PHYSICAL REVIEW A VOLUME 53, NUMBER 6 JUNE 1996

Interference of two condensates

William Hoston[1,2] and L. You[1]
[1]*Institute for Theoretical Atomic and Molecular Physics, Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, MS 14, Cambridge, Massachusetts 02138*
[2]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*
(Received 7 February 1996)

We propose an experimental scheme to probe the relative global phase of two condensates and thus demonstrate the spontaneous symmetry breaking of global gauge invariance in Bose-Einstein condensation. A positive measurement would justify the standard view of the condensate wave function as a coherent state when there are interparticle interactions. [S1050-2947(96)09306-7]

PACS number(s): 03.75.Fi, 05.30.Jp, 32.80.Pj

With the realization of Bose-Einstein condensation (BEC) by various groups [1–3] comes a variety of opportunities for further work with these systems. Among the possibilities are studies elucidating the nature of the condensate and its formation. In this paper we propose to study the breaking of global gauge symmetry which is believed to be necessary for the condensation phase transition [4,5]. Recently, Javanainen *et al.* have addressed similar issues for noninteracting condensates using an atom counting formulation [6]. Our focus will be on interacting systems.

The second quantized Hamiltonian describing a system of interacting bosons in a background potential $V_t(\mathbf{r})$ is

$$\mathcal{H}=\int d\mathbf{r}\Psi^\dagger(\mathbf{r})\left(-\frac{\hbar^2}{2M}\nabla^2+V_t(\mathbf{r})\right)\Psi(\mathbf{r}) +\frac{1}{2}\int d\mathbf{r}\int d\mathbf{r}'\Psi^\dagger(\mathbf{r})\Psi^\dagger(\mathbf{r}')V(\mathbf{r}-\mathbf{r}')\Psi(\mathbf{r}')\Psi(\mathbf{r}). \tag{1}$$

In what follows we will use $V(\mathbf{r})\to u_0\delta(\mathbf{r})$ with $u_0=4\pi a_{sc}\hbar^2/M$. Here $a_{\rm sc}$ is the s-wave scattering length of the atom-atom interaction potential $V(\mathbf{r})$. This shape-independent approximation can be justified in a homogeneous system [7,8]. Motivation of its validity in the inhomogeneous case is beyond this paper but we note that in these systems at temperatures orders of magnitude higher than the critical temperature for BEC already only s-wave scattering contributes to the ground state-ground state collision processes [9]. The Hamiltonian is seen to be invariant under the global gauge transformation $\Psi(\mathbf{r})\to e^{i\theta}\Psi(\mathbf{r})$ where θ is a real number independent of position. This symmetry, however, is not manifested by the ground state of the system or its order parameter [4]. The order parameter of the BEC transition, also called the condensate wave function, is believed to be $\psi(\mathbf{r})=\langle\Psi(\mathbf{r})\rangle$. Above the transition temperature, where $\psi(\mathbf{r})=0$, the order parameter trivially shows the global phase symmetry. Below the transition temperature $\psi(\mathbf{r})\neq 0$ and the global phase symmetry is broken. One would expect that during repeated experiments producing Bose-Einstein condensates there would be no correlation between the phases of the resulting condensates. We propose a simple experimental scheme to demonstrate this difference in phase.

The trap used by the MIT group produces two separate condensates each time Bose-Einstein condensation is achieved. This is due to the way in which the leak of the quadrupole magnetic trap is circumvented [3]. Let us write the order parameter of the two condensates in the form $\psi_L e^{i\theta_L}$ and $\psi_R e^{i\theta_R}$ with ψ_L and ψ_R real functions. In the experimental setup the potential barrier between the two condensates is practically infinite and it is assumed that no tunneling occurs between the two condensates. As a result, θ_L and θ_R are assumed to be different. We believe this phase difference can be observed by studying the interaction between the two condensates.

The physical picture is the following. Two independent condensates are formed within the two wells of the MIT trap. The trapping potential is removed at time $t=0$ leaving the two condensates to ballistically expand. There will be a time when the two condensates begin to overlap. What will happen?

