the detection of the ground-state occupation of a weak electronic transition, which can be measured with extremely high sensitivity [22].

Alternatively to the tomographic methods, more complicated homodyne detection schemes could be used for determining the quantum state of light. For example, eight-port schemes [23,24] allow one to determine the quantum state in terms of the Q function [25,26]. For nonideal detectors the recorded distributions are further smoothed [27], so that s-parametrized quasiprobability distributions [28] are measured, which are broader than the Q function. More recently it has been shown that the same information is accessible by using a six-port homodyne detection scheme [29].

In the present paper we will show that a simple unbalanced homodyne detection scheme is feasible to reconstruct the quantum state of light from the measured photocounting statistics. The reconstruction is particularly simple for a representation of the quantum states in terms of s-parametrized quasiprobability distributions, which are obtained by summing up the measured counting statistics with appropriate weight factors. This approach includes the deconvolution needed for any realistic detector. Numerical simulations demonstrate the applicability of the method. Eventually we will show that classical noise of the local oscillator is not crucial for the technique under study.

The paper is organized as follows. In Sec. II we introduce the unbalanced detection scheme considered in our approach. The quantum-state reconstruction from the photoelectron statistics recorded in the unbalanced scheme is studied in Sec. III, including some numerical simulations. The effects of local oscillator noise in this unbalanced reconstruction method are treated in Sec. IV. A summary and some conclusions are given in Sec. V.

II. UNBALANCED DETECTION SCHEME

Let us consider the unbalanced homodyne detection scheme given in Fig. 1. A beam splitter combines the signal field with the local oscillator field. The superimposed light can be described by the beam splitter transformation [30]

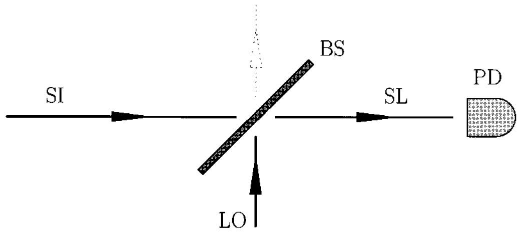

FIG. 1. Unbalanced homodyne detection scheme for the reconstruction of the quantum state of light; BS, beam splitter; LO, local oscillator; SI, signal field; SL, superimposed light field; and PD, photodetector.

$$\hat{a}_{sl} = T\hat{a} + R\hat{b}; \tag{1}$$

$\hat{a}$, $\hat{b}$, and $\hat{a}_{sl}$ are the photon annihilation operators of the signal field, the local oscillator field, and the superimposed field, respectively. T and R are the complex amplitude transmission and reflection coefficients of the beam splitter, respectively, which obey the familiar relations

$$|T|^2 + |R|^2 = 1, \tag{2}$$

$$\arg(T) - \arg(R) = \pm \pi/2. \tag{3}$$

This yields the photon number operator of the superimposed light field as

$$\hat{n}_{sl} = |T|^2 \left(\hat{a} + \frac{R}{T}\hat{b} \right)^\dagger \left(\hat{a} + \frac{R}{T}\hat{b} \right). \tag{4}$$

The probability p_n of recording n counts with a photodetector of quantum efficiency ζ is given by [1–3]

$$p_n = \left\langle : \frac{(\zeta \hat{n}_{sl})^n}{n!} e^{-\zeta \hat{n}_{sl}} : \right\rangle, \tag{5}$$

where the :: symbol denotes normal ordering. We assume that the local oscillator is prepared in a coherent state $|\beta\rangle$, $\hat{b}|\beta\rangle = \beta|\beta\rangle$. Thus we may rewrite the counting statistics as

$$p_n(\alpha;\eta) = \left\langle : \frac{[\eta \hat{N}(\alpha)]^n}{n!} e^{-\eta \hat{N}(\alpha)} : \right\rangle, \tag{6}$$

where $\hat{N}(\alpha)$ is the displaced (signal-field) number operator,

$$\hat{N}(\alpha) = \hat{D}(\alpha) \hat{a}^\dagger \hat{a} \hat{D}(\alpha)^\dagger, \tag{7}$$

$\hat{D}(\alpha)$ being the coherent displacement operator. The coherent amplitude α reads as

$$\alpha = -\frac{R}{T}\beta, \tag{8}$$

and the overall quantum efficiency η is given by

$$\eta = \zeta |T|^2. \tag{9}$$

In order to keep the overall efficiency η as large as possible, the beam splitter should fulfill the relation $|R|^2 \ll 1$.

