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Theory of the Kapitza-Dirac effect in strong radiation fields

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By applying the stationary wave functions for an electron interacting with a circularly polarized standing-wave photon field, and a nonperturbative scattering theory, we obtain a theoretical description of the Kapitza-Dirac effect in strong radiation fields. The theory predicts that large momentum transfers can occur, when two propagating light waves have the same angular momentum and opposite helicity, as observed by Bucksbaum, Schumacher, and Bashkansky as a half process in multiphoton ionization. The theory and possible experiments to detect the full Kapitza-Dirac effect in electron scattering from a strong radiation field are discussed. The theory also predicts that with the participation of ponderomotive energy, the incident electron beam, after interacting with a standing photon wave, may have symmetric reflection, asymmetric reflection, penetration, and refraction. Transition rate formulas of these processes are presented. [S1050-2947(96)00206-5]

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I. INTRODUCTION

In 1933, Kapitza and Dirac [1] pointed out that when an electron crosses a standing-wave light beam with an incident angle θ_i, a reflection of the electron motion may occur due to a stimulated light emission. The momentum of the incident electron $\mathbf{P}_i$, the photon energy ω, and the incident angle are related by Bragg's law

$$|\mathbf{P}_i|\cos\theta_i=\omega, \tag{1}$$

where natural units $\hbar=c=1$ are used throughout this paper. This effect is known as the Kapitza-Dirac (KD) effect. Several experiments [2–5] and theories [5–8] have attempted to show this effect. In the original work of Kapitza and Dirac, the photon numbers absorbed and emitted are both 1, so the total transferred photon number is 2. The resulting small momentum transfer requires an incident angle θ_i, which is so close to $\pi/2$ that the reflection is difficult to observe.

At first sight, one might think that it is easy to generalize the argument made by Kapitza and Dirac to multiphoton cases. But there are some theoretical difficulties: (1) Multiphoton processes are significant only when the radiation field is strong, but in a strong radiation field, the ponderomotive potential energy becomes significant [9]. With the participation of ponderomotive energy, the argument made by Kapitza and Dirac no longer holds; (2) since multiphoton transition rates typically decrease exponentially with the transferred photon number, it seems impossible to observe the KD effect with large momentum transfer.

Recently, in a standing-wave multiphoton ionization experiment [10], Bucksbaum, Schumacher, and Bashkansky showed that the angular distribution of photo-electrons has a large splitting angle when the two light beams have the same angular momentum. They interpreted this effect as a KD effect in a strong radiation field with more than 500 transferred photons, and attributed this effect to the periodicity of the ponderomotive potential energy. In a recent paper [11] (referred to as I) we developed a nonperturbative quantum electrodynamical scattering theory for standing-wave multiphoton ionization which agrees well with experiment. We proved that the large angle splitting is due to a large momentum transfer between the two oppositely traveling modes through a scattering process by the ponderomotive potential energy. Thus we confirmed the interpretation by Bucksbaum, Schumacher, and Bashkansky that the splitting process is a KD effect in a strong radiation field. However, an ionization process is a half-scattering process. The question addressed here is, can this kind of KD effect exist as a full-scattering process? If the answer is yes, one could expect that an electron beam may be reflected with large momentum transfer by crossing a standing light wave.

The scattering wave functions in this paper are treated as solutions of the Lippmann-Schwinger (LS) equation [12,13]. In this treatment, one needs to express propagators in terms of a complete set of energy eigenstates with corresponding eigenvalues. The nonperturbative quantum electrodynamic (NPQED) approach to multiphoton processes [14–17,11] developed in recent years possesses unique advantages in the treatment of interactions between atoms, electrons, and strong radiation fields. In the NPQED theory, photons and electrons are treated as elementary particles on the same footing in an isolated system where the radiation field is no longer an external field. Stationary wave functions with corresponding energy eigenvalues for an electron interacting with single-mode [14,16], standing-wave [11], and general multimode photon fields [17] have been obtained by solving the Schrödinger and Dirac equations directly. The use of these stationary wave functions to formulate propagators appearing in the LS equation has been discussed thoroughly [15]. An experimental verification for this scattering approach was worked out in I.

This paper is organized as follows. Section II is devoted to developing a nonperturbative scattering method to calculate the transition matrix elements. In Sec. III we extend the approach in paper I to the full-scattering process. We prove that symmetric reflections with a large momentum transfer can occur when the two circularly polarized light beams have the same angular momentum and opposite helicity. We also prove that with the participation of ponderomotive potential energy, asymmetric reflections, penetrations, and refractions of the electron beam can occur. The transition rates for these processes are also given. In Sec. IV we give a brief discussion on some subtle questions in the theory.

II. TRANSITION MATRIX ELEMENTS

Starting with a complete set of eigenstates of an interacting Hamiltonian, in this section we develop a nonperturbative expansion technique to evaluate the transition matrix elements due to the interaction.

We assume that a scattering state satisfies the Lippmann-Schwinger equation [12,13]

$$\Psi_i^+ = \phi_i + \frac{1}{\mathcal{E}_i - H_0 + i\epsilon} V\Psi_i^+ , \tag{2}$$

where H_0 is the noninteracting Hamiltonian, V is the interaction, and ϕ_i is the eigenstate of H_0. We use i to stand for the initial state, and f for the final state. By projecting Ψ_i^+ onto a final plane wave ϕ_f, from Eq. (2) we have

$$\Omega_{fi} \equiv \langle \phi_f | \Psi_i^+ \rangle = \delta_{fi} + \frac{1}{\mathcal{E}_i - \mathcal{E}_f + i\epsilon} \langle \phi_f | V | \Psi_i^+ \rangle, \tag{3}$$

i.e.,

$$\Omega_{fi} - \delta_{fi} = -i\pi\delta(\mathcal{E}_f - \mathcal{E}_i) \langle \phi_f | V | \Psi_i^+ \rangle + \mathrm{P} \frac{1}{\mathcal{E}_i - \mathcal{E}_f} \langle \phi_f | V | \Psi_i^+ \rangle, \tag{4}$$

where Ω_{fi} is called the Møller operator matrix element and P stands for the principal value. By limiting transitions to the energy shell, one gets

$$\Omega_{fi} - \delta_{fi} = -i\pi\delta(\mathcal{E}_f - \mathcal{E}_i) T_{fi} , \quad T_{fi} \equiv \langle \phi_f | V | \Psi_i^+ \rangle \quad \text{for } \mathcal{E}_i = \mathcal{E}_f . \tag{5}$$