We suggest the following answer. Since there are no obvious dissipation mechanisms, we assume that each condensate maintains its macroscopic coherence during ballistic expansion. This assumption is consistent with the work of Holland *et al.* [10]; however, further work on the kinetics of condensates is needed for its proper justification. When the two condensates occupy the same region of space, interactions between the condensates should either destroy their coherence or bring about a uniform condensate. The time scale for such a transformation should be the time needed for a significant number of collisions to take place, on order of the thermalization time. For times much shorter than this we believe it is reasonable to treat the condensates as independent with the sum of their wave functions approximating the total amplitude of the system. Thus, the probability density will be of the form

$$\rho(\mathbf{r},t,\theta)=|\psi_L(\mathbf{r},t)+\psi_R(\mathbf{r},t)e^{i\theta}|^2. \tag{2}$$

This will produce an interference pattern in space.

To demonstrate this explicitly, we work with a one-dimensional model of the system—the essential physics remains the same in the three-dimensional case. First, the condensate wave function of a set of bosons in a harmonic trapping potential must be found. We assume it is the solution of the nonlinear Schrödinger equation [11]

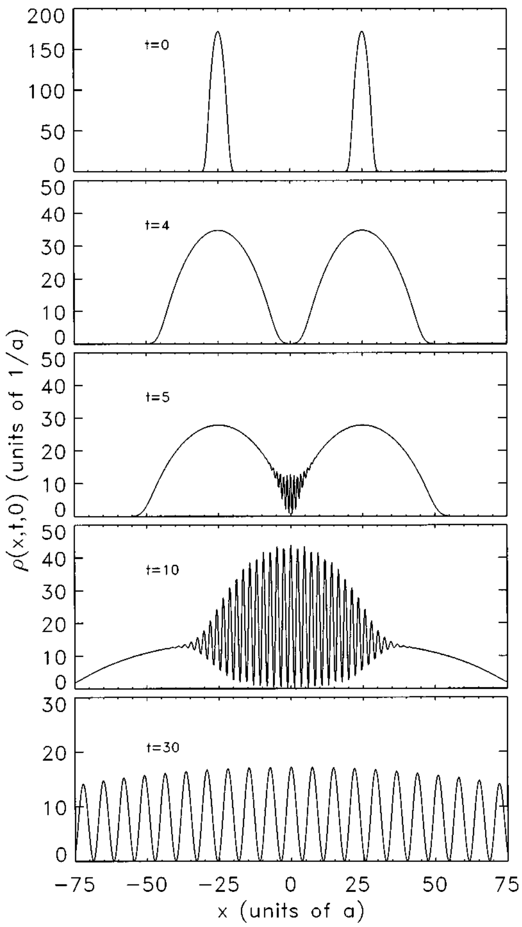

FIG. 1. The time development of the interference pattern of two condensates as they ballistically expand through one another.

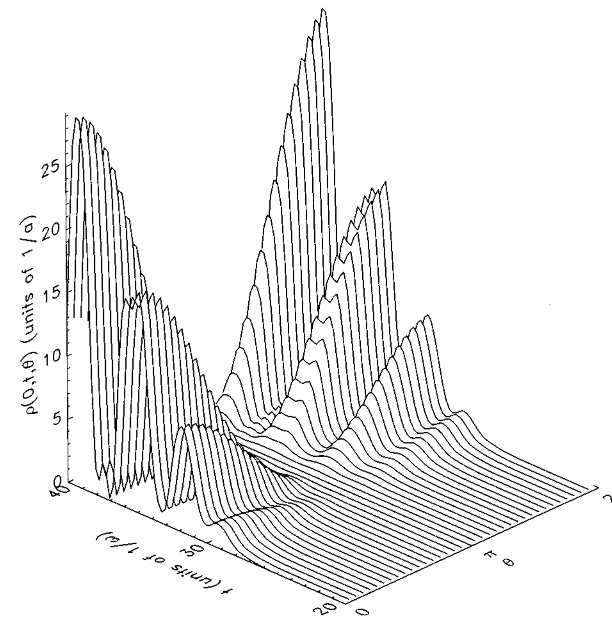

FIG. 2. The time development of the density at $x=0$ due to the interference for various values of the relative phase. In practice this density is proportional to the total number of photons scattered. Therefore, resonant light can be used such that each atom can scatter many photons during the time it is inside the light sheet located in the plane $x=0$.

$$\left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2}+\frac{1}{2}M\omega^2x^2+u_0|\psi(x)|^2\right]\psi(x)=\mu\psi(x), \quad (3)$$

which is solved numerically. Details on how various groups obtain solutions can be found elsewhere [12–14]. For our purposes, once a solution is found, a duplicate can be made, given a phase and translated to represent a second condensate. In this way we represent the two condensates of the MIT trap. We explore the time evolution of the ballistic expansion by using Eq. (3) with no trapping potential and $\mu\psi$ replaced by $i\hbar(\partial\psi/\partial t)$. As was stated previously, the two condensates are propagated *independently* and it is assumed that the sum of the amplitudes will give a reasonable approximation to the total amplitude. Also in keeping with our preceding discussion we assume the nonlinear Schrödinger equation is valid for the description of condensates undergoing ballistic expansion [10].

