Transverse mode competition in a CO_2 laser

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Pattern formation resulting from transverse mode competition has been observed in CO_2 lasers with a large transverse section and a stable near-degenerate optical cavity. The pattern properties ruled by transverse hole burning are analyzed experimentally as a function of the Fresnel number, the frequency intermode spacing, and the symmetry breaking induced by the astigmatic cavity. It is shown that mode competition imposes selection rules amid modes belonging to the largest transverse mode family allowed to oscillate. [S1050-2947(96)05405-4]

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Transverse patterns in lasers have been observed since the earliest days of laser physics, as, e.g., in 1964 when they were reported for the first time on the transverse structures of a HeNe laser [1], but transverse dynamics studies developed only in the past decade. Two approaches have been followed, depending essentially on the number of transverse degrees of freedom of the system, i.e., on the Fresnel number. At low Fresnel number, it has been shown that modal expansion of the field on a suitable basis of empty cavity modes is well adapted to explain the main properties of the various stationary and dynamical regimes [2,3]. At high Fresnel number, Coullet *et al.* demonstrated theoretically the existence of optical turbulence induced by defects, also called optical vortices, and suggested describing complex spatiotemporal dynamics as a function of such vortices [4]. Unfortunately, although phase singularities similar to optical vortices are common in the transverse patterns of lasers, and may form complex disordered patterns [3,5–8], optical turbulence in lasers has not yet been experimentally evidenced. Complex patterns have also been observed in a liquid-crystal device with optical feedback [9], and turbulence has been evidenced in optical oscillators with photorefractive gain [10].

In CO_2 lasers, the limiting factor of the experimental analysis is the detection, as there is no technical solution to record patterns at a cadence of 1 MHz or higher, which is the typical scale of the dynamics. Therefore, the observations on laser transverse patterns are limited to the time averaged intensity [8]. The preliminary results of [8] showed that among a wide variety of patterns, the transverse profile of the CO_2 laser could exhibit self-organization, even at Fresnel numbers as large as 40. We show in this paper that in this situation, it is still possible to describe experimentally the patterns as a function of the modes of the empty cavity. Such an analysis allows us to evidence that patterns are combinations of a few modes among those present in the gain profile. The selection mechanism is shown to be transverse spatial hole burning, in good agreement with recent theoretical studies [11]. These results provide an alternative interpretation of laser patterns to that given by Feng *et al.* in terms of standing waves [12].

The experimental setup is essentially the one described in [8]. The detection consists in phosphorescent plates and a video camera. Unfortunately, as this system has a low resolution and is nonlinear, it provides pattern intensity distributions with a typical uncertainty of 20%. Another important point is the presence in the cavity of Brewster windows introducing astigmatism. This induces that (i) the cylindrical symmetry of the cavity is broken to a rectangular symmetry, so that the Hermite-Gauss basis $TEM_{m,n}$ becomes relevant, and (ii) the frequency degeneracy of modes having the same $q=m+n$ index is lifted. Pertinent parameters to characterize the cavity are the generalized Fresnel number $\mathcal{N}_F$ and the ratio R_ν of the free spectral range to the transverse mode spacing. The former is a measure of the transverse degrees of freedom and the latter rules the interactions between the transverse modes. In [8], it was shown that for $R_\nu \approx 15$ and values of $\mathcal{N}_F$ up to 30, the laser exhibits ordered time averaged intensity patterns [Figs. 1(a)–(c)] with the following properties: (i) patterns may be described as lattices of dark (or bright) spots distributed on concentric rings. For sake of simplicity, dark (bright) spots will be referred to as holes (spots) in the following. (ii) Along the two main axes x and y, the patterns exhibit either a row of bright spots or a row of holes. Thus, a pattern may be denoted by the symbol xy_n, where x and y can take the value h (holes) or s (spots) depending on what is found on the respective axis, and n is the number of spot rings. For example, the patterns shown in

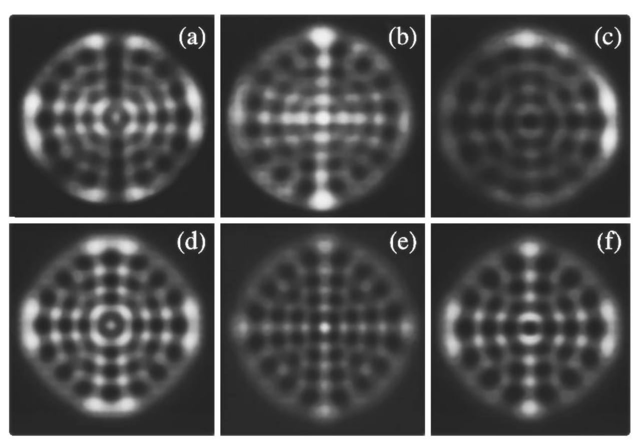

FIG. 1. Comparison of experimental (a)–(c) and reconstructed (d)–(f) patterns hh_4, ss_4, and hs_4 in the CO_2 laser. Patterns (d)–(f) follow the rules given in Table II.