The transition rate in momentum space $dW/d^3\mathbf{P}_f$ is obtained by

$$\frac{dW}{d^3\mathbf{P}_f} = 2\pi\delta(\mathcal{E}_f - \mathcal{E}_i) |T_{fi}|^2 = \frac{4}{T} |\Omega_{fi} - \delta_{fi}|^2, \tag{6}$$

where T is total interaction time. In deriving Eq. (6), we need a formula to treat the square of the energy δ function. In later sections, we need to treat the squares of momentum δ functions. These formulas are [18]

$$\left(\frac{2\pi}{T} \delta(\mathcal{E}_f - \mathcal{E}_i) \right)^2 = \frac{2\pi}{T} \delta(\mathcal{E}_f - \mathcal{E}_i), \quad \left(\frac{(2\pi)^3}{V_e} \delta^3(\mathbf{P}_f - \mathbf{P}_i) \right)^2 = \frac{(2\pi)^3}{V_e} \delta^3(\mathbf{P}_f - \mathbf{P}_i), \tag{7}$$

where V_e is the normalization volume for the free electron with momentum $\mathbf{P}_f$ or $\mathbf{P}_i$. The angular distribution of the transition rate can be obtained by integrating both sides of Eq. (6) along the radial direction of $\mathbf{P}_f$, therefore we have

$$\frac{dW}{d\Omega} = \int_0^\infty d|\mathbf{P}_f| \frac{V_e}{(2\pi)^3} \mathbf{P}_f^2 2\pi\delta(\mathcal{E}_f - \mathcal{E}_i) |T_{fi}|^2. \tag{8}$$

The scattering wave functions are derived from another form of the Lippmann-Schwinger equation

$$\Psi_i^+ = \frac{i\epsilon}{\mathcal{E}_i - H + i\epsilon} \phi_i , \tag{9}$$

where H is the full Hamiltonian of the interacting system. Using the complete set of eigenstates of H as the basis to expand the right hand side of Eq. (9), one gets

$$\Psi_i^+ = \sum_{(\mu, \mathcal{E}_\mu = \mathcal{E}_i)} |\Psi_\mu\rangle\langle\Psi_\mu|\phi_i\rangle + \sum_{(\mu, \mathcal{E}_\mu \neq \mathcal{E}_i)} \frac{\epsilon^2}{(\mathcal{E}_i - \mathcal{E}_\mu)^2 + \epsilon^2} |\Psi_\mu\rangle\langle\Psi_\mu|\phi_i\rangle + \sum_{(\mu, \mathcal{E}_\mu \neq \mathcal{E}_i)} \frac{i\epsilon(\mathcal{E}_i - \mathcal{E}_\mu)}{(\mathcal{E}_i - \mathcal{E}_\mu)^2 + \epsilon^2} |\Psi_\mu\rangle\langle\Psi_\mu|\phi_i\rangle. \tag{10}$$

The leading term of the right hand side of the above equation agrees with an earlier theoretical result [15], and has been verified in standing-wave multiphoton ionizations by the experiment of Bucksbaum, Schumacher, and Bashkansky and Guo and Drake [10,11]. The second term yields a Lorentzian distribution of the near-resonance energies. The second and the third terms could be significant for cases with a finite time duration of the light pulse in near-resonance transitions. In this paper we calculate the transition matrix elements by constructing the Møller operator matrix elements, so we keep only the leading term in calculations as in I. Thus the initial scattering state in the standing light wave case has the form

$$\Psi_i^+ = \sum_{(\mu, \mathcal{E}_\mu = \mathcal{E}_i)} |\Psi_\mu\rangle\langle\Psi_\mu|\phi_i ; l_1, l_2\rangle, \tag{11}$$

where l_1 and l_2 are the initial free photon numbers of the two traveling modes, and the initial free state for the electron-photon system is

$$|\phi_i ; l_1, l_2\rangle = V_e^{-1/2} e^{i\mathbf{P}_i \cdot \mathbf{r}} |l_1, l_2\rangle. \tag{12}$$

The Møller operator matrix element in this kind of scattering state has the form

$$\Omega_{fi} = \sum_{(\mu, \mathcal{E}_\mu = \mathcal{E}_i)} \langle \phi_f ; m_1, m_2 | \Psi_\mu\rangle\langle\Psi_\mu|\phi_i ; l_1, l_2\rangle. \tag{13}$$

III. ELECTRON REFLECTION AND REFRACTION BY STANDING WAVES

The scattering treatment developed in the preceding section is quite general. In this section we restrict the discussion to an interacting system including an electron and a circularly polarized standing light wave in the following two cases.

A. Case I. Two light beams with the same angular momentum and opposite helicity

The existence of this type of KD effect is indicated by the experiment of Bucksbaum, Schumacher, and Bashkansky, where two circularly polarized laser beams of 1064-nm wavelength with the same angular momentum propagating along opposite directions are applied to xenon gas. An unusual deep peak splitting was found in the photoelectron angular distributions. In this experiment, the initial state of the electron is an atomic bound state. The angular distribution peak splittings can be interpreted as an absorption of photons from one mode with an emission to the other mode when the photoelectron leaves the radiation field. This mechanism is the same as the one described in the original paper of Kapitza and Dirac except the transferred photon numbers are much larger. To perform a full KD scattering process, one needs to send a free electron beam instead of a bound-state beam to strike the standing light wave. Thus the initial scattering state expressed by Eq. (11) is the proper one.