It will be seen in the following that for reconstructing the quantum state of the signal field from the photocounting statistics recorded in an unbalanced scheme, the dependence of this measured statistics on the coherent amplitude α and the value of the overall quantum efficiency η are of relevance. For this reason we have introduced in Eq. (6) these dependencies explicitly in the notation of the counting statistics.

III. QUASIPROBABILITY DISTRIBUTIONS FROM PHOTOELECTRON STATISTICS

For relating the statistics measured in this scheme to the quantum state of the signal field in terms of the s-parametrized quasiprobability distributions $P(\alpha;s)$, we start with the relation [28]

$$P(\alpha;s) = \frac{2}{\pi(s'-s)} \int d^2\beta \, \exp\left(-\frac{2|\alpha-\beta|^2}{s'-s} \right) P(\beta;s'). \tag{10}$$

It allows one to express any s-parametrized distribution in terms of another distribution of parameter s', provided that $s < s'$. Choosing $s' = 1$, where $P(\beta;1)$ is the Glauber-Sudarshan distribution, this result yields any quasiprobability distribution with $s < 1$, including the Wigner function $(s = 0)$ and the Q function $(s = -1)$. In this particular case the right-hand side of Eq. (10) can be rewritten as a normally ordered expectation value of the form

$$P(\alpha;s) = \frac{2}{\pi(1-s)} \left\langle : \exp\left(-\frac{2}{1-s} \hat{N}(\alpha) \right) : \right\rangle. \tag{11}$$

Comparing this result with Eq. (6) we see that the quasiprobability distribution at the phase-space point α is formally related to the zero-count probability p_0 of a photodetector with efficiency $\eta_v = 2/(1-s)$:

$$P(\alpha;s) \equiv \frac{2}{\pi(1-s)} p_0\left(\alpha; \frac{2}{1-s} \right). \tag{12}$$

In general this efficiency η_v may only be considered as those of a virtual photodetector, since it may attain unphysical values larger than unity when one is interested in quasiprobability distributions with ordering parameters $s > -1$. For example, the direct determination of the Wigner function from the zero-count distributions would require a virtual detector of efficiency $\eta_v = 2$. For determining the Q function we arrive at $\eta_v = 1$. Consequently, this distribution could be measured directly with a perfect detector $(\eta = 1)$ by recording the probability of zero counts in dependence on the complex amplitude of the local oscillator. More realistically, the zero-count probability of a detector of overall efficiency η directly yields the quasiprobability distributions for $s = 1 - 2/\eta$. However, quasiprobability distributions of this type are rather smooth and the most interesting structures of the quantum states are hidden therein. For this reason it is of great interest to obtain the distributions with $s > 1 - 2/\eta$ from measured quantities.

Although in practice zero-count probabilities with efficiencies $\eta_v \geq 1$ are not accessible, they can be reconstructed from the full photoelectron statistics measured with a realistic photodetector of efficiency $\eta < 1$. The solution of this problem turns out to be rather simple. The exponent appearing in Eq. (11) is decomposed into the sum of the term $-\eta\hat{N}(\alpha)$ containing the physical efficiency η and a residual term as follows:

$$P(\alpha;s)=\frac{2\pi^{-1}}{1-s}\left\langle :\exp\left[-\frac{2-\eta(1-s)}{1-s}\hat{N}(\alpha)\right]e^{-\eta\hat{N}(\alpha)}:\right\rangle. \tag{13}$$

