Collisions of dressed ground-state atoms

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(Received 10 January 1996)

We study collisions between dressed ground-state atoms in a full coupled-channel approach. In this way we examine collisions in the microwave trap, in the cryogenic hydrogen maser, and in the context of rf-induced evaporative cooling. The results for dressed H atoms confirm earlier results obtained within the so-called degenerate internal states approximation. [S1050-2947(96)10806-4]

PACS number(s): 32.80.Pj, 42.50.Vk

INTRODUCTION

The concept of dressed atoms [1] has been shown to be a fruitful approach in the field of trapping and cooling atoms. It has been very useful in designing and explaining new cooling schemes such as Sisyphus cooling [2] and trapping methods such as the microwave trap [3]. The success of the recent Bose-Einstein condensation (BEC) experiments [4–6] is based to a great extent on evaporative cooling where the rf field dresses the atoms and causes atoms with the highest kinetic energy to go to an untrapped spin state. By lowering the frequency of the rf field the effective depth of the trapping potential well is lowered leaving the remaining atoms in the trap with lower and lower kinetic energy. One of the crucial questions in connection with the application of an rf or microwave field is whether the field dressing will turn ‘‘good’’ collisions, i.e., elastic collisions, causing a new thermal equilibrium, into (partly) ‘‘bad’’ collisions, i.e., collisions which open up inelastic spin channels leading to unwanted density decay and heating.

The idea to use a dressed-state picture to describe collisions of cold atoms in a radiation field was proposed in connection with an applied near-resonant laser field by Julienne [7]. Subsequently, coupled-channel calculations for such cold optical collisions have been successfully carried out using dressed states [8,9]. However, collisions where the atoms are dressed by rf or microwave fields have thus far only been calculated in the so-called degenerate internal state (DIS) approximation [10]. This is a fundamentally different situation: whereas for optical collisions the coupling with neighboring manifolds occurs through spontaneous emission, for the present low frequency radiation field such a coupling takes place via the exchange interatomic interaction or via the magnetic dipole interaction. Also, in this case the atoms interact only through electronic ground-state potentials, which are well known for H, Li [11], and Na [12]. In this paper we present results of full coupled-channel calculations of these dressed atom collisions. We consider three cases: the microwave trap for H, the cryogenic H maser, and Na atoms in a static magnetic trap including an rf field for forced evaporative cooling [6].

I. METHOD

A. Dressed states

For a general background we briefly recapitulate the dressed state concept [13]. We consider a two level system, consisting of an internal atomic ground state and a single internal excited state which are coupled by an rf or microwave field. We denote the upper atomic eigenstate by $|e\rangle$ and the lower one by $|g\rangle$. The atom can be described in terms of a fictitious spin-1/2 system, where the state $|e\rangle$ is considered as the spin-up state and $|g\rangle$ as the spin-down state. Taking the zero point of energy halfway between the two levels, the (internal) Hamiltonian can be written as

$$H_{\rm at}=\frac{1}{2}\hbar\omega_0\sigma_z, \tag{1}$$

with σ_z a Pauli spin matrix. The part of the Hamiltonian which represents the single-mode radiation field can be written as

$$H_f=\hbar\omega_c a^\dagger a, \tag{2}$$

with $a^\dagger$ and a the creation and annihilation operators for a photon in the mode with frequency ω_c. Finally, the interaction of the field with the atom is written as

$$H_{{\rm at},f}=\frac{\hbar\omega_r}{2\sqrt{\overline{n}}}(a^\dagger\sigma_-+a\sigma_+), \tag{3}$$

where ω_r is the so-called Rabi frequency and $\overline{n}$ stands for the average number of photons. When there is no interaction between the radiation field and the atom, the eigenstates of the total Hamiltonian are the so-called manifolds separated by energy $\hbar\omega_c$. The manifolds consist of states $|g,N\rangle$ and $|e,N-1\rangle$ with energy difference $\hbar\delta$, where N stands for the photon number and $\delta=\omega_c-\omega_0$ for the detuning.

With the interaction included the eigenstates become superpositions of the states $|e,N-1\rangle$ and $|g,N\rangle$:

$$|1N\rangle=\cos\theta|e,N-1\rangle+\sin\theta|g,N\rangle, \tag{4}$$

$$|2N\rangle=-\sin\theta|e,N-1\rangle+\cos\theta|g,N\rangle,$$

with eigenvalues

$$E_{1N} = \left(N - \frac{1}{2}\right)\hbar\omega_c + \frac{1}{2}\hbar\Omega, \tag{5}$$

$$E_{2N} = \left(N - \frac{1}{2}\right)\hbar\omega_c - \frac{1}{2}\hbar\Omega,$$

in which $\sin 2\theta = \omega_r/\Omega$, $\cos 2\theta = -\delta/\Omega$, and $\Omega = \sqrt{\delta^2 + \omega_r^2}$.

B. Collisions of dressed atoms

In this section we will look at collisions of dressed atoms. Reference [7] contains the essential new ideas on the role of dressed states in collisions, while Refs. [8,9] describe the implementation in a coupled-channel scheme. The effective Hamiltonian for the collision of two atoms in a radiation field is

$$H = \frac{\vec{p}^{\,2}}{2\mu} + V^c + V^d + \sum (H_{\rm at} + H_{\rm at,f}), \tag{6}$$

where V^c is the central (Coulomb) interaction and V^d the magnetic dipole interaction. Both $H_{\rm at}$ and $H_{{\rm at},f}$ were introduced in the preceding section. When the atoms are far apart the eigenstates of H are the two-atom dressed states which can be described by the following symmetrized and normalized states:

$$|\{11\}N\rangle = \cos^2\theta|\{ee\},N-2\rangle + \frac{1}{\sqrt{2}}\sin 2\theta|\{eg\},N-1\rangle + \sin^2\theta|\{gg\},N\rangle,$$

$$|\{12\}N\rangle = -\frac{1}{\sqrt{2}}\sin 2\theta|\{ee\},N-2\rangle + \cos 2\theta|\{eg\},N-1\rangle + \frac{1}{\sqrt{2}}\sin 2\theta|\{gg\},N\rangle, \tag{7}$$