The generalized quantum field Volkov solutions and their energy eigenvalues in the current case were derived in I as

$$\Psi_\mu = V_e^{-1/2} e^{i[\mathbf{P}_\mu - \mathbf{k}(N_{a_1} - N_{a_2})]\cdot \mathbf{r}} \sum_j |n_1 + j, n_2\rangle_c J_{-j}(\zeta) e^{-ij\varphi_\mu},$$

$$\mathscr{E}_\mu = \frac{\mathbf{P}_\mu^2}{2m_e} + (n_1 + \tfrac{1}{2})\omega + (n_2 + \tfrac{1}{2})\omega + 2u_p\omega. \tag{14}$$

The terms in Eq. (14) are defined as follows. The photon state $|n_1, n_2\rangle_c$ is defined by

$$|n_1, n_2\rangle_c = \frac{(c_1^\dagger)^{n_1}}{\sqrt{n_1!}} \frac{(c_2^\dagger)^{n_2}}{\sqrt{n_2!}} |0,0\rangle. \tag{15}$$

The photon operators c_1 and c_2 represent the recombined two normal modes related to the original modes by

$$c_1 = \frac{1}{\sqrt{2}} (a_1 + a_2),$$

$$c_2 = \frac{1}{\sqrt{2}} (a_1 - a_2). \tag{16}$$

The notations N_{a_1} and N_{a_2} stand for the number of operators of the original photon modes and are defined by

$$N_{a_i} = \frac{1}{2} (a_i a_i^\dagger + a_i^\dagger a_i), \quad i = 1,2. \tag{17}$$

The argument of the Bessel function is defined by

$$\zeta = \frac{2\sqrt{2}|e|\Lambda}{m_e\omega} |\mathbf{P}_\mu \cdot \boldsymbol{\epsilon}|. \tag{18}$$

The angle $\varphi_\mu = \arctan\{(P_\mu)_y/(P_\mu)_x\}$. The polarization vector is defined by

$$\boldsymbol{\epsilon} = \frac{1}{\sqrt{2}} (\boldsymbol{\epsilon}_x + i\boldsymbol{\epsilon}_y), \tag{19}$$

2Λ is the classical amplitude for the vector potential $\mathbf{A}$ of each photon mode, and $2u_p\omega$ is the total ponderomotive potential energy.

Now, we calculate the first overlap factor in the Møller operator matrix element in Eq. (13),

$$\langle \Psi_\mu | \phi_i ; l_1, l_2\rangle = \frac{(2\pi)^3}{V_e} \delta(\mathbf{P}_i - \mathbf{P}_\mu + l_1\mathbf{k} - l_2\mathbf{k}) \times \sum_j \langle n_1 + j, n_2|_c l_1, l_2\rangle J_{-j}(\zeta) e^{ij\varphi_i}. \tag{20}$$

The photon part can be evaluated in the large-photon-number limit, as described in detail in the Appendix of I, to obtain

$$\langle n_1, n_2|_c l_1, l_2\rangle = 2^{-l} \left[\binom{2l - n_2}{l - n_2/2} \binom{n_2}{n_2/2} \right]^{1/2} (-1)^{n_2/2} \times \cos(\Delta\gamma)\, \delta_{l_1 + l_2, n_1 + n_2}, \tag{21}$$

where

$$2l = l_1 + l_2,$$

$$\Delta \equiv l_2 - l_1 \quad (\ll l), \tag{22}$$

$$\gamma = \cos^{-1} \left(\frac{2l - n_2}{2l} \right)^{1/2}.$$

By noticing that n_1 appears only in the indices of the Kronecker δ, the above expression can be generalized to

$$\langle n_1 + j, n_2|_c l_1, l_2\rangle = 2^{-l} \left[\binom{2l - n_2}{l - n_2/2} \binom{n_2}{n_2/2} \right]^{1/2} (-1)^{n_2/2} \times \cos(\Delta\gamma)\, \delta_{l_1 + l_2, n_1 + n_2 + j}. \tag{23}$$

The other overlap factor in the Møller operator matrix element can be evaluated in a similar way to obtain

$$\langle \phi_f ; m_1, m_2 | \Psi_\mu \rangle = \frac{(2\pi)^3}{V_e} \delta(\mathbf{P}_\mu - \mathbf{P}_f + m_2\mathbf{k} - m_1\mathbf{k}) \times \sum_{j'} \langle m_1, m_2 | n_1 + j', n_2\rangle_c \times J_{-j'}(\zeta) e^{-ij'\varphi_f} \tag{24}$$

and

$$\langle m_1, m_2 | n_1 + j', n_2\rangle_c = 2^{-l} \left[\binom{2l - n_2}{l - n_2/2} \binom{n_2}{n_2/2} \right]^{1/2} (-1)^{n_2/2} \times \cos(\Delta'\gamma)\, \delta_{m_1 + m_2, n_1 + n_2 + j'}, \tag{25}$$

where

$$\Delta' \equiv m_2 - m_1 \quad (\ll l). \tag{26}$$

The Møller operator matrix element has the explicit form

$$\Omega_{fi}=\frac{(2\pi)^3}{V_e}\,\delta(\mathbf{P}_i-\mathbf{P}_f+\Delta'\mathbf{k}-\Delta\mathbf{k})\times\sum_{j,j'}J_{-j}(\zeta)e^{ij\varphi_i}J_{-j'}(\zeta)e^{-ij'\varphi_f}F,\qquad(27)$$

where the factor F is

$$F\equiv\sum_{n_2}\langle m_1,m_2|n_1+j',n_2\rangle_c\langle n_1+j,n_2|_c l_1,l_2\rangle=\frac{1}{\pi}\left(\frac{\sin[(\Delta'+\Delta)\pi/2]}{\Delta'+\Delta}+\frac{\sin[(\Delta'-\Delta)\pi/2]}{\Delta'-\Delta}\right)\times\delta_{l_1+l_2-m_1-m_2,j-j'}.\qquad(28)$$

The evaluation of F is described in the Appendix. The δ symbol in Eq. (28) gives the value of the net transferred photon number $j-j'$. From Eqs. (23) and (25), we know

$$j=l_1+l_2-n_1-n_2,\qquad j'=m_1+m_2-n_1-n_2.\qquad(29)$$

Thus we can say that j is the photon number absorbed by the electron when it enters the radiation field, and j' the photon number emitted when it leaves the field.