Figs. 1(a)–(c) are denoted hh_4, ss_4, and hs_4, respectively. (iii) Each ring contains $2(2l+i+1)$ equidistant holes, where l is the ring index and $i=0$ (1) for hs and sh (hh and ss) patterns. These patterns obey the following scaling laws: (i) the number n of rings is proportional to $\mathcal{N}_F$, (ii) the number of holes (or spots) evolves as $\mathcal{N}_F^2$, and (iii) the distance between successive rings is proportional to $\mathcal{N}_F^{-1/2}$.

These scaling laws are also displayed by eigenmodes of the empty cavity [13]. This is a good indication that the experimental patterns can be expanded on such a basis, probably as a function of a few modes. Unfortunately, in the present experiment, there is no straightforward projection method. Indeed, our experimental setup does not provide a stable enough local oscillator to analyze the output field amplitude of the laser. Moreover, as the two-dimensional (2D) detectors have a nonlinear response, methods such as least square applied on the intensity pattern need careful interpretation.

However, as mentioned above, the presence of astigmatism in the cavity allows us to limit our investigations to the Hermite-Gauss basis. If we use the extra hypothesis that, because of the frequency degeneracy lift, all modes have a different frequency, it becomes possible to use a least square method to build up time averaged patterns adding intensities of modes. Indeed, if ξ modes coexist, the total time averaged intensity $\langle I\rangle$ will be

$$\langle I\rangle=\left\langle\left|\sum_{k=1}^{\xi}|f_k|A_k e^{i\omega_k t}\right|^2\right\rangle \tag{1a}$$

$$=\left\langle\sum_k |f_k|^2 I_k\right\rangle+\left\langle\sum_i\sum_{j\neq i}\|f_i||f_j|A_iA_j e^{i(\omega_i-\omega_j)t}|\right\rangle, \tag{1b}$$

where $f_k(t)$ is the complex modal amplitude of the kth mode with spatial distribution $A_k(x,y)$ and $I_k=|A_k|^2$. If $\omega_i\neq\omega_j$ for $j\neq i$, the second term in Eq. (1b) vanishes and the total average intensity pattern is just the sum of the intensity of each mode. In this situation, the concept of a relevant basis is fundamental. Indeed, if all modes have different frequencies in a basis, this is not necessarily the case in another one. Thus, the hypothesis that the Hermite-Gauss basis is the relevant one, is essential. This conjecture is confirmed by the results of the least square analyses, as the residues are always larger when another basis, as, e.g., Laguerre-Gauss, is used.

Starting from these hypotheses, expansion coefficients for each mode in the pattern have been determined by a least square method. A typical result is shown in Table I, where only the strongest modes are reported. These results must be taken with great care because of detector nonlinearity. However, it appears clearly in Table I that 75% of the energy is concentrated in six modes whose coefficients are larger than half the largest one. These modes belong to the same family, with $q\approx\mathcal{N}_F$. This last property has already been observed experimentally in a high power CO_2 laser [14] and was also encountered in a theoretical treatment of the laser just above threshold [11]. Taking into account the imperfections of the detection, we finally found that patterns of the type xy_n can be reconstructed with a good approximation using few modes obeying the following rules: (i) all modes belong to the same family, with $q\approx\mathcal{N}_F$; (ii) in this family, only modes following the rules of Table II are present; (iii) all modes have equal weight. Some examples of reconstructed patterns are given in Figs. 1(d) and 1(e) and show excellent agreement with the experimental observations of Figs. 1(a)–1(c).

TABLE I. Coefficients α of the modal expansion applied on the Hermite-Gauss modes $TEM_{m,n}$, obtained by least square fit of the pattern shown in Fig. 1(a). α is normalized to the weight of the strongest mode and only modes with $\alpha>0.05$ are reported. σ is the standard deviation.

m,n	α	σ
7,1	1.00	0.28
0,8	0.93	0.15
1,7	0.82	0.28
5,3	0.69	0.38
3,5	0.57	0.38
8,0	0.55	0.15
6,2	0.40	0.34
2,6	0.34	0.34
0,0	0.09	0.07
0,1	0.07	0.12
4,4	0.06	0.39
1,4	0.06	0.38