$$|\{22\}N\rangle = \sin^2\theta|\{ee\},N-2\rangle - \frac{1}{\sqrt{2}}\sin 2\theta|\{eg\},N-1\rangle + \cos^2\theta|\{gg\},N\rangle,$$

with energies

$$E_{\{11\}N} = (N-1)\hbar\omega_c + \hbar\Omega,$$

$$E_{\{12\}N} = (N-1)\hbar\omega_c, \tag{8}$$

$$E_{\{22\}N} = (N-1)\hbar\omega_c - \hbar\Omega.$$

These states correspond to the incoming and outgoing asymptotic two-body collision channels [7]. In this basis the coordinate-space representation of the scattering wave function is given by

$$\langle\vec{r}|\Psi\rangle = \sum_{\{ij\}Nlm} \frac{F_{\{ij\}Nlm}}{r} i^l Y_{lm}(\hat{r})|\{ij\}N\rangle. \tag{9}$$

To find collision properties such as the S matrix one has to integrate the Schrödinger equation, which can be written as [14,15]

$$\left(\frac{-\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + E_{iN} + E_{jN} - E\right)F_{\{ij\}Nlm}(r) = -\sum_{\{i'j'\}N'l'm'} C_{\{ij\}Nlm,'\{i'j'\}N'l'm'}(r)F_{\{i'j'\}N'l'm'}(r), \tag{10}$$

where E_{iN} are the energies of the individual dressed atoms and

$$C_{lm\{ij\}N,l'm'\{i'j'\}N'}(r) = i^{l'-l}\langle lm\{ij\}N|V^c(r) + V^d(\vec{r})|l'm'\{i'j'\}N'\rangle. \tag{11}$$

By transforming to the bare state basis this matrix can be evaluated in terms of bare-state coupling-matrix elements. In this respect the central and dipolar interaction only couple bare states which have the same value of N. Details of the numerical calculation can be found in Refs. [14,15]. There is no fundamental difference with coupled-channels calculations for atoms trapped in static magnetic traps except that the number of coupled channels can be larger in the dressed state case because a dressed state is usually a superposition of different bare states. Furthermore, if a specific channel is coupled with a channel in a manifold with different N and in turn this latter channel is coupled with channels in other manifolds, the channel under consideration is coupled to these other channels in higher order. In principle an infinite number of channels might be necessary to calculate scattering properties of some definite channel. However, our results fortunately show that the inclusion of channels to which a specific incoming channel is coupled directly is sufficient in practice to calculate the needed scattering properties. In physical terms this means that the collision takes place on a time scale too short for the rf interaction to cause such an indirect transition.

C. Dressed atom collisions in the DIS approximation

In calculating collisional transitions between atomic states dressed by a radiation field, a considerable simplification occurs when the DIS approximation is applied [10]. One then neglects the Rabi oscillation during collisions. This makes it possible to express the elastic and inelastic scattering amplitudes between pairs of dressed states in terms of amplitudes between bare states, writing out both the initial and final dressed states by means of (7). S-matrix elements for transitions between bare atomic states follow from a much simpler fieldless coupled-channel collisional calculation. We will refer to this as the dressed-DIS approximation to distinguish it from the normal bare-DIS approximation [15] in which the bare atomic coupled-channel problem is further reduced to separate triplet and singlet potential scattering problems by neglecting the hyperfine splitting. Since the rigorous coupled-channel atomic S-matrix elements are known from previous papers [15,16], we restrict ourselves here to the less far-reaching dressed DIS. Its range of validity is difficult to assess rigorously. Roughly, for the approximation to apply, the radial range where the exchange interaction $V_S - V_T$ is comparable to $\hbar\omega_r$ should be so narrow that the system behaves diabatically in passing through this range to short distances and back. Classically speaking, the precession angle of the fictitious spin analog system should be small during these passages. Note that the system need not be diabatic with respect to the hyperfine precession. The latter would correspond to the bare DIS. Taking into account that the photon number N is conserved the dressed-DIS S-matrix element for a collision with incoming state $|\{22\}N\rangle$ and outgoing state $|\{12\}N\rangle$ can for instance be written out as

$$S_{\{12\}N,\{22\}N} = \frac{1}{\sqrt{2}} \sin 2\theta \big(-(\sin^2\theta) S_{\{ee\},\{ee\}} - (\cos 2\theta) S_{\{eg\},\{eg\}} + (\cos^2\theta) S_{\{gg\},\{gg\}} \big). \tag{12}$$

The dressed-DIS approximation has in the past been used to calculate the loss rate of atoms trapped in a resonant microwave trap [10]. These results give us the opportunity to check our program and to investigate the validity of the dressed-DIS approximation in this context.

II. THE MICROWAVE TRAP

The principle of the microwave trap is based on the creation of a trapping potential for a specific dressed state by applying a position-dependent microwave amplitude [10] with a local maximum in free space. The advantage of a microwave trap over an optical trap is the extremely low spontaneous emission rate. The advantage compared to a static magnetic trap is that atoms can be trapped in the lowest energy state so that spin relaxation can be reduced. Recently, trapping of Cs atoms in a microwave trap has been demonstrated [17].