Next we determine the values for Δ and Δ', and set the constraints for j, j', and the incident angle θ_i. Consider the entry process first. Energy conservation in the entry process gives

$$\frac{\mathbf{P}_\mu^2}{2m_e}+(2u_p-j)\omega=\frac{\mathbf{P}_i^2}{2m_e}.\qquad(30)$$

Combining the above equation with momentum conservation determined by Eq. (20),

$$\mathbf{P}_\mu=\mathbf{P}_i-\Delta\mathbf{k},\qquad(31)$$

we have the quadratic equation for Δ,

$$\Delta^2-2\Delta|\mathbf{P}_i|\omega^{-1}\cos\theta_i+2m_e\omega^{-1}(2u_p-j)=0,\qquad(32)$$

with two solutions

$$\Delta_{1,2}=|\mathbf{P}_i|\omega^{-1}\cos\theta_i\mp[\mathbf{P}_i^2\omega^{-2}\cos^2\theta_i-2m_e\omega^{-1}(2u_p-j)]^{1/2}.\qquad(33)$$

Thus we have $\mu=1,2$ in Eq. (30). The existence condition for Δ from the above equations is

$$(2u_p-j)\omega\leqslant\frac{\mathbf{P}_i^2}{2m_e}\cos^2\theta_i.\qquad(34)$$

This gives the constraint between j and the incident angle θ_i. There are three cases according to the values of j: (a) $j>2u_p$, extra-absorption case, or accelerating case; (b) $0\leqslant j\leqslant 2u_p$, absorption case, or decelerating case; (c) $j<0$, emission case, or extra-decelerating case. For simplicity, we consider the case (b) first. This is the ordinary case. We have treated a similar case in I, but in an exit process for photoelectrons. In the following analysis, we always assume $0<\theta_i\leqslant\pi/2$. The geometric relations in momentum space of the entry process are shown in Fig. 1.

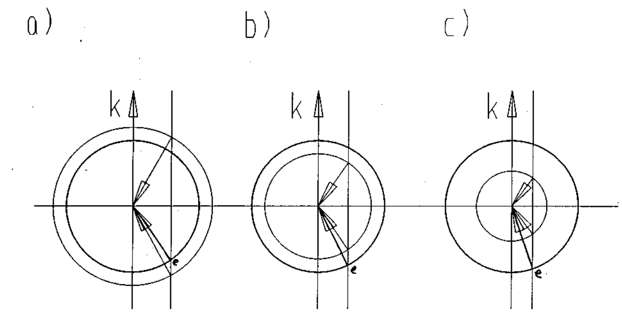

FIG. 1. Momentum space of the entry process, (a) $j>2u_p$, extra-absorption case, or accelerating case; (b) $0\leqslant j\leqslant 2u_p$, absorption case, or decelerating case; (c) $j<0$, emission case, or extra-decelerating case. By setting $2m_e=1$, the circles represent the kinetic energy sphere of the photoelectron. The lighter one is for the electron in the radiation field, and the darker for the free electron. The arrows represent momentum vectors. The notations are for all figures in this paper.

Case (b). When an electron enters the radiation field, the electron needs to absorb an integer number j of photons to contribute $j\omega$ energy as an integer part to the total ponderomotive potential energy $2u_p\omega$. Since $2u_p\omega$ as a number depends only on the properties of the field, it can be determined whether or not the electron enters the field. The remaining noninteger part of the ponderomotive energy $(2u_p-j)\omega$ will be obtained from a reduction of the kinetic energy of the electron. In this case, the electron absorbs photons. Since the electron and the field both contribute their energies to the interaction energy, except when $j=2u_p$, the electron is decelerated. From Eqs. (30) and (33) we have the following inequalities:

$$\sqrt{2m_e(2u_p-j)\omega}\leqslant|\mathbf{P}_i|,\qquad\theta_i\leqslant\cos^{-1}\left((2u_p-j)\omega\Big/\frac{\mathbf{P}_i^2}{2m_e}\right)^{1/2},\qquad|\mathbf{P}_\mu|\leqslant|\mathbf{P}_i|,\qquad\theta_1\geqslant\theta_i,\qquad(35)$$

by specifying $\theta_1\leqslant\pi/2$.

Case (a). In this case, when the electron enters the field, the electron absorbs extra photons beyond the number needed to form the ponderomotive potential energy. The extra energy $(j-2u_p)\omega$ turns into electron kinetic energy. The electron is accelerated. Thus we have the following inequalities:

$$|\mathbf{P}_\mu|>|\mathbf{P}_i|,\qquad\theta_1<\theta_i\quad\text{for}\quad\theta_1\leqslant\frac{\pi}{2}.\qquad(36)$$

The incident angle θ_i has no restriction in this case.

Case (c). In this case, when the electron enters the field, the electron emits photons. All ponderomotive energy and

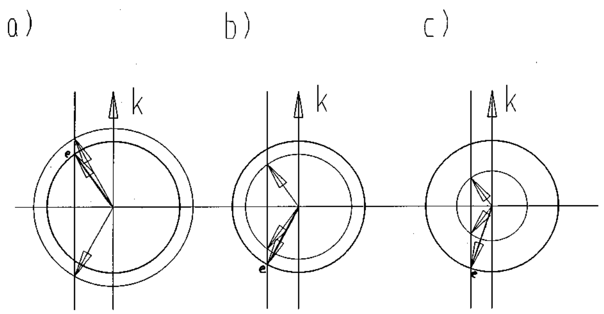

FIG. 2. Momentum space of the exit process, (a) $j'>2u_p$, extra-emission case, or decelerating case; (b) $0\leqslant j'\leqslant 2u_p$, emission case, or accelerating case; (c) $j'<0$, absorption case, or extra-accelerating case.

