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Accelerating light clocks

F. V. Kowalski
Department of Physics, Colorado School of Mines, Golden, Colorado 80401
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The behavior of light clocks which undergo constant acceleration in flat space-time is considered. Time dilation, the Doppler shift, and clock synchronization are shown to be different from that of comoving unaccelerated clocks. These results are discussed in relation to the nature of time in accelerating systems. [S1050-2947(96)01306-6]

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I. INTRODUCTION

The time dilation of inertial clocks is a consequence of the postulates of special relativity. This is a property of space time and therefore not dependent on any particular clock mechanism. In general, the period of an accelerating clock *is* dependent on acceleration. This is a result of the influence that acceleration has on the clock dynamics and is perhaps best illustrated using a pendulum clock.

Nevertheless, it seems reasonable to assume that a unique underlying structure of time exists for accelerating systems, independent of these dynamical effects. The standard assumption about this nature of time is that it is unaffected by acceleration [1]. In other words, the rate at which an accelerated clock ticks is equal to that of a comoving unaccelerated clock. This assumption, which is in addition to those of special relativity, is denoted ACP for the accelerated-clock principle [2].

As an example of the consequences of the ACP, consider a grid of spatially separated clocks which is synchronized in an inertial frame S and is then accelerated into another inertial frame S', while the clocks maintain their distance of separation in frame S. The ACP predicts that these clocks are no longer synchronized in S' although they remain synchronized to an observer in S [3].

Alternate clock hypotheses and their effects are discussed by Mainwaring and Stedman [2]. However their discussion is limited to a comparison of the infinitesimal time interval of an accelerating clock with the corresponding infinitesimal time interval of a comoving inertial clock. Effects which are either quadratic in these time intervals or depend on the size of the clock are overlooked when only infinitesimal time intervals are considered.

A fundamental limit on the ACP is described by Mashhoon [4]. He argues that deviations from the predictions of the ACP will be manifest for length and time scales which are comparable to the acceleration scale of the observer.

The purpose of this work is to consider a time structure different from that given by the ACP. If this is to be clock independent then a study of clock dynamics will most likely not be fruitful in formulating a new hypothesis. Similarly, in any experimental verification of this structure, the effect that acceleration has on the dynamics of a particular clock mechanism must either be negligible or neutralized.

As an example, consider a clock made from a mass and spring. As the spring is stretched by the acceleration, the nonlinearities in the spring constant will effect the period. Such effects, which are not uniform even among similar clocks, will have to be calibrated and removed from the consideration of the structure of time.

Note, however, that in describing the behavior of clocks in special relativity no recourse is made to dynamics. The nature of time is found using a light clock, which is kinematic in nature and relies directly on the postulates of the theory for its mechanism.

In a similar manner, a study of an accelerating light clock could yield the structure of time in accelerating systems. Here, this assumption will be applied separately to linear and circular motion.

Before embarking on this program it is worth considering how the results of the calculation can be confirmed experimentally. The justification for time dilation comes from a comparison of the decay rates of stationary and moving particles. Also, the Doppler shift is used in Mössbauer spectroscopy to compare the rates of nuclear clocks in both linear and circular motion. Therefore both time dilation and the Doppler shift need to be examined.

II. LINEAR MOTION

To facilitate the time dilation calculation, a simple model of the light clock is chosen. In the inertial frame in which the clock is initially at rest, denoted by the unprimed frame, the clock consists of a source separated from a plane mirror by a distance L_0. An emitted pulse of light returns to the source in a time $\Delta T = 2L_0/c$ which results in one tick of the clock.

This light clock is then accelerated at a constant rate a and acquires speed v, as seen in the unprimed frame. It is in this frame that the properties of the light clock are determined. To facilitate the calculation, aL_0/c^2 and v/c are considered small and the period is calculated as a power series expansion in these parameters.

A. Time dilation

1. Vertical light clock

This clock is now accelerated in a direction perpendicular to the light ray of the inertial light clock described above. Such a clock is called a vertical light clock. In addition, a spherical light pulse is sent from the source to the mirror so that some fraction of the emitted pulse returns to the source, which is displaced from its original position due to the acceleration.

The relation between the period ΔT as seen in this inertial frame and the proper period $\Delta T'$ of the accelerating clock is derived closely following the standard time dilation calculation of a light clock in special relativity [5].

In this derivation it is assumed that the angle of incidence is equal to the angle of reflection for a ray of light which reflects from a mirror which is accelerating in a direction tangential to its surface. Such a result is true for a mirror which moves at an arbitrary but constant speed tangentially to its surface [6]. Experimental verification of this assumption to order v^2/c^2, under conditions of centripetal acceleration of magnitude 4×10^3 m/sec^2, has been obtained [7].