In the following we will concentrate on the microwave trap for hydrogen. A hydrogen atom has four hyperfine states which are enumerated from low to high energy as $|a\rangle$, $|b\rangle$, $|c\rangle$, and $|d\rangle$, see Fig. 1. At high magnetic field electron spin resonance is possible between the $|b\rangle$ and $|c\rangle$ states and between the $|a\rangle$ and $|d\rangle$ states. Here we will only consider the pair $|b\rangle$ and $|c\rangle$ as the two-level system to compare our results with Ref. [10]. We thus have $|g\rangle \equiv |b\rangle$ and $|e\rangle \equiv |c\rangle$. From the preceding section it is clear that atoms in the dressed state $|2N\rangle$ have a decreased internal energy in a region of increased rf power and will thus be trapped. We are therefore interested in two-body collisions with the initial channel $\{22\}N$, given in Eq. (7). In spin-exchange decay,

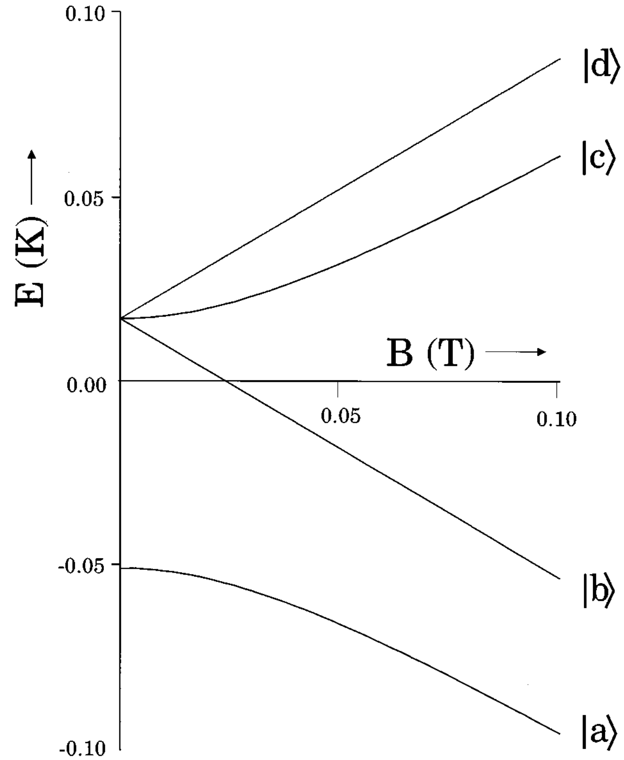

FIG. 1. Hyperfine diagram for ground-state atomic hydrogen.

which is governed by the central interaction, the total two-atom M_F value is conserved as well as the photon number. It is easily seen that this allows only transitions within the same manifold to take place. These processes are endothermal and can be neglected if Ω is large enough [10]. The dipole interaction, on the other hand, which permits an exchange of angular momentum between internal and external degrees of freedom and therefore does not conserve M_F, causes transitions to lower manifolds. More specifically, transitions to all two-body states (7) with $\Delta N = -1, -2$ are allowed. It is easy to see that in the DIS approximation the S-matrix element for the decay from $|\{22\}N\rangle$ to $|\{11\}N-1\rangle$ is given by

$$S_{\{11\}N-1,\{22\}N} = \frac{1}{\sqrt{2}} \sin^2\theta \sin 2\theta \big(S_{\{bc\},\{cc\}} - S_{\{bb\},\{bc\}} \big). \tag{13}$$

For large magnetic field values the S-matrix element $S_{\{bb\},\{bc\}}$ is equal to $-S_{\{bc\},\{cc\}}$ and all S-matrix elements for the decay from $\{22\}N$ in this situation can be expressed in terms of $S_{\{bc\},\{cc\}}$ and $S_{\{bb\},\{cc\}}$. From the experimental point of view it is more interesting to discuss rate constants. These can be defined in terms of S-matrix elements by [15]

$$G_{\alpha\beta\to\alpha'\beta'} = \left\langle \frac{\pi}{\mu k} \sum_{l',m'} \sum_{l,m} \left| S_{\{\alpha'\beta'\}l'm',\{\alpha\beta\}lm} - \delta_{\{\alpha'\beta'\}l'm',\{\alpha\beta\}lm} \right|^2 \right\rangle_{\text{th}}, \tag{14}$$

with k the wave number in the initial channel. The decay of the density of trapped atoms in the $|2N\rangle$ state is given by

$$\frac{dn_{2N}}{dt}=-G^{\text{eff}}n_{2N}^2, \tag{15}$$

and the effective rate constant calculated in the dressed-DIS approximation is given by

$$\begin{aligned}G^{\text{eff}}(B,T)=&2\left(\sin^4\theta[2\sin^2 2\theta+(1-4\cos^2\theta)^2]\right.\\&\left.+\frac{1}{2}\sin^2 2\theta\cos^2\theta\right)G_{cc\to bc}(B,T)\\&+2\left(\sin^4\theta\left[\sin^4\theta+\frac{1}{2}\sin^2 2\theta\right.\right.\\&\left.\left.+\cos^4\theta\right]\right)G_{cc\to bb}(B,T).\end{aligned} \tag{16}$$

Note that Eq. (16) differs from the final expression derived in Ref. [10]. This is because in Ref. [10] no account was taken of the fact that transitions to states $|\{12\}N-1\rangle$ and $|\{12\}N-2\rangle$ cause a gain of kinetic energy that is large enough for the final atoms to leave the trap so that two atoms instead of one are lost. Also transitions to $|\{22\}N-1\rangle$ and $|\{22\}N-2\rangle$ were not included in Ref. [10]. The contributions of various terms to the total rate constant and the total effective rate according to the dressed-DIS approximation (16) are shown in Fig. 2 for a magnetic field strength of 5 T as in Ref. [10]. Clearly, in this approximation G^{eff} does not depend on δ and ω_r separately, but only on their ratio which determines θ. This is also the result of full coupled-channel calculations, provided ω_r is not too large. The dots indicate coupled-channel results for an arbitrarily low Rabi frequency $\omega_r=2\pi\times10^7$ sec^{-1}. It is seen that they agree rather well with the approximate equation (16).

To study the breakdown of the dressed-DIS approximation we calculated the rate constant for the decay from $|\{22\}N\rangle$ to $|\{11\}N-1\rangle$ for zero detuning as a function of the Rabi frequency, again for $B=5$ T. Note that this transition is dominant for the decay from $|\{22\}N\rangle$. In Fig. 3 the results are shown. Clearly, at all practical values of the Rabi frequency the discrepancy is negligible. A discrepancy of order 10% starts to build up only above unrealistic ω_r values where the Rabi precession becomes significant during a collision. Note also that at such ω_r values the microwave magnetic field is no longer negligible compared to the static field. At $\omega_r = 2\pi\times3\times10^{10}$ sec^{-1} the energy distance of the initial and final dressed levels has dropped to only 15% of the distance for zero Rabi frequency. At even higher frequencies the level $|\{11\}N-1\rangle$ crosses the $|\{22\}N\rangle$ level and the decay channel becomes closed. In Fig. 4 the total effective rate G^{eff} for $\delta=0$ is shown as a function of the Rabi frequency. The steps in G^{eff} as a function of ω_r are caused by the same effect of closing of channels. The overall conclusion, however, is that the DIS approximation where the Rabi oscillation is neglected during a collision works very well in the experimentally interesting circumstances.