the emitted photon energy are obtained from a reduction of the electron kinetic energy. In this case, we have the following inequalities:

$$(2u_p+|j|)\omega\leqslant\frac{\mathbf{P}_i^2}{2m_e},$$

$$\theta_1>\theta_i, \tag{37}$$

$$\theta_i\leqslant\cos^{-1}\left((2u_p+|j|)\omega \bigg/ \frac{\mathbf{P}_i^2}{2m_e}\right)^{1/2}.$$

Now, we consider the exit process. Energy conservation in the exit process is

$$\frac{\mathbf{P}_\mu^2}{2m_e}+(2u_p-j')\omega=\frac{\mathbf{P}_f^2}{2m_e}. \tag{38}$$

Momentum conservation from Eq. (24) is

$$\mathbf{P}_\mu=\mathbf{P}_f-\Delta'\mathbf{k}, \tag{39}$$

we have a quadratic equation for Δ',

$$\Delta'^2-2\Delta'|\mathbf{P}_f|\omega^{-1}\cos\theta_f+2m_e\omega^{-1}(2u_p-j')=0. \tag{40}$$

The existence condition for Δ' from the above equations is

$$(2u_p-j')\omega\leqslant\frac{\mathbf{P}_f^2}{2m_e}\cos^2\theta_f. \tag{41}$$

This gives the constraint between j' and the scattering angle θ_f. There are, as in the entry process, also three cases according to the values of j': (a) $j'>2u_p$, extra-emission case, or decelerating case; (b) $0\leqslant j'\leqslant 2u_p$, emission case, or accelerating case; (c) $j'<0$, absorption case, or extra-accelerating case. The geometric relations in momentum space for the exit process are shown in Fig. 2. Here we do not need to go through the whole analysis, as in the treatment of the entry process, since it may not be the simplest way to establish the final-state properties by going through the intermediate states. A set of conditions for the final states can be obtained from relations linking directly to the initial state.

Momentum and energy conservation relations between the initial and the final states are

$$\mathbf{P}_i=\mathbf{P}_f-(\Delta'-\Delta)\mathbf{k},$$
$$\frac{\mathbf{P}_i^2}{2m_e}=\frac{\mathbf{P}_f^2}{2m_e}+(j'-j)\omega. \tag{42}$$

By canceling $\mathbf{P}_f$ in the above equations, a quadratic equation for $(\Delta'-\Delta)$ is obtained:

$$(\Delta'-\Delta)^2+2(\Delta'-\Delta)|\mathbf{P}_i|\omega^{-1}\cos\theta_i-2m_e\omega^{-1}(j-j')=0, \tag{43}$$

which will be used to establish the value of Δ' directly from parameters of the initial state.

The momentum δ function has the following factorization:

$$\begin{aligned}\delta\{\mathbf{P}_f-[\mathbf{P}_i+(\Delta'-\Delta)\mathbf{k}]\}&=\mathbf{P}_f^{-2}\delta(|\mathbf{P}_f|-|\mathbf{P}_i+(\Delta'-\Delta)\mathbf{k}|)\\&\times\delta\left(\cos\theta_f-\frac{|\mathbf{P}_i|\cos\theta_i+(\Delta'-\Delta)\omega}{|\mathbf{P}_i+(\Delta'-\Delta)\mathbf{k}|}\right)\\&\times\delta(\varphi_f-\varphi_i).\end{aligned} \tag{44}$$

For different final states, there are the following two cases: (1) $f=i$, the penetration case; (2) $f\neq i$, the reflection and refraction cases.

To treat these cases, we need to prove a necessary and sufficient condition for $f=i$, that is,

$$\Delta'=\Delta. \tag{45}$$

The necessary condition is trivial, since from the photon wave part, the conditions $l_1=m_1$ and $l_2=m_2$ alone must lead to Eq. (45), while the sufficient condition is not. Suppose we have $\Delta'=\Delta$, which yields $j=j'$ by Eq. (43). From Eq. (29), we get $m_1+m_2=l_1+l_2$, which combining with the assumed condition immediately leads to $m_1=l_1$ and $m_2=l_2$. Thus the final and initial photon waves are the same. Returning to Eq. (42), we have $\mathbf{P}_i=\mathbf{P}_f$. Thus the electron waves are also the same. The statement has thus been proven.

We need to express δ_{fi} in an explicit form for later use as

$$\begin{aligned}\delta_{fi}&=\frac{(2\pi)^3}{V_e}\,\delta(\mathbf{P}_f-\mathbf{P}_i)\delta_{m_1l_1}\delta_{m_2l_2}\\&=\frac{(2\pi)^3}{V_e}\,\mathbf{P}_f^{-2}\delta(|\mathbf{P}_f|-|\mathbf{P}_i|)\\&\times\delta(\cos\theta_f-\cos\theta_i)\delta(\varphi_f-\varphi_i)\delta_{m_1l_1}\delta_{m_2l_2}.\end{aligned} \tag{46}$$

Now, we discuss the two cases individually. The geometric relations for the following cases are shown in Fig. 3.

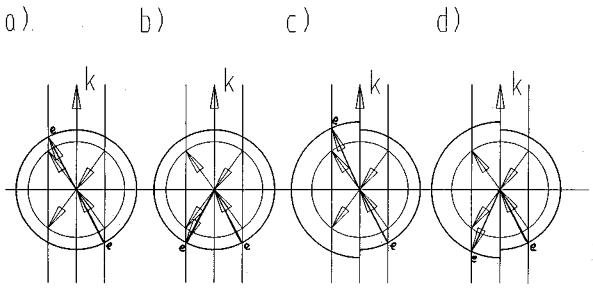

FIG. 3. Momentum space of the whole process, (a) penetration; (b) symmetric reflection; (c) refraction; (d) asymmetric reflection.