Expanding the residual term into a power series we get

$$P(\alpha;s)=\frac{2}{\pi(1-s)}\sum_{n=0}^{\infty}\left[-\frac{2-\eta(1-s)}{\eta(1-s)}\right]^{n}p_n(\alpha;\eta), \tag{14}$$

which yields the quasiprobabilities as a weighted sum over the homodyne counting distributions $p_n(\alpha;\eta)$ [31]. This result allows one to reconstruct quasiprobability distributions for $s>1-2/\eta$ from the data measured with a realistic detector. The simple summation replaces phase-space integrations over the measured data as used in recent experiments [13–16]. Whereas in these experiments the chosen grid of measured data essentially determines the quality of reconstruction, our method allows the reconstruction of the distribution in each point of the phase space independently. Thus the resolution can be improved point by point without repeating the whole reconstruction.

From the theoretical point of view Eq. (14) would allow one to determine any quasiprobability distribution with $s<1$ by homodyne photocounting measurements with a realistic detector. In practice, however, the data are noisy and the quality of the reconstruction crucially depends on this noise. In the limits $s\to 1$ and/or $\eta\to 0$ the weighting factors appearing in Eq. (14) in the sum over the counting statistics are seen to become divergent. Thus the noise is amplified and the method is expected to fail. Another practical problem consists in the fact that presently available photomultipliers have small efficiencies [32], so that the applicability of our method might be presently limited to the reconstruction of rather smooth quasiprobability distributions. However, since only probabilities for rather small numbers of counts are desired [cf. Eqs. (12) and (14)], a possible solution of this problem could be the use of an array of highly efficient avalanche photodiodes [33]. This renders it possible to defocus the light to be measured in order to achieve a sufficiently small probability that a single photodiode is illuminated by more than one photon within its response time. Processing the individual outputs of the photodiodes allows the measurement of the photoelectron statistics close to the level of high quantum efficiency of the individual avalanche photodiodes.

Let us give some numerical simulations to demonstrate the possibilities of reconstructing quasiprobability distributions from the photoelectric counting statistics in unbalanced homodyne detection. The method works very well when the s values fulfill the condition $s\leq 1-1/\eta$, so that the weighting factors improve the convergence of the series. When the equal sign holds, an efficiency of $\eta=0.5$ is feasible to reconstruct the Q function. The simulation of such an example is given in Fig. 2, where each counting statistics is simulated with a sample of 10^3 events. We demonstrate the method for an odd coherent state [34,35],

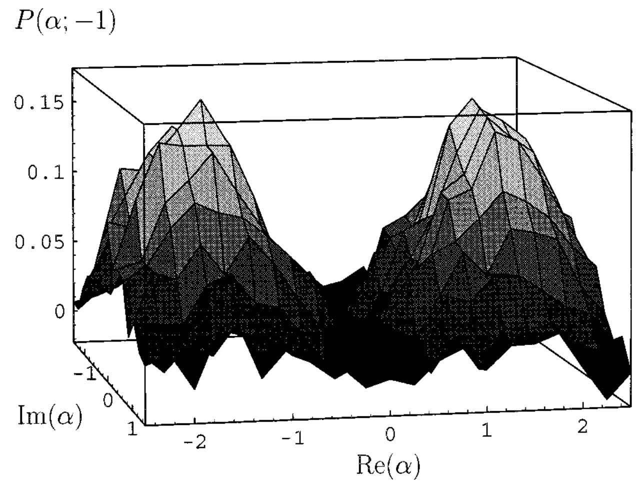

FIG. 2. Simulation of the reconstruction of the Q function for an odd coherent state with $\alpha=1.6$, quantum efficiency $\eta=0.5$, and 10^3 sampling events for each point of the 21×15 grid.

$$|\alpha_-\rangle=N(|\alpha\rangle-|-\alpha\rangle), \tag{15}$$

with $|\alpha\rangle$ being a coherent state and N a normalization constant. States of this type exhibit quantum interferences giving rise to negative values and sharp structures in the Wigner function, so that their reconstruction necessitates particular care. In the Q function, however, these effects are lost. In Fig. 3 we consider the situation for an efficiency of $\eta=0.8$ and reconstruct the distribution with $s=-0.25$. In this case the quantum interferences are seen clearly.