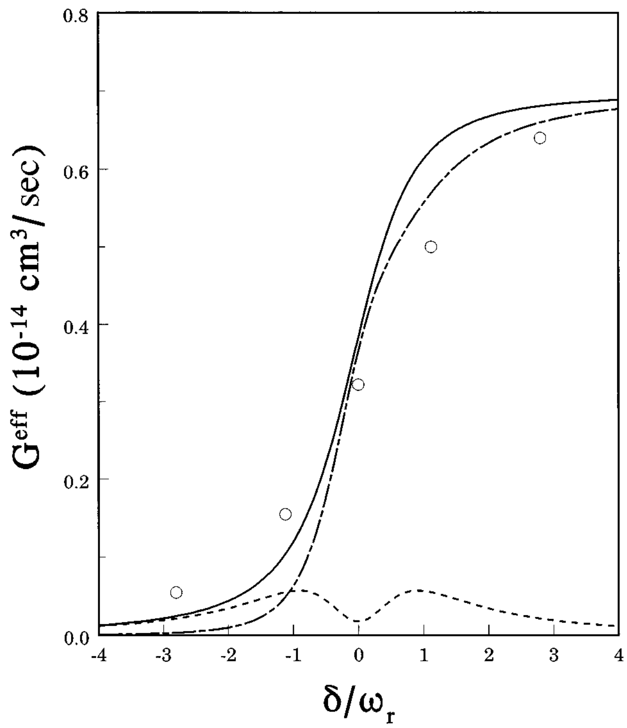

FIG. 2. Effective rate constant for dipolar relaxation as a function of the ratio of detuning and Rabi frequency. Solid line: DIS expression for total effective rate. Dashed line: Contribution of the decay to the $|\{22\}N-1\rangle$ and $|\{22\}N-2\rangle$ states. Dashed-dotted line: Contribution of the decay to the $|\{11\}N-1\rangle$, $|\{11\}N-2\rangle$, $|\{12\}N-1\rangle$, and $|\{12\}N-2\rangle$ states. Dots: results of full dressed-state coupled-channels calculation. The magnetic field value is 5 T as in Ref. [10].

III. THE CRYOGENIC HYDROGEN MASER

Another device for which a dressed state approach could in principle be useful is the cryogenic H maser. Hydrogen masers are the most stable frequency reference for averaging times from a few seconds to 1 h. The basic elements are a dissociator where H atoms are formed from H_2, a state selector to select only the upper hyperfine levels for creating a population inversion and a storage bulb which is located inside a microwave cavity tuned to the zero field transition between the a and c hyperfine levels of the H atom. This $\Delta f=1$, $\Delta m_f=0$ hyperfine transition at about 1420 MHz is used because its frequency is only weakly dependent on magnetic field near $B=0$ (see Fig. 1). Although atoms stay inside the storage bulb for about 1 s, realizing an extremely long interaction time with the electromagnetic field, wall collisions limit the reproducibility and the accuracy of H masers.

Another limitation stems from the interatomic collisions. Two-body collisions shift the frequency from the unperturbed hyperfine transition frequency. At an early stage in the development of hydrogen masers these collisional effects were calculated in the bare DIS approximation [18–20], in this case ignoring not only the influence of the microwave

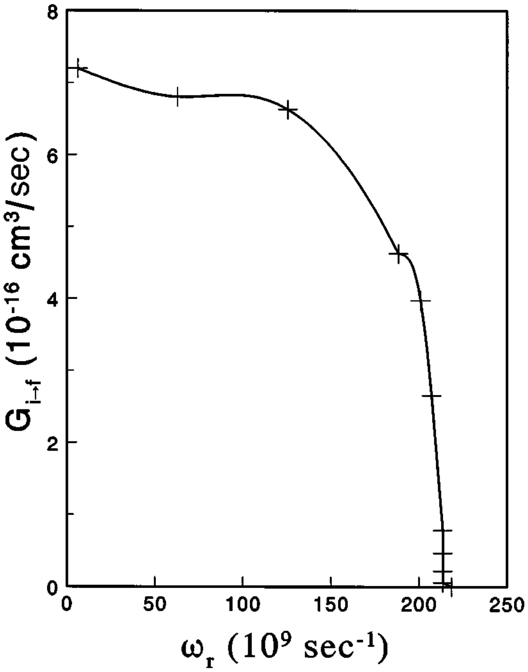

FIG. 3. Dipolar rate constant $G_{\{22\}N\to\{11\}N-1}$ for $\delta=0$ as a function of Rabi frequency ω_r, illustrating breakdown of dressed-DIS approximation.

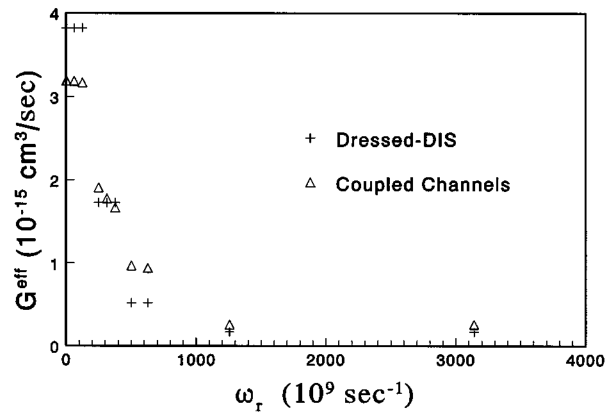

FIG. 4. Rate constant G^{eff} for $\delta=0$ as a function of Rabi frequency ω_r calculated in the dressed-DIS approximation (crosses) and with the coupled-channels method (triangles).

field during collisions but also that of the hyperfine precession.