1. f=i, penetration case

From Eqs. (5), (27), and (28), we have [see Fig. 3(a)]

$$-i\pi\delta(\mathcal{E}_f-\mathcal{E}_i)T_{ii}=\frac{(2\pi)^3}{V_e}\mathbf{P}_f^{-2}\delta(|\mathbf{P}_f|-|\mathbf{P}_i|)\times\delta(\cos\theta_f-\cos\theta_i)\,\delta(\varphi_f-\varphi_i)\times\delta_{m_1l_1}\delta_{m_2l_2}\left(\frac{1}{2}\sum_j J_{-j}^2(\zeta)-1\right)\Bigg|_{f=i}. \tag{47}$$

The angular distribution of the transition rate according to Eqs. (6)–(8) and (27) is

$$\frac{dW}{d\Omega}=\frac{4}{T}\left(1-\frac{1}{2}\sum_j J_{-j}^2(\zeta)\right)^2\delta(\cos\theta_f-\cos\theta_i)\times\delta(\varphi_f-\varphi_i). \tag{48}$$

The transition rate in this case is interpreted as the rate of non-penetration. The time length T can be evaluated as the ratio of the width L of the photon beam to the transverse velocity of the electron beam, i.e.,

$$T=L/(v\ \sin\theta_i). \tag{49}$$

The differential cross section for the electron can be obtained by dividing the differential rate by the light beam intensity,

$$\frac{d\sigma}{d\Omega}=\frac{dW}{d\Omega}I^{-1}=\frac{4\sin\theta_i}{\rho L}\left(1-\frac{1}{2}\sum_j J_{-j}^2(\zeta)\right)^2\times\delta(\cos\theta_f-\cos\theta_i)\,\delta(\varphi_f-\varphi_i), \tag{50}$$

where ρ is the density of background photon numbers. Equation (50) provides an example which shows the relation between the cross section and the differential rate. In the rest of the paper, we only give the differential rates for various cases.

2. f≠i, reflection and refraction cases

From Eqs. (5) and (27) we have

$$-i\pi\delta(\mathcal{E}_f-\mathcal{E}_i)T_{fi}=\frac{(2\pi)^3}{V_e}\mathbf{P}_f^{-2}\delta(|\mathbf{P}_f|-|\mathbf{P}_i+(\Delta'-\Delta)\mathbf{k}|)\times\delta\left(\cos\theta_f-\frac{|\mathbf{P}_i|\cos\theta_i+(\Delta'-\Delta)\omega}{|\mathbf{P}_i+(\Delta'-\Delta)\mathbf{k}|}\right)\times\delta(\varphi_f-\varphi_i)\sum_{jj'}J_{-j}(\zeta)J_{-j'}(\zeta)e^{i(j-j')\varphi_i}F. \tag{51}$$

The angular distribution of the transition rate for fixed $q\equiv j-j'$ according to Eqs. (6)–(8), and (27) is

$$\frac{dW_q}{d\Omega}=\frac{4}{T}\mathcal{T}_q^2(\zeta)\,\delta\left(\cos\theta_f-\frac{|\mathbf{P}_i|\cos\theta_i+(\Delta'-\Delta)\omega}{|\mathbf{P}_i+(\Delta'-\Delta)\mathbf{k}|}\right)\times\delta(\varphi_f-\varphi_i)F^2, \tag{52}$$

where the function $\mathcal{T}_q(\zeta)$ is defined by

$$\mathcal{T}_q(\zeta)=\sum_j J_{-j}(\zeta)J_{q-j}(\zeta). \tag{53}$$

For the $q=0$ case, Eq. (43) has the nonvanishing solution

$$\Delta'-\Delta=-2|\mathbf{P}_i|\omega^{-1}\cos\theta_i, \tag{54}$$

which gives a symmetric reflection [see Fig. 3(b)]. The transition matrix and the differential rate are

$$-i\pi\delta(\mathcal{E}_f-\mathcal{E}_i)T_{fi}=\frac{(2\pi)^3}{V_e}\mathbf{P}_f^{-2}\delta(|\mathbf{P}_f|-|\mathbf{P}_i|)\times\delta(\cos\theta_f+\cos\theta_i)\,\delta(\varphi_f-\varphi_i)\sum_j J_{-j}^2(\zeta)F \tag{55}$$

and

$$\frac{dW_0}{d\Omega}=\frac{4}{T}\mathcal{T}_0^2(\zeta)\,\delta(\cos\theta_f+\cos\theta_i)\,\delta(\varphi_f-\varphi_i)F^2. \tag{56}$$

For the $q\neq0$ case, Eq. (43) has two solutions. The solution

$$\Delta'-\Delta=-|\mathbf{P}_i|\omega^{-1}\cos\theta_i+(\mathbf{P}_i^2\omega^{-2}\cos^2\theta_i+2m_e\omega^{-1}q)^{1/2} \tag{57}$$

corresponds to the refraction case, since $\mathbf{P}_f$, as well as $\mathbf{P}_i$, has a positive component in the $\mathbf{k}$ direction in this case [see Fig. 3(c)].

The solution

$$\Delta'-\Delta=-|\mathbf{P}_i|\omega^{-1}\cos\theta_i-(\mathbf{P}_i^2\omega^{-2}\cos^2\theta_i+2m_e\omega^{-1}q)^{1/2} \tag{58}$$

corresponds to the asymmetric reflection case, since $\mathbf{P}_f$ has a negative component in the $\mathbf{k}$ direction in this case [see Fig. 3(d)].

B. Case II. Two light beams with the same helicity and opposite angular momentum

The generalized quantum field Volkov solutions and their energy eigenvalues in this case are [14,10]

$$\Psi_{\mathbf{P}n_1,n_2}(\mathbf{r}) = V_e^{-1/2} e^{i\mathbf{P}\cdot\mathbf{r}} \sum_{j_1=-n_1, j_2=-n_2}^{\infty} |n_1+j_1, n_2+j_2\rangle \times \mathscr{T}_{j_1 j_2}(\zeta)^* e^{-i(j_1-j_2)(\mathbf{k}\cdot\mathbf{r}+\varphi)},$$
$$\mathscr{E} = \mathbf{P}^2/2m_e + (n_1+\tfrac{1}{2})\omega + (n_2+\tfrac{1}{2})\omega + 2u_p\omega. \tag{59}$$