To interpret this behavior, let us consider the ‘‘free energy’’ Ξ of the laser defined as the difference between the total energy of modes and the ‘‘mode overlapping energy’’:

$$\Xi=\sum_{i=1}^{\xi}C_{ii}-\sum_i\sum_{j\neq i}C_{ij}\quad\text{with}\quad C_{ij}=\int\int dx\,dy\,I_iI_j, \tag{2}$$

where ξ is the number of modes and I_i the intensity of the ith mode. With an adequate normalization of the free energy, the first member in Eq. (2) reduces to ξ and Eq. (2) becomes

$$\Xi=\xi-\sum_i\sum_{j\neq i}\frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}}. \tag{3}$$

We have calculated Ξ for all possible combinations of modes with $q<10$. It appears that the combinations obeying the selection rules of Table II correspond to maxima of Ξ. Figure 2 shows as an example the values taken by Ξ for all the combinations of modes of the $q=4$ family. The selection rules given in Table II indicate that two patterns may exist in this family, corresponding to m,n both even or odd, i.e., hh_2 and ss_2. The two maxima in Fig. 2 correspond to these

TABLE II. $TEM_{m,n}$ modes needed to reconstruct the xy_n patterns.

Type	m	n	Additional modes
hh	odd	odd	$TEM_{0,q}+TEM_{q,0}$
ss	even	even	
hs	even	odd	$TEM_{q,0}$
sh	odd	even	$TEM_{0,q}$

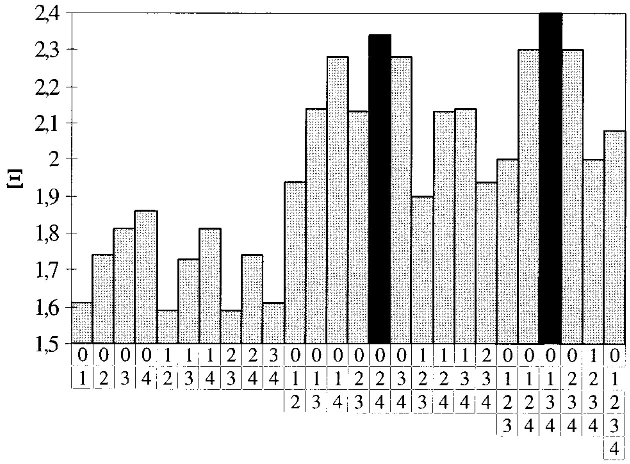

FIG. 2. Values of Ξ for the different combinations of modes in the $q=4$ family. The combinations are indicated on the x axis with the m value of each $\mathrm{TEM}_{m,n}$ mode. n may be deduced from the relation $n+m=q=4$. For example, the first column with $m=0$ and $m=1$ corresponds to the superposition of modes $\mathrm{TEM}_{0,4}$ and $\mathrm{TEM}_{1,3}$. The two emphasized values (black columns), which are the highest values taken by Ξ, correspond to the patterns hh_2 and ss_2.

two particular patterns. So it is clear that transverse modes associate in the laser to maximize energy and simultaneously minimize overlapping between their intensity distribution. This ''transverse hole burning'' has to be compared with its counterpart for longitudinal modes, where the arrangement of modes occurs also through a coupling of the intensities of each mode. The origin of this coupling can be found qualitatively in a simple model of ring class B laser at resonance. In such a laser, the model proposed by Lugiato *et al.* [15] using a modal expansion of the field gives

$$\frac{dD(x,y,\tau)}{d\tau}=-\gamma\left[D(x,y,\tau)-1+\left|\sum_i f_i(\tau)A_i(x,y)\right|^2 D(x,y,\tau)\right], \quad (4a)$$

$$\frac{df_i(\tau)}{d\tau}=\theta_{ii}f_i(\tau)+\sum_{j\neq i}\theta_{ij}f_j(\tau), \quad (4b)$$

with

$$\theta_{ii}=\kappa\left[2C\int\int dxdyA_i^2(x,y)D(x,y,\tau)-(1+ia_i)\right], \quad (4c)$$

$$\theta_{ij,i\neq j}=\kappa 2C\int\int dxdyA_j(x,y)A_i^*(x,y)D(x,y,\tau), \quad (4d)$$

where $D(x,y,\tau)$ is the population inversion. κ and γ are the relaxation rates of the field and of the population inversion, respectively. Relaxation rates and time are in units of the relaxation rate of the polarization. $2C$ is the pump parameter and $a_i=(\nu_i-\nu_R)\kappa^{-1}$ gives the losses of the mode i of frequency ν_i. ν_R is an arbitrary reference frequency. Note that θ_{ii} represents the well known ''gain minus losses'' term of the mode i while θ_{ij} is a cross saturation term between modes i and j.