From these results it was found that by a procedure called spin-exchange tuning it is possible to get rid of the frequency shift due to the combined effect of spin-exchange collisions and cavity pulling, the total frequency offset having the form

$$\delta\omega=[\Delta+\gamma\langle v\lambda_0^{\mathrm{DIS}}\rangle_{th}(1+\Delta^2)]\Gamma, \tag{17}$$

where λ_0^{DIS} is the bare-DIS value of a frequency-shift cross section, v is the collision velocity, Γ is the full atomic linewidth, γ is a constant dependent on cavity parameters, and $\Delta=Q_c(\omega_c/\omega-\omega/\omega_c)$ equals twice the ratio of the cavity detuning to the cavity resonance width. The spin-exchange tuning procedure consists of choosing Δ, so that $\delta\omega$ does not vary with atomic density via the collisional contribution Γ_c to the linewidth Γ, a crucial result to avoid instability due to density fluctuations. By the elimination of $\delta\omega$ the instability due to thermal noise became the main limiting factor of the H maser. From this perspective, cooling down the H maser to liquid helium temperatures promised to lead to an increase of the stability by about three orders of magnitude [21–24].

This exciting prospect turned into a less favorable one when it was shown [25,26] that a more rigorous treatment of spin-exchange collisions including the hyperfine precession of spins during the collision leads to an additional, hyperfine-induced, frequency shift not simply proportional to Γ. An earlier semiclassical calculation [18] for thermal energies had already led to a hyperfine-induced shift of a type with less grave consequences. In view of its importance it became a priority to measure the hyperfine-induced shift and its dependence on the partial densities of the hyperfine states. Recently, an experiment has been carried out in an H maser operating at room temperature [27] and two other experiments in a cryogenic H maser [28,29]. In each of the three experiments the order of magnitude of the theoretically predicted hyperfine-induced shift was confirmed. With respect to its sign, however, the experiments may form the first indication of a discrepancy. Moreover, one of the collisional linewidth parameters also appears to disagree with theory [30].

An effect that has been neglected in the existing theoretical treatment is the influence of the radiation field on the collisions. In fact, this treatment is based on the dressed-DIS approximation. An obvious question, fitting within the framework of this present paper, is therefore whether deviations from the dressed DIS can be held responsible for the discrepancy. Estimating the strength of the microwave field by assuming that it gives rise to a π pulse on the atomic oscillators during their residence time in the storage bulb, we find a Rabi frequency ω_r of the order of a few sec^{-1}. Taking into account the short duration of a collision ($\leqslant$ 1 nsec), this cannot be considered a serious option for an explanation of the discrepancy. The smallness of the deviations from the dressed DIS is confirmed by a coupled-channel calculation along the lines of Sec. I. Corrections to S-matrix elements and to the frequency-shift cross sections λ_0, λ_1, and λ_2 [25,26] turn out to be too small to be of importance.

In this connection, we point out that in principle there are good reasons in favor of the case of a dressed-state basis for the description of atomic collisions in the H maser, despite the weakness of the microwave field: the detuning δ of the frequency ω_m of the maser field relative to ω_{at} is much smaller than the already small value of ω_r, as is easily shown making use of the equation for the spin-exchange cavity tuning. As a consequence, the asymptotic states in the incoming and outgoing collision channels are dressed states. We are therefore forced to deal with a quantum Boltzmann equation with both the density matrix and the S matrix in a dressed-state basis. This leads to the equation for the rate of change of the spin-density matrix for dressed states, which we denote by symbols with bars, describing the collisional time evolution of the density matrix element $\rho_{\overline{\kappa\kappa'}}$ for the coherence of the $\overline{\kappa}$ and $\overline{\kappa'}$ states:

$$\dot{\rho}_{\overline{\kappa}\,\overline{\kappa'}}\Big|_{coll}=n\sum_{\overline{\lambda}\,\overline{\mu}\,\overline{\mu'}\,\overline{\nu}\,\overline{\nu'}}{}' \quad \rho_{\overline{\mu}\,\overline{\mu'}}\rho_{\overline{\nu}\,\overline{\nu'}}\sqrt{(1+\delta_{\overline{\kappa}\,\overline{\lambda}})(1+\delta_{\overline{\mu}\,\overline{\mu'}})(1+\delta_{\overline{\kappa'}\,\overline{\lambda}})(1+\delta_{\overline{\nu}\,\overline{\nu'}})}$$
$$\times\sum_{l}\ (2l+1)\left\langle \frac{\pi\hbar}{m_H k}\left[S^{l}_{\{\overline{\kappa}\,\overline{\lambda}\},\{\overline{\mu}\,\overline{\mu'}\}}(k)S^{l*}_{\{\overline{\kappa'}\,\overline{\lambda}\},\{\overline{\nu}\,\overline{\nu'}\}}(k)-\delta_{\{\overline{\kappa}\,\overline{\lambda}\},\{\overline{\mu}\,\overline{\mu'}\}}\delta_{\{\overline{\kappa'}\,\overline{\lambda}\},\{\overline{\nu}\,\overline{\nu'}\}}\right]\right\rangle_{\text{therm}}. \tag{18}$$

The product of S-matrix elements in this equation describes the contribution of collisions in which an atom in a superposition of the $\overline{\kappa}$ and $\overline{\kappa'}$ states undergoes a collision with another atom. The derivation of the equation, as well as a discussion of its physical meaning, is given in Ref. [26]. The prime on the summation sign indicates the subsidiary condition $\epsilon_{\overline{\kappa}}-\epsilon_{\overline{\mu}}-\epsilon_{\overline{\nu}}=\epsilon_{\overline{\kappa'}}-\epsilon_{\overline{\mu'}}-\epsilon_{\overline{\nu'}}$, i.e., energy conservation in each of the collisions described by the two S-matrix elements. Here we take into account that the two spatial atomic states corresponding to the two spin states occurring as subscripts in a ρ component have to be equal in view of spatial homogeneity. Note that the energy conservation condition refers to the dressed atomic energies, i.e., to the sum of $H_{\text{at}}+H_{\text{at},f}+H_f$ for the two atoms together. Expressing dressed states in bare states by means of the dressed-DIS equations of Sec. I [Eq. (7)] and noting that energy conservation in terms of dressed states is to a very good approximation equivalent to energy conservation in terms of bare states in view of the smallness of $H_{\text{at},f}$, we recover the usual expression for the rate of change of the spin-density matrix, except for a number of additional terms associated with a transition to a lower manifold, i.e., associated with the absorption of a microwave photon during a collision. Classically speaking, the latter contributions are due to an energy nonconserving transition caused by the time-dependent interaction term with the cavity field. Also these additional contributions turn out to be negligible.