It is important to note that the condition $s\leq 1-1/\eta$ gives no ultimate limit for the reliability of our method. For larger s values one usually needs a larger sample of data. That is, even the Wigner function can be obtained from the data recorded with a nonideal detector. For example, in Fig. 4 we have simulated the reconstruction of the Wigner function for a quantum efficiency of $\eta=0.95$ using a sample of 5×10^3

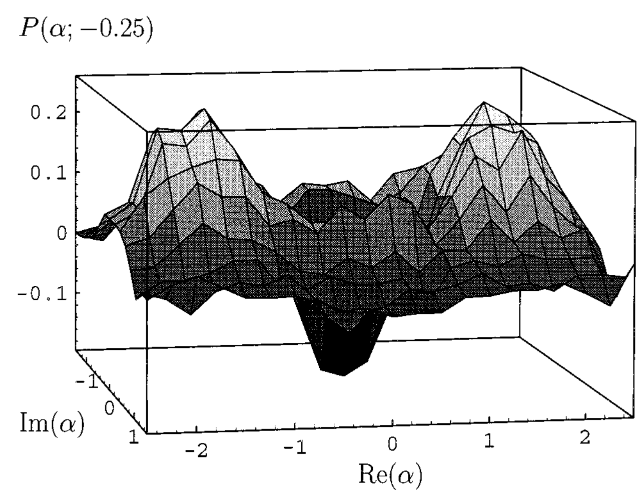

FIG. 3. Simulation of the reconstruction of $P(\alpha;s=-0.25)$ for the same state as in Fig. 2, quantum efficiency $\eta=0.8$, and 10^3 sampling events for each point of the grid. Quantum interferences are seen clearly, which lead to negative values of the distribution.

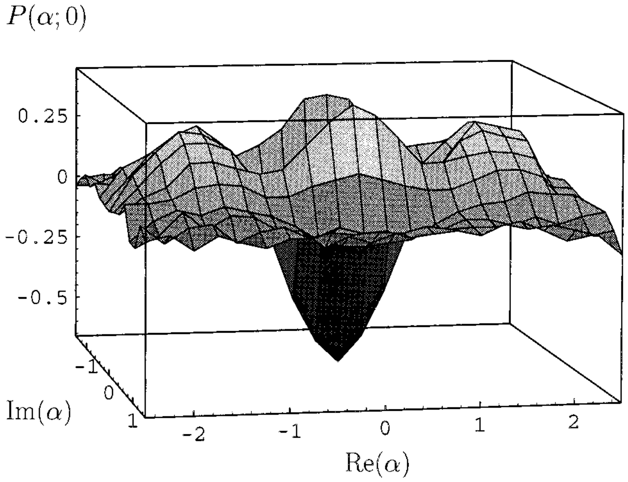

FIG. 4. Simulation of the reconstruction of the Wigner function for the same state as in Fig. 2, quantum efficiency $\eta=0.95$, and 5×10^3 sampling events for each point of the grid. The negative values of the distribution are a signature of the nonclassical properties of the state.

events. Note that the simulated distributions throughout are in suitable agreement with the corresponding exact ones.

IV. LOCAL OSCILLATOR NOISE

Let us finally consider the influence of the classical noise of the local oscillator on the method under consideration. This point is of great importance since it was the reason for the development of balanced homodyne detection. However, the noise problems in unbalanced schemes are closely related to the use of a strong local oscillator. In the context of homodyne correlation measurements it has recently been shown that the noise effects of the local oscillator become meaningless when the strength of the local oscillator is comparable to those of the signal field [36]. This is just the relevant situation in our scheme of quantum state reconstruction based on unbalanced homodyne detection. To understand this point, we first give a simple, suggestive argument. Let us return to the formal expression of the quasiprobability distributions in terms of the zero-count probabilities in Eq. (12). Usually the quantum state under consideration is localized in a certain phase-space area. The leading contributions to the zero-count probability are expected to arise when the local oscillator just displaces the phase-space distribution of the signal field towards the origin of the phase space. To achieve this, the part of the local oscillator incident on the detector [characterized by the value α in Eq. (8)] should be comparable to the strength of the signal field and is therefore weak.