To put in evidence that in such a system, overlapping occurs through the mode intensities, let us consider the stationary state of Eq. (4), just above threshold. In this situation, Eq. (4c) gives for mode i [15]:

$$\sum_k |f_k|^2\int\int dxdyI_iI_k=1-\frac{1}{2C}. \quad (5)$$

Note that at this point of the calculation, it already appears that the coupling between modes occurs through their intensity. Equation (5) is a system of ξ equations with ξ variables $f_k^2=X_k$:

$$\sum_k C_{ik}X_k=1-\frac{1}{2C}. \quad (6)$$

Direct calculation of the C_{ik} coefficients for mode families up to $q=8$ shows that their sum over k is quasiconstant. As the right-hand side of Eqs. (6) is the same for all equations, it results that the unknown quantities X_k are almost equal: $X_k=f_k^2\approx f^2$. Adding the ξ equations (6), we obtain:

$$f^2=\xi\left(1-\frac{1}{2C}\right)\left(\sum_i\sum_k C_{ik}\right)^{-1}. \quad (7)$$

The maximum for the total energy of the pattern is obtained if ξf^2 is maximum. Starting from Eq. (7) and separating in the denominator the terms C_{ii} and $C_{ik,k\neq i}$, we obtain at the first order in C_{ik}:

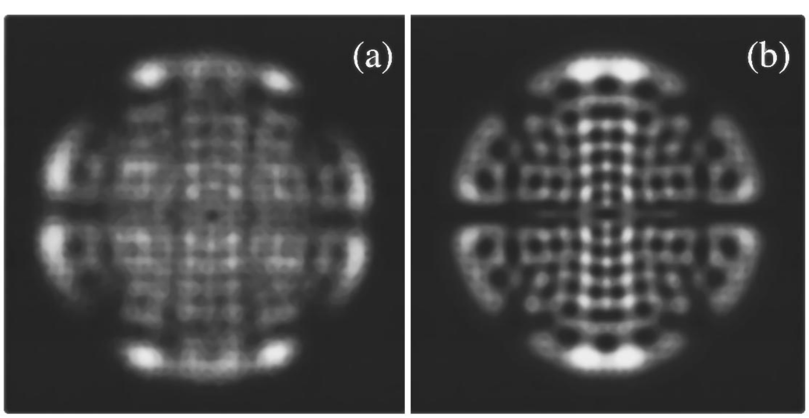

FIG. 3. (a) Experimental imperfect circular lattice observed for parameters slightly different from those of the hs_6 pattern. (b) Numerically reconstructed pattern using the hs_6 rules modified as indicated in text.

$$\Xi' = \xi f^2 = \frac{1}{\zeta}\left(\xi - \frac{2}{\zeta}\sum_i \sum_{k\neq i} C_{ik}\right), \tag{8}$$

where

$$\zeta = \frac{1}{\xi}\sum_i C_{ii}. \tag{9}$$

This equation is an unnormalized version of Eq. (3). Although it has been established in a situation far from the experimental conditions, it shows that the coupling between modes occurs through their intensity rather than their field amplitude, and that they arrange together following a principle of transverse hole burning, minimizing the overlap between modes, in spite of the fact that experimental patterns are not stationary. Indeed, such a principle is rather intuitive in a stationary pattern but is surprising in a pattern where modes could be not present at the same time, and so could minimize their energy overlapping through, e.g., a winner-takes-all dynamics.

Let us recall that regular xy_n patterns as those of Fig. 1 are not the only ones observed, but coexist with a wide variety of disordered ones. Usually, starting from an ordered pattern and changing a control parameter such as the cavity length, the transition from an xy_n distribution to a disordered one occurs through changes in the intensity distribution, whose first steps give rise to patterns with lattice defects [8]. These patterns remain symmetrical with respect to the x and y axes but do not form regular arrays (Fig. 3). With a method similar to the previous one, we observed that these patterns can be derived from xy_n ones by suppressing or substituting some modes of the same q family. For example, the pattern shown in Fig. 3(a) results from the substitution of modes $TEM_{13,0}$, $TEM_{6,7}$, $TEM_{4,9}$, and $TEM_{2,11}$ by modes $TEM_{3,10}$ and $TEM_{1,12}$ in pattern hs_6, as shown in Fig. 3(b).

A global scenario of the morphogenesis in the CO_2 laser may be proposed on the basis of the results presented here. For a given Fresnel number, possible patterns are the xy_n ones. We have shown that these patterns may be described as a function of the modes of the empty cavity, through a drastic selection due to transverse hole burning. But these patterns form islands of order in the parameter space. Starting from these points, order disappears progressively through mode substitutions. For parameters far from those of xy_n patterns, modes arrange in a way probably different from that described in this paper. The composition of these highly disordered patterns will be investigated in the near future, together with the temporal dynamics of all patterns, which could give important informations on the temporal arrangement of modes.

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