IV. STATIC MAGNETIC TRAP + RF FIELD

The last case we will consider is collisions of atoms in a static magnetic trap including a rf field for forced evaporative cooling. In this device a magnetic field gradient is created in free space and in this way the atoms in so-called low-field seeking states experience a trapping potential whereas atoms in high-field seeking states will not. By applying a rf field with certain frequency ω_{rf}, the atoms are dressed and the adiabatic potential for the trappable low-field seeking state will turn over at the specific magnetic field where $\hbar\omega_{\text{rf}}=g\mu_B B_0$. Assuming $B=0$ at the center of the trap, i.e., the specific situation in the present Bose-Einstein condensation experiments, atoms with kinetic energy exceeding $\hbar\omega_{\text{rf}}$ can thus leave the trap. By lowering the frequency of the rf field, atoms with lower and lower kinetic energy escape, whereby the atom cloud left in the trap is cooled. This cooling principle made it possible recently to obtain the needed critical temperatures to cross the BEC phase transition line for Rb, Li, and Na [4–6]. Temperatures as low as 20 nK have been reported [4]. One of the main concerns was that by dressing the atoms the exchange interaction instead of the much weaker dipole interaction (of relevance in the microwave trap) could cause collisional trap loss even for states for which this process is negligible in the conventional magnetic trap. This is because dressing causes eigenstates to become superpositions of different bare states, thus introducing components with various total M_f which could cause decay to untrapped states.

Preliminary results of calculations of these processes have been used to estimate loss rates for rf-induced evaporative cooling in the Na setup [31]. In this paper too we will focus on the specific case of ultracold Na atoms in the lower hyperfine manifold. For Na there are three states in this manifold so that we have to consider the dressed states as superpositions of three independent bare states instead of two as in the case of H. This is also essential for understanding the effective modification of the edge of the trap potential induced by the rf field.

We start by considering the limiting case of a weak rf field ($\omega_r\to 0$), i.e., the dressed-DIS approximation. In addition we take the static field B in the low range of linear Zeeman splittings. The bare-state S matrix can then be evaluated in terms of elastic scattering in each of the channels with $F=0$ and 2, where F ($\vec{F}=\vec{I}+\vec{S}$) is the total conserved

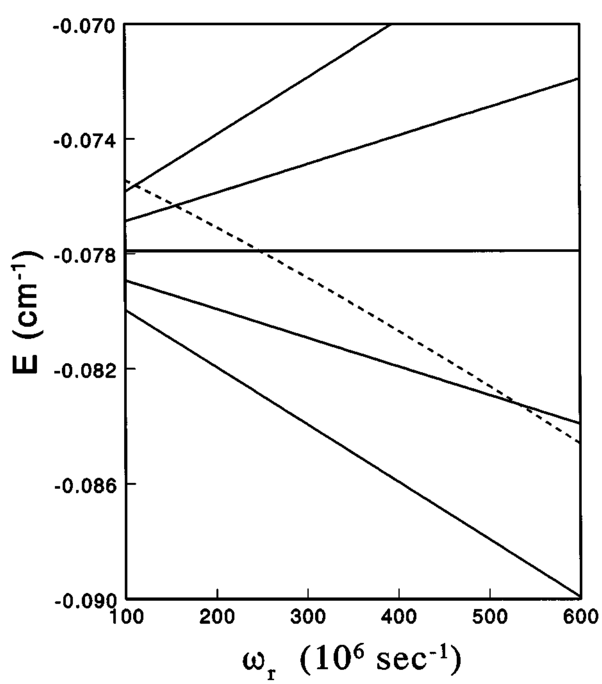

FIG. 5. Field-split energies of the $|v,l(SI)F\rangle=|15,0(13)2\rangle$ level for detuning equal to 44.3×10^{6} sec^{-1} (bottom of the trap) as a function of ω_r calculated treating $H_{\text{at},f}$ as a first order perturbation. Also shown is the threshold energy of the collision channel (dashed line).

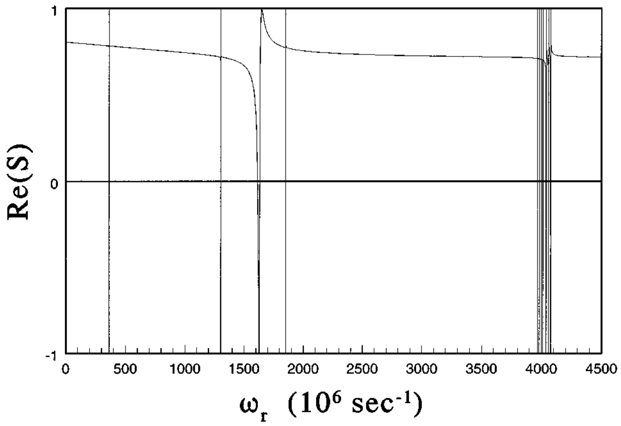

FIG. 6. Real part of elastic S-matrix element for collision energy $E=100$ μK as a function of ω_r for detuning of 44.3×10^6 sec^{-1} (bottom of trap).