A more rigorous treatment of the effects of local oscillator noise requires a statistical averaging over the value of α in Eq. (14). For this purpose let us consider a simple fluctuation model, where the fluctuations of the local oscillator are described by a Gaussian (Glauber-Sudarshan) P function of the type

$$P(\beta)=\frac{1}{\pi n_{\mathrm{th}}}\exp\left(-\frac{|\beta-\overline{\beta}|^2}{n_{\mathrm{th}}}\right). \tag{16}$$

In this model the local oscillator amplitude (of mean value $\overline{\beta}$) is superimposed by thermal noise characterized by the mean thermal photon number n_{th}. We suppose n_{th} to be proportional to $|\overline{\beta}|^2$, so that for decreasing amplitudes of the local oscillator its fluctuations also decrease.

In order to include the classical fluctuations of the local oscillator in the theory, we have to average the measured photoelectron statistics $p_n(\alpha;\eta)$ over the noise distribution $P(\beta)$. Since the averaged statistics $\overline{p_n(\alpha;\eta)}$ for general η values can be given as

$$\overline{p_n(\alpha;\eta)}=\sum_{m=0}^{\infty}\binom{n+m}{n}\eta^n(1-\eta)^m\overline{p_{n+m}(\alpha;1)}, \tag{17}$$

we may confine ourselves to the case of a perfect detector with $\eta=1$,

$$\begin{aligned}\overline{p_n(\alpha;1)}&=\overline{\left\langle :\frac{\hat{N}(\alpha)^n}{n!}e^{-\hat{N}(\alpha)}:\right\rangle}\\&=\int d^2\beta P(\beta)\left\langle :\frac{\hat{N}(\alpha)^n}{n!}e^{-\hat{N}(\alpha)}:\right\rangle.\end{aligned} \tag{18}$$

After some lengthy but straightforward algebra we derive from Eqs. (16) and (18) an expression for the averaged counting statistics in the form

$$\begin{aligned}\overline{p_m(\alpha;1)}&=\sum_{n=0}^{\infty}F_{m,n}(n_{\mathrm{fl}})\left\langle :\frac{\hat{N}(\overline{\alpha})^n}{n!}e^{-\hat{N}(\overline{\alpha})}:\right\rangle\\&=\sum_{n=0}^{\infty}F_{m,n}(n_{\mathrm{fl}})p_n(\overline{\alpha};1).\end{aligned} \tag{19}$$

The matrix $F_{m,n}(n_{\mathrm{fl}})$ describes the effects of the fluctuations of the local oscillator and is given by

$$F_{m,n}(n_{\mathrm{fl}})=\frac{n_{\mathrm{fl}}^{|m-n|}}{1+n_{\mathrm{fl}}}P_n^{(|m-n|,0)}\left(\frac{1+n_{\mathrm{fl}}{}^2}{1-n_{\mathrm{fl}}{}^2}\right)\times\begin{cases}\dfrac{(1-n_{\mathrm{fl}})^n}{(1+n_{\mathrm{fl}})^m} & \text{for} \quad m\geqslant n\\ \dfrac{(1-n_{\mathrm{fl}})^m}{(1+n_{\mathrm{fl}})^n} & \text{for} \quad m<n,\end{cases} \tag{20}$$

with $P_n^{(k,l)}(x)$ being the Jacobi polynomials and the quantity $n_{\mathrm{fl}}=(|R|/|T|)^2 n_{\mathrm{th}}$ being that part of the thermal photons incident on the detector. Note that the fluctuation matrix fulfills the relation

$$\lim_{n_{\mathrm{fl}}\to0}F_{m,n}(n_{\mathrm{fl}})=\delta_{m,n}, \tag{21}$$

so that we retain an identical transformation when the noise vanishes.