Consider now the case where the accelerating clock has no initial velocity. That is, at $t=0$ a pulse is emitted and the initial speed of the clock is zero. The trajectory of the ray which is emitted from and received by the source is triangular in nature, the hypotenuse of which has a base of length $a(\Delta T)^2/4$, and a height L_0. The equation which determines the transit time is then given by

$$c\Delta T=2\{[a(\Delta T)^2/4]^2+L_0^2\}^{1/2}. \tag{1}$$

Changing to the dimensionless parameter $w=c\Delta T/2L_0$, using a power series expansion in the parameter $a^2L_0^2/c^4$, and then rewriting the result in terms of ΔT yields

$$\Delta T=\frac{2L_0}{c}\left[1+\frac{a^2L_0^2}{2c^4}+\cdots\right]. \tag{2}$$

In this calculation the constraint $2aL_0/c^2<1$ is needed in order to obtain a real value for ΔT.

In the following calculations only terms up to those proportional to c^{-3} will be maintained. To this approximation, the time ΔT, observed in the instantaneously comoving inertial frame of the accelerating clock is the same as that measured in the rest frame of the unaccelerated clock. This period is assumed to be the time registered by the accelerating clock $\Delta T'$. Such a result appears to be consistent with the ACP. However this is deceiving since the period of an accelerating clock, as viewed from the unprimed frame, does not agree with the predictions of the ACP when the clock has an initial velocity, as is now discussed.

Next, consider these events of emission and reception of the pulse in the unprimed frame, when the clock has an instantaneous initial velocity v. Again the trajectory of the ray which is emitted from and received by the source is triangular in nature. However now the hypotenuse has a base of length $v\Delta T/2+a(\Delta T)^2/4$ while the height is again L_0. The equation which determines the transit time is then given by,

$$c\Delta T=2\{[v\Delta T/2+a(\Delta T)^2/4]^2+L_0^2\}^{1/2}. \tag{3}$$

Again, changing to the dimensionless parameter $w=c\Delta T/2L_0$, using a power series expansion in the parameters aL_0/c^2 and v/c, and rewriting the result in terms of ΔT yields

$$\Delta T\approx\Delta T'\left[1+\frac{av\Delta T'}{2c^2}+\frac{v^2}{2c^2}\right], \tag{4}$$

where $\Delta T'=2L_0/c$. As before, only terms up to c^{-3} have been maintained.

2. Horizontal light clock

A horizontal light clock is one in which the acceleration is in a direction parallel to the light ray of the inertial light clock described above. The round trip transit time for this configuration is now calculated assuming an initial velocity v (the case with zero initial velocity is given by this result with $v=0$).

The relevant assumptions, approximations, and definitions of the previous subsection are maintained. However in this case an additional assumption, about the separation of the source and mirror, is needed. There are two prominent choices: the proper separation of these clock components is maintained during acceleration or their separation in the inertial frame S is maintained. Only the former choice is considered here.

In frame S, acceleration of the light clock, in which the proper length between the components is maintained, requires that the acceleration be different for the source and mirror by a factor of $a/(1-aL_0/c^2)$, where L_0 is the proper distance between the components [8]. However, since it is assumed that aL_0/c^2 is small, the correction due to the spatial dependence of the acceleration is of higher order in the power series expansion for the period and is neglected. In this approximation, the acceleration of both components appears to be the same in frame S.

There is also a conceptual problem in defining proper length for an object accelerating in this manner [9]. Simultaneously in frame S, different parts of the accelerating object move at different speeds. This issue will be dealt with by approximating the length of the accelerating object with that of a comoving but unaccelerated object.

Let the constant acceleration vector point in the direction from the source to the mirror. The transit time of the pulse from the source to the mirror T_{out} is then given by

$$cT_{\mathrm{out}}\approx L_0(1-v^2/c^2)^{1/2}+vT_{\mathrm{out}}+\frac{a(T_{\mathrm{out}})^2}{2}. \tag{5}$$

The return transit time T_{return} is given by

$$L_0[1-(v+aT_{\mathrm{out}})^2/c^2]^{1/2}-cT_{\mathrm{return}}\approx(v+aT_{\mathrm{out}})T_{\mathrm{return}}+\frac{a(T_{\mathrm{return}})^2}{2}. \tag{6}$$

Changing to the dimensionless parameters $w_{\mathrm{out}}=cT_{\mathrm{out}}/2L_0$ and $w_{\mathrm{return}}=cT_{\mathrm{return}}/2L_0$ in the appropriate equations above, using a power series expansion in the parameters aL_0/c^2 and v/c, and then rewriting the result in terms of T_{out} and T_{return} yields

$$\Delta T=T_{\mathrm{out}}+T_{\mathrm{return}}\approx\Delta T'\left[1-\frac{aL_0}{2c^2}+\frac{3av\Delta T'}{4c^2}+\frac{v^2}{2c^2}\right], \tag{7}$$

where $\Delta T' = 2L_0/c$, is the period registered in the frame of the accelerating vertical clock. Note that the periods of the horizontal and vertical light clocks differ, in the instantaneously comoving inertial frame ($v=0$), by the term proportional to $aL_0/2c^2$.

B. Doppler shift

Next, consider the Doppler shift, which will be calculated following the standard derivation used in special relativity [10]. However, in the case to be studied here, an electromagnetic wave is emitted by an accelerating source.