The generalized Bessel functions are defined as

$$\mathscr{T}_{j_1 j_2}(\zeta) = \sum_s J_{-j_1+s}(\zeta_1) J_{-j_2+s}(\zeta_1) J_{-s}(\zeta_5), \tag{60}$$

where

$$\zeta_1 = \frac{2|e|\Lambda}{m_e\omega}|\mathbf{P}\cdot\boldsymbol{\epsilon}|,$$
$$\zeta_5 = \frac{e^2\Lambda^2}{m_e\omega} = u_p. \tag{61}$$

The overlap factor to the initial plane wave is

$$\langle \Psi_\mu | \phi_i; l_1, l_2 \rangle = \frac{(2\pi)^3}{V_e} \delta(\mathbf{P}_i - \mathbf{P}_\mu + j_1\mathbf{k} - j_2\mathbf{k}) \times \mathscr{T}_{j_1 j_2}(\zeta) e^{i(j_1-j_2)\varphi},$$
$$j_1 \equiv l_1 - n_1, \quad j_2 \equiv l_2 - n_2. \tag{62}$$

Energy and momentum conservations in the entry process look the same as in case I,

$$\frac{\mathbf{P}_\mu^2}{2m_e} + (2u_p - j)\omega = \frac{\mathbf{P}_i^2}{2m_e},$$
$$\mathbf{P}_\mu = \mathbf{P}_i - \Delta\mathbf{k}, \tag{63}$$

but with slightly different meanings of the symbols

$$j = j_1 + j_2,$$
$$\Delta = j_2 - j_1. \tag{64}$$

The overlap factor to the final plane wave is

$$\langle \phi_f; m_1, m_2 | \Psi_\mu \rangle = \frac{(2\pi)^3}{V_e} \delta(\mathbf{P}_\mu - \mathbf{P}_f + j_2'\mathbf{k} - j_1'\mathbf{k}) \times \mathscr{T}_{j_1' j_2'}(\zeta)^* e^{-i(j_1'-j_2')\varphi}, \tag{65}$$
$$j_1' \equiv m_1 - n_1, \quad j_2' \equiv m_2 - n_2.$$

Energy and momentum conservations in the exit process also look the same as in case I,

$$\frac{\mathbf{P}_\mu^2}{2m_e} + (2u_p - j')\omega = \frac{\mathbf{P}_f^2}{2m_e},$$
$$\mathbf{P}_\mu = \mathbf{P}_f - \Delta'\mathbf{k}, \tag{66}$$

but with the definitions

$$j' = j_1' + j_2',$$
$$\Delta' = j_2' - j_1'. \tag{67}$$

The Møller operator matrix element is

$$\Omega_{fi} = \frac{(2\pi)^3}{V_e} \delta(\mathbf{P}_i - \mathbf{P}_f + \Delta'\mathbf{k} - \Delta\mathbf{k}) \times \sum_{j,j'} \mathscr{T}_{j_1 j_2}(\zeta) e^{-i\Delta\varphi} \mathscr{T}_{j_1' j_2'}(\zeta)^* e^{i\Delta'\varphi}. \tag{68}$$

The angular distribution of the transition rate for fixed Δ and Δ' in $f \neq i$ case is

$$\frac{dW}{d\Omega} = \frac{4}{T} |\mathscr{T}_{j_1 j_2}(\zeta)|^2 |\mathscr{T}_{j_1' j_2'}(\zeta)|^2 \times \delta\left(\cos\theta_f - \frac{|\mathbf{P}_i|\cos\theta_i + (\Delta'-\Delta)\omega}{|\mathbf{P}_i + (\Delta'-\Delta)\mathbf{k}|} \right) \delta(\varphi_f - \varphi_i). \tag{69}$$

As pointed out in I, the rate for this kind of transition process with large momentum transfer is too small to observe. The reason is as follows. The ponderomotive parameter $2u_p$ usually is not an integer. When the electron enters the field, j photons are absorbed by the electron from the field to form a part of the ponderomotive energy. The remaining part $(2u_p - j)\omega$ is obtained from a reduction of the kinetic energy of the electron [see Eq. (30) or (63)]. Due to the large ratio of m_e/ω, if $(2u_p - j)$ is of magnitude 1, the photon mode changing number Δ to provide the momentum transfer [see Eq. (31)] is of the order of $\sqrt{2m_e/\omega}$ [see Eq. (33)]. In the experiment of Bucksbaum, Schumacher, and Bashkansky, Δ is of the order of 1000–2000. The transition rate in case I is proportional to Δ^{-2}, while in case II it is proportional to a function like $\zeta^{-\Delta}$, since the indices of the Bessel functions j and j' are of the magnitude of $\Delta/2$. This is the reason one can observe effects of large momentum transferred in case I, not in case II.

IV. DISCUSSIONS

The first thing we want to discuss is the case without participation of ponderomotive potential. In the derivation we did not see explicitly Bragg's relation, Eq. (1), originally pointed out by Kapitza and Dirac. Our general treatment should include the original KD effect as a special case. To fulfill this task, we consider an ideal situation where the laser beam intensity is adjusted such that the ponderomotive parameter is an integer, i.e.,

$$2u_p = j. \tag{70}$$

Thus Eq. (32) has two solutions,

$$\Delta_1 = 0, \tag{71a}$$

$$\Delta_2 = 2|\mathbf{P}_i|\omega^{-1}\cos\theta_i. \tag{71b}$$

If we set $\Delta_2=2$, which is the simplest case of the second solution, Eq. (71b) is exactly Bragg's relation, Eq. (1), which yields the original KD effect. If the exit process has a full ponderomotive decay such that

$$j = j', \tag{72}$$

from Eq. (43), we have two solutions,

$$\Delta' - \Delta = 0, \tag{73}$$

$$\Delta' - \Delta = -2|\mathbf{P}_i|\omega^{-1}\cos\theta_i.$$

Combining with Eqs. (71), actually we have four combined solutions,

$$\begin{gathered}\Delta = 0, \quad \Delta' = 0,\\ \Delta = 2|\mathbf{P}_i|\omega^{-1}\cos\theta_i, \quad \Delta' = 2|\mathbf{P}_i|\omega^{-1}\cos\theta_i,\end{gathered} \tag{74}$$

and

$$\begin{gathered}\Delta = 2|\mathbf{P}_i|\omega^{-1}\cos\theta_i, \quad \Delta' = 0,\\ \Delta = 0, \quad \Delta' = -2|\mathbf{P}_i|\omega^{-1}\cos\theta_i.\end{gathered} \tag{75}$$