In Fig. 5 we show a simulation of the reconstruction of the Wigner function with a nonideal detector and in the presence of rather strong fluctuations of the local oscillator: $n_{\mathrm{fl}}/|\overline{\alpha}|^2=0.1$. For this purpose we apply

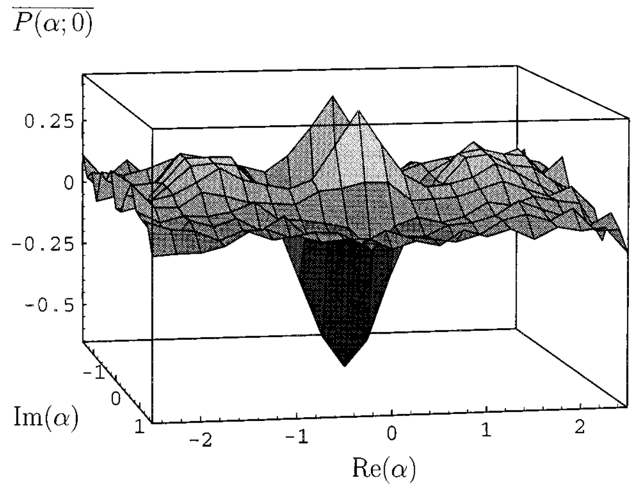

FIG. 5. Simulation of the reconstruction of the Wigner function for the same situation as in Fig. 4, but with a noisy local oscillator of relative intensity fluctuations $n_{\mathrm{fl}}/|\overline{\alpha}|^2 = 0.1$.

$$\overline{P(\alpha;s)} = \frac{2}{\pi(1-s)} \sum_{n=0}^{\infty} \left[-\frac{2-\eta(1-s)}{\eta(1-s)} \right]^n \overline{p_n(\alpha;\eta)}, \tag{22}$$

with $\overline{p_n(\alpha;\eta)}$ from Eqs. (17) and (19). Since the fluctuations grow with increasing local oscillator amplitude, the reconstruction of the quasiprobability in the origin of phase space is not disturbed, while with increasing distance from the origin the distribution is smeared out by the averaging effect of the fluctuations. In order to show their effects clearly, in the example chosen we have assumed much stronger fluctuations than would usually occur in experiments with stabilized lasers.

V. SUMMARY AND CONCLUSIONS

In the present paper we have shown that the full information on the quantum state of light can be determined from the photocounting statistics recorded by an unbalanced homodyne detection scheme. This is the simplest scheme presently known that allows one to derive the full quantum statistical information on a radiation mode from the measured data. The reconstruction procedure is particularly simple when the quantum state is represented in terms of the s-parametrized quasiprobability distributions of Cahill and Glauber. We have found that any such quasiprobability distribution can be formally expressed in terms of the zero-count probability of a virtual detector of a quantum efficiency that may exceed the ideal value of one. The limit of an ideal detector would directly allow one to measure the Q function by recording the zero-count probability as a function of the complex amplitude of the local oscillator. For a realistic detector the quasiprobability distribution recorded in this manner is more smooth than the Q function.

We further show that the zero-count distribution of a virtual detector with a quantum efficiency larger than one can be expressed in terms of a weighted sum of the counting statistics recorded with a realistic detector of quantum efficiency smaller than one. This gives a very simple expression for the s-parametrized quasiprobability distributions in terms of measured quantities. Results of simulations show that the deconvolution of data can be included in the reconstruction in appropriate limits, depending on the number of sampling events. In particular we demonstrate that the method allows one to reconstruct distributions that contain significantly more structures than the Q function. When a large quantum efficiency can be realized it even becomes possible to reconstruct the Wigner function in a rather simple manner. In view of the fact that photomultipliers have small efficiencies, present technology may limit the method to the reconstruction of rather smooth quasiprobability distributions. A way to overcome these limitations could be the appropriate use of an array of highly efficient avalanche photodiodes.

Another important problem in the context of unbalanced homodyning is the fact that classical noise of the local oscillator may substantially disturb the measured statistics. In our method the leading contributions to the quasiprobability distributions are observed with a weak local oscillator so that the classical noise effects are very small. This intuitive argument is confirmed by detailed calculations of the noise effects, which are based on a particular model of the superposition of the coherent local oscillator by thermal noise.

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