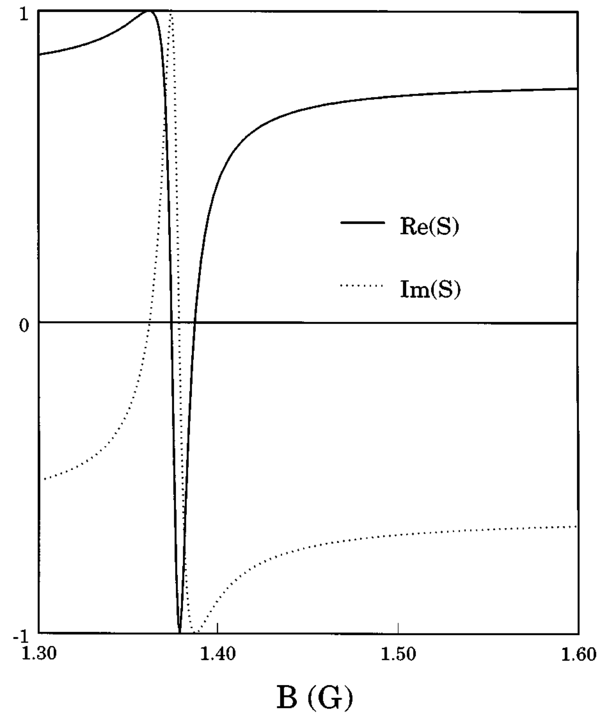

FIG. 7. Elastic S-matrix element for collision energy $E=100$ μK as a function of magnetic field for the specific Rabi frequency equal to 3.6×10^8 sec^{-1}.

spin quantum number, $\vec{I}$ being the total (two-atom) nuclear spin, and $\vec{S}$ the total electron spin. Note that $F=1$ is forbidden for s-wave scattering because of Bose symmetry, since it would correspond to an antisymmetric spin state. Values for the scattering lengths $a_{F=0}$ and $a_{F=2}$ for ultracold collisions in the $F=0$ and 2 channels can be derived from the Na+Na triplet and singlet potentials obtained [12] from experimental data. We find

$$a_{F=0}=77\pm^{61}_{16}a_0, \quad a_{F=2}=86\pm^{65}_{23}a_0. \tag{19}$$

Note that the determination of these scattering lengths is a multichannel problem, despite the fact that only elastic channels are open. Calling the single-atom dressed states

$$|\overline{iN}\rangle=\alpha_{\bar{i}}|-1,N-1\rangle+\beta_{\bar{i}}|0,N\rangle+\gamma_{\bar{i}}|1,N+1\rangle \tag{20}$$

with $\bar{i}$ $[=(-1,0,1)]$ numbering the dressed states and $-1,0,1$ in the right-hand side the $fm_f=$ 1$-$1, 10, and 1+1 bare single atom hyperfine states, the S-matrix elements for the transition between two-atom dressed states can be found in terms of $a_{F=0}$ and $a_{F=2}$. For most of the transitions we find a vanishing result. Inelastic transitions occur only between two two-atom dressed states. This restriction turns out to be due to a selection rule which can be understood as follows. In the case of three bare single-atom basis states the dressed-state system cannot in general be represented in the form of a fictitious spin in an effective magnetic field. For that to be the case the energy splittings have to be equal. In the present situation, however, in between collisions each of the atoms behaves as an elementary spin 1 particle, the external constant + rotating magnetic fields being too weak to decouple the electronic and nuclear spins. During collisions this does happen, but only via virtual transitions to closed channels. Since the rf field is practically constant during a collision, the total two-atom spin projection $M_{F,\mathrm{eff}}$ is conserved, together with F. As a consequence, the collision process is most easily described in terms of purely elastic scattering in a basis with F and $M_{F,\mathrm{eff}}$ as good quantum numbers. Also, the scattering is described by the same $a_{F=0}$ and $a_{F=2}$ scattering lengths. This picture is confirmed by the rigorous coupled-channel calculations. It fully applies to the actual experimental circumstances with a static field B_0 of order 1 mT or less. We conclude that rf-induced exchange relaxation does not occur in this case.

We now turn to a search for resonances occurring in the collision of two atoms in the 1-1 state. The presence of such a resonance would be of major importance for the study of Bose-Einstein condensation as in principle the scattering length can then be changed arbitrarily [32,33]. In analogy to atoms in a static magnetic trap it should be possible to have a bound state in a closed channel crossing the energy threshold of the colliding atoms due to the rf interaction. Na is a good candidate for a resonance at relatively low values of the rf field as there is a bound state very close to the continuum, which is responsible for the large positive value of the $fm_f=$ 1-1 scattering length [34]. Due to the rf interaction the energy levels of the bound states split and some of them cross the collision threshold, see Fig. 5. We consider an experimental setup comparable to the one used at MIT and consider the specific case of $B_0=1$ mT. In Fig. 6 the real part of the elastic S-matrix element as a function of ω_r for atoms colliding at the center of the trap, corresponding to a detuning of 44.3×10^6 sec^{-1}, is shown. Note that we define ω_r to be equal to $\langle f=1,\ m_f=-1|H_{\mathrm{at},f}|f=1,\ m_f=0\rangle/\hbar$. From the figure it is seen that the position of the first resonance is found not very far from where the first order picture predicted a possible resonance for the specific state with $M_{F,\mathrm{eff}}=-1$. Note that higher order perturbation terms lead to deviation from the linear behavior in Fig. 5. We estimate the width of this resonance, $\Delta\omega_r$, at 3500 sec^{-1} for a collision energy of 100 μK. The width behaves as $\sqrt{E}$ and will therefore be equal to about 630 sec^{-1} at the temperatures at which BEC has been observed [6]. At even higher values of ω_r more resonances show up. Note the broad resonance ($\Delta\omega_r = 1.4\times10^6$ sec^{-1} at 100 μK) for a Rabi frequency of 1.6 $\times 10^9$ sec^{-1}. It turns out that the Rabi frequency at which these resonances are found is too high to treat $H_{\text{at},f}$ as a first order perturbation. A complete determination from which specific resonances result requires a calculation along the lines of Ref. [34], i.e., taking the rf interaction fully into account in a coupled-channel method. However, this goes beyond the scope of this publication and will be dealt with in a forthcoming paper. In Fig. 7 we finally show the elastic S-matrix element for the same first resonance as a function of magnetic field (or position or detuning) for a specific value of the Rabi frequency. It is clear that this specific resonance will turn up very locally with an estimated width of $\Delta B \approx 0.014$ G at 100 μK. For the broad resonance, however, we find a width of 1.2 G at a collision energy of 1 nK. This means that for a collision energy of 2 μK, the temperature at which BEC has been observed, this resonance will be visible over the whole trap.