Equation (74) describes two different penetrations via different generalized intermediate states. The first one is a straightforward penetration, while the second has a reflection in the entry process and a successive reflection which turns the electron back to the original motion in the exit process. Equation (75) describes two different reflections. The first one has a reflection in the entry process, while the second does in the exit process. From Eq. (75),

$$|\mathbf{P}_i|\cos\theta_i = n\omega, \tag{76}$$

where n is an integer equal to $\Delta/2$ or $-\Delta'/2$. This is exactly the direct generalization of the original KD effect. Since there is no ponderomotive energy transfer or ponderomotive energy scattering, from Eq. (30), the electron in the field and the free electron have the same kinetic energy. Thus a reflection will occur due to an absorption from one mode and an emission to another mode with the same number of photons. The restriction for the incident angle is also removed by Eq. (34), and a small momentum transfer is allowed. If $\Delta=20$ to 30, both case I and case II will result in observable effects from considerations of the magnitude of the Bessel functions, and the amount of transferred momentum. A difficulty in this kind of experiment is to produce standing waves with certain intensities, such that the ponderomotive parameter $2u_p$ is exactly an integer or close to an integer. The difficulty might be removed by developments both in technique and theories.

The second thing to discuss is the meaning of the condition of $\Delta\neq 0$ for the initial state. One may think that $\Delta\neq 0$ is unphysical and the initial states should be assumed such that the two initial light beams have an equal photon number. To resolve this problem, let us look back to how the condition $\Delta\neq 0$ originated. At the beginning, we assume that the initial free state has free photon numbers l_1 and l_2, without any restriction on their difference. When we project the free state onto a certain generalized quantum field Volkov state, $\Delta\neq 0$ occurs to satisfy energy and momentum conservation. Thus the condition $\Delta\neq 0$ is a requirement for forming a generalized quantum field Volkov state as an intermediate state with energy and momentum conservation. When an electron hits a standing-wave light beam, it does not pick up all surrounding photons, rather it picks up the right number of photons in each mode to form the intermediate states. The photons passed by do not participate in any interaction and reaction with the electron, thus will not appear in the formalism at all. A good example is shown by the simplest case of Eq. (75). One cannot say that the $\Delta=0$ with $\Delta'=2$ case is more physical than the $\Delta=2$ with $\Delta'=0$ case, since two photon difference is meaningless in the determination of the light beam intensity. The difference between these two cases lies just in the different requirements for formation of the different intermediate states. With this interpretation, we have confidence that the initial condition $\Delta\neq 0$ does not lead to an unphysical situation.

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APPENDIX

The evaluation of the factor F is as follows. According to the definition

$$F \equiv \sum_{n_2} \langle m_1, m_2 | n_1 + j', n_2\rangle_c \langle n_1 + j, n_2 |_c l_1, l_2\rangle,$$

we need to evaluate the overlap factor

$$\langle n_1, n_2 |_c l_1, l_2\rangle = \left(\frac{(l_1+l_2-n_2)!\,n_2!}{2^{l_1+l_2} l_1!\,l_2!}\right)^{1/2} \delta_{l_1+l_2, n_1+n_2} \times \sum_s (-1)^s \binom{l_1}{n_2-s}\binom{l_2}{s}.$$

In the Appendix of I, we showed that, if $\Delta \equiv l_2 - l_1 \ll 2l \equiv l_1 + l_2$, then

$$\begin{aligned}\sum_s (-1)^s \binom{l_1}{n_2-s}\binom{l_2}{s} &\equiv (-1)^{n_2/2} \sum_p (-1)^p \binom{\Delta}{2p}\binom{l_1}{n_2/2-p}\\ &\approx (-1)^{n_2/2}\binom{l_1}{n_2/2}\left(\frac{2l}{2l-n_2}\right)^{\Delta/2}\cos(\Delta\gamma).\end{aligned}$$

We also showed in the same limiting condition

$$\left(\frac{(l_1+l_2-n_2)!n_2!}{2^{l_1+l_2}l_1!l_2!}\right)^{1/2}\binom{l_1}{n_2/2}$$
$$\approx 2^{-l}\left[\binom{2l-n_2}{l-n_2}\binom{n_2}{n_2/2}\right]^{1/2}\left(\frac{2l}{2l-n_2}\right)^{-\Delta/2}.$$

Combining the above two expressions, we have

$$\langle n_1,n_2|_c l_1,l_2\rangle = 2^{-l}\left[\binom{2l-n_2}{l-n_2}\binom{n_2}{n_2/2}\right]^{1/2}(-1)^{n_2/2}$$
$$\times\cos(\Delta\gamma)\,\delta_{l_1+l_2,n_1+n_2}. \tag{A1}$$

Using this result we simplify F,

$$F=2^{-2l}\sum_{n_2}\binom{2l-n_2}{l-n_2}$$
$$\times\binom{n_2}{n_2/2}\cos(\Delta'\gamma)\cos(\Delta\gamma)\,\delta_{m_1+m_2-l_1-l_2,j'-j}. \tag{A2}$$

By introducing a new variable $x\equiv n_2/2$, the sum can be changed to an integral,

$$F=\pi^{-1}\int_0^l \frac{dx}{(l-x)^{1/2}x^{1/2}}\cos(\Delta'\gamma)\cos(\Delta\gamma)$$
$$\times\,\delta_{m_1+m_2-l_1-l_2,j'-j}. \tag{A3}$$

By changing variable x to $\gamma=\cos^{-1}\sqrt{1-x}$, F is simplified as

$$F=\pi^{-1}\int_0^{\pi/2} d\gamma\,\cos(\Delta'\gamma)\cos(\Delta\gamma)\,\delta_{m_1+m_2-l_1-l_2,j'-j}. \tag{A4}$$

After carrying out the integration, we get the result in Eq. (28).

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