Plots of the probability density (2) based on this model are given in Figs. 1 and 2 for two cases of interest. In both figures lengths are scaled by the size of the harmonic trapping potential, $a=\sqrt{\hbar/2M\omega}$, times are scaled by the oscillator frequency, $1/\omega$, and energies by $\hbar\omega$. Each condensate has $N_0=10^3$ particles, a scattering length of $a_{sc}=10^{-3}a$, and chemical potential $\mu=4.46(\hbar\omega)$. The strength of the two-body interaction is given by $u_0^{1D}=4\pi\hbar^2a_{sc}/Ma^2$. These values are reasonable for approximating the MIT trap in this model. In Fig. 1 we show the time evolution of the spatial part of the probability density (2) as given by our one-dimensional model of the experiment. Initially the two condensates were located at $x=\pm 25a$ and were assumed to be independent. Their ballistic expansion starts with the sudden turning off of the trap potential at $t=0$. One sees that, as the two condensates begin to overlap, an interference pattern develops. This pattern appears to be extremely robust and maintains its form during ballistic expansion. Figure 2 represents a prediction that may be easier to measure. There the density at the point of reflection symmetry ($x=0$ in Fig. 1) as a function of time and relative phase between the two condensates is plotted. Such a measurement could be made by using a laser to probe the density of atoms in a region of space. All of the plots in Figs. 1 and 2 represent times up to about five or six oscillation periods of the original trapping potential.

We have proposed a measurement that would test our understanding of Bose-Einstein condensation as a state with broken global gauge invariance.

We acknowledge very fruitful discussions with V. Kharchenko and W. Ketterle. This work is supported by NSF through a grant for the Institute for Theoretical Atomic and Molecular Physics at Harvard University and Smithsonian Astrophysical Observatory.

[1] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).
[2] C. C. Bradley, C. A. Sackett, J. J. Tollett, and R. G. Hulet, Phys. Rev. Lett. **75**, 1687 (1995).
[3] K. B. Davis, M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
[4] P. W. Anderson, *Basic Notions of Condensed Matter Physics* (Benjamin-Cummings, Menlo Park, CA, 1984).
[5] J. F. Annett, Contemp. Phys. **36**, 423 (1995).
[6] J. Javavainen and S. M. Yoo, Phys. Rev. Lett. **76**, 161 (1996).
[7] K. Huang, *Statistical Mechanics* (Wiley, New York, 1963); S. Flugge, *Practical Quantum Mechanics* (Springer, New York, 1974).
[8] S. T. Beliaev, Zh. Éksp. Teor. Fiz. **34**, 417 (1958) [Sov. Phys. JETP **7**, 289 (1958)]; see also N. M. Hugenholtz and D. Pines, Phys. Rev. **116**, 48 (1959).
[9] A. J. Moerdijk, W. C. Stwalley, R. G. Hulet, and B. J. Verhaar, Phys. Rev. Lett. **72**, 40 (1995); J. R. Gardner, R. A. Cline, J. D. Miller, D. J. Heinzen, H. M. J. M. Boesten, and B. J. Verhaar, *ibid.* **74**, 3764 (1995); R. Côté, A. Dalgarno, and R. G. Hulet, *ibid.* **74**, 3581 (1995).
[10] M. Holland and J. Cooper, Phys. Rev. A **53**, R1954 (1996).
[11] V. L. Ginzburg and L. P. Pitaevskii, Zh. Éksp. Teor. Fiz. **34**, 1240 (1958) [Sov. Phys. JETP **7**, 858 (1958)]; L. P. Pitaevskii, *ibid.* **13**, 451 (1961); E. P. Gross, J. Math. Phys. **4**, 195 (1963).
[12] M. Edwards and K. Burnett, Phys. Rev. A **51**, 1382 (1995); P. A. Ruprecht, M. J. Holland, K. Burnett, and M. Edwards, *ibid.* **51**, 4704 (1995).
[13] L. You, M. Lewenstein, R. J. Glauber, and J. Cooper, Phys. Rev. A **53**, 329 (1996).
[14] W. Hoston and L. You (unpublished).