CONCLUSIONS

This paper deals with the most rigorous treatment of atomic collisions in a rf or microwave field so far. It treats the collision partners as atoms dressed by the field and takes the coupling of an unrestricted number of dressed-state manifolds into account in a full coupled-channel approach. Previous calculations for the cases of the hydrogen microwave trap and the cryogenic ac H maser have been carried out in the dressed-DIS approximation which neglects spin precessions in the external field during the collision. Our present rigorous treatment confirms the results of these previous treatments for realistic external field strengths. A third application is associated with the recent Bose-Einstein condensation experiments in which rf-induced evaporative cooling played an important role. We address the question whether the rf field might turn ‘‘good’’ collisions into ‘‘bad’’ collisions. This turns out to be not the case. Considering in particular the specific case of the Na rf BEC experiments at MIT we find resonances in the collision of atoms trapped in the energetically lowest (dressed) state for experimentally accessible values of the Rabi frequency, which may have interesting applications in future experiments: a change in Rabi frequency provides a handle to change the macroscopic properties of the Bose condensate.

ACKNOWLEDGMENTS

This work is part of a research program of the Stichting voor Fundamenteel Onderzoek der Materie, which is financially supported by the Nederlandse Organisatie voor Wetenschappelijk Onderzoek.

[1] J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **2**, 1707 (1985).
[2] J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **6**, 2023 (1989).
[3] C.C. Agosta and I.F. Silvera, in *Proceedings of the International Conference on Spin-Polarized Quantum Systems, Torino, Italy, 1988*, edited by S. Stringari (World Scientific, London, 1989).
[4] M.H. Anderson, J.R. Ensher, M.R. Matthews, C.E. Wieman, and E.A. Cornell, Science **269**, 198 (1995).
[5] C.C. Bradley, C.A. Sackett, J.J. Tollett, and R.G. Hulet, Phys. Rev. Lett. **75**, 1687 (1995).
[6] K.B. Davis, M.-O. Mewes, M.R. Andrews, N.J. van Druten, D.S. Durfee, D.M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
[7] P.S. Julienne, Phys. Rev. Lett. **61**, 698 (1988).
[8] H.M.J.M. Boesten, B.J. Verhaar, and E. Tiesinga, Phys. Rev. A **45**, 4879 (1992).
[9] H.M.J.M. Boesten and B.J. Verhaar, Phys. Rev. A **49**, 4240 (1994).
[10] C.C. Agosta, I.F. Silvera, H.T.C. Stoof, and B.J. Verhaar, Phys. Rev. Lett. **62**, 2361 (1989).
[11] A.J. Moerdijk, W.C. Stwalley, R.G. Hulet, and B.J. Verhaar, Phys. Rev. Lett. **72**, 40 (1994).
[12] A.J. Moerdijk and B.J. Verhaar, Phys. Rev. Lett. **73**, 518 (1994).
[13] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions* (John Wiley & Sons, New York, 1992).
[14] R.M.C. Ahn, J.P.H.W. van der Eijnde, and B.J. Verhaar, Phys. Rev. B **27**, 5424 (1983).
[15] H.T.C. Stoof, J.M.V.A. Koelman, and B.J. Verhaar, Phys. Rev. B **38**, 4688 (1988).
[16] A. Lagendijk, I.F. Silvera, and B.J. Verhaar, Phys. Rev. A **33**, 626 (1986).
[17] R.J.C. Spreeuw, C. Gerz, L.S. Goldner, W.D. Phillips, S.L. Rolston, C.I. Westbrook, M.W. Reynolds, and I.F. Silvera, Phys. Rev. Lett. **72**, 3162 (1994).
[18] S.B. Crampton, Phys. Rev. **158**, 57 (1967).
[19] P.L. Bender, Phys. Rev. **132**, 2154 (1963).
[20] L.C. Balling, R.J. Hanson, and F.M. Pipkin, Phys. Rev. **133**, A607 (1964).
[21] S.B. Crampton, W.D. Phillips, and D. Kleppner, Bull. Am. Soc. **23**, 86 (1978).
[22] R.F.C. Vessot, M.W. Levine, and E.M. Mattison, NASA Technical Memorandum No. 78104, 1978 (unpublished).
[23] W.D. Hardy and M. Morrow, J. Phys. (Paris) Colloq. **42**, C8-171 (1981).
[24] A.J. Berlinsky and W.D. Hardy, NASA Conference Publication No. 2220, 1982.
[25] B.J. Verhaar, J.M.V.A. Koelman, H.T.C. Stoof, O.J.T. Luiten, and S.B. Crampton, Phys. Rev. A **35**, 3825 (1987).
[26] J.M.V.A. Koelman, S.B. Crampton, H.T.C. Stoof, O.J.T. Luiten, and B.J. Verhaar, Phys. Rev. A **38**, 3535 (1988).
[27] R.L. Walsworth, I.F. Silvera, E.M. Mattison, and R.F. Vessot, Phys. Rev. A **46**, 2495 (1992).
[28] M.E. Hayden, M.D. Hürlimann, and W.N. Hardy, IEEE Trans.

Instr. Measurement **42**, 314 (1993).
[29] M.E. Hayden, M.D. Hürlimann, and W.N. Hardy, Phys. Rev. A **53**, 1589 (1996).
[30] M.E. Hayden and W.N. Hardy, Phys. Rev. Lett. **76**, 2041 (1996).
[31] W. Ketterle (private communication).
[32] E. Tiesinga, B. J. Verhaar, and H.T.C. Stoof, Phys. Rev. A **47**, 4114 (1993).
[33] A.J. Moerdijk, B. J. Verhaar, and A. Axelsson, Phys. Rev. A **51**, 4852 (1995).
[34] A.J. Moerdijk and B. J. Verhaar, Phys. Rev. A **51**, R4333 (1995).