Let the source accelerate either directly toward or away from the receiver which is in an inertial frame. The source and receiver are fixed with respect to the coordinate axes in the primed and unprimed frames, respectively. Consider the wavelength of an electromagnetic wave emitted by the source, as seen in the unprimed frame. Also, in this frame let the source emit a wavecrest at time T_1 and then emit the next wavecrest at time T_2, where $\Delta T = T_2 - T_1$.

The wavelength is then determined as follows: $c\Delta T$ is the distance a wavecrest moves while $v\Delta T + a(\Delta T)^2/2$ is the distance the source moves before the next wavecrest is emitted. The wavelength is then $\lambda = (c \mp v \mp a\Delta T/2)\Delta T$, where the minus and plus signs correspond to the source approaching or receding from the receiver, respectively.

Now ΔT is not the period of the wave as seen in the inertial frame. It is the period of the accelerating clock as seen in the inertial frame. For motion at constant speed the period of the moving clock is related to its rest or proper period via the time dilation formula.

Although it is not a harmonic wave, a given wavecrest of this wave propagates at the phase velocity in a vacuum due to the dispersion relation $\omega = ck$, where $k = 2\pi/\lambda$. To maintain this relation, the frequency received ν for the wave whose wavelength is defined above must obey $\nu = c/\lambda$ or

$$\nu = \frac{1}{1 \mp v/c \mp a\Delta T/2c}\,\frac{1}{\Delta T}. \tag{8}$$

The frequency of the clock in the primed frame is given by $\nu_0 = 1/\Delta T'$, where $\Delta T'$ is the time interval between emission of consecutive wavecrests in the primed frame. To determine the Doppler effect a relation between the clock periods in the two frames $\Delta T'$ and ΔT is needed. The ACP yields the relation $\Delta T = \Delta T'\gamma$, where $\gamma = [1-(v/c)^2]^{-1/2}$, while this relation for a light clock is given by Eq. (4).

Consider first the Doppler shift using the ACP. Inserting $\Delta T = \Delta T'\gamma$ and $\nu_0 = 1/\Delta T'$ into Eq. (8) and expanding to terms proportional to c^{-2} yields

$$\nu \approx \nu_0\left[1 \pm \frac{v}{c} + \frac{v^2}{2c^2}\right] + \frac{a}{2c}\left[\mp 1 + \frac{2v}{c} + \frac{a}{2\nu_0 c}\right], \tag{9}$$

where the minus and plus signs correspond to the source receding or approaching the receiver, respectively.

Next consider the Doppler shift using the result for the accelerating vertical light clock. Insertion of Eq. (4) with the substitution $\nu_0 = 1/\Delta T'$ into Eq. (8) and expanding to terms proportional to c^{-2} yields

$$\nu \approx \nu_0\left[1 \pm \frac{v}{c} + \frac{v^2}{2c^2}\right] + \frac{a}{2c}\left[\mp 1 + \frac{v}{c} + \frac{a}{2\nu_0 c}\right]. \tag{10}$$

The first bracketed term in the two previous equations is the Doppler shift given in special relativity while two of the remaining terms are independent of both the frequency and the size of the clock.

Note that in the Doppler relations the instantaneous velocity of the source is $v = at$. The receiver will measure a wavecrest which is delayed by the transit time between the source and receiver. The finite speed of propagation of the wavecrest is accounted for by using the retarded time $t_r = t - x/c$ in the expression for the instantaneous velocity.

C. Spatially separated clocks

The synchronization of spatially separated inertial clocks is frame dependent. It is therefore of interest to study this synchronization issue for accelerating light clocks.

Again, a simple clock model is chosen in which three clocks, aligned in a linear array, are at rest in an inertial frame S and then accelerate into another inertial frame S'. The acceleration vector is in the direction parallel to the line connecting the clocks. Initially, each clock is separated from the next one by a distance H. The master clock, located in the middle, generates timing light pulses for the other clocks. That is, reception of a light pulse from the master clock triggers the generation of the light pulse which is then used in the clock mechanism as described above.

Again there are two prominent choices for the acceleration: the proper separation of the clocks is maintained during acceleration or their separation in the inertial frame S is maintained. The latter choice will be dealt with first.

Consider the clock behavior as seen from frame S, where the clocks maintain their separation distance H. After having received a few pulses from the master clock, the clocks accelerate simultaneously in frame S. Upon acquiring speed they are no longer synchronized in frame S, since the trailing clock receives a given pulse from the master clock before the leading clock. After the acceleration has terminated, their lack of synchronization in frame S is just that given in special relativity by spatially separated clocks which are synchronized in frame S'.

To better illustrate this, let the master clock emit a total of N pulses. Also, once the clocks reach frame S', let the time spent there be long compared with the transit time of a pulse from the master clock to the other clocks, before the Nth pulse is emitted.

Consider the time registered by each clock just before receiving the Nth pulse from the master clock. The trailing and leading clocks have both registered $N-1$ ticks. However, the trailing clock receives the Nth pulse before the leading clock, as seen in frame S. In frame S', on the other hand, the Nth pulse is received simultaneously by the front and rear clocks and therefore, in this frame the clocks remain synchronized.

In this discussion it has been assumed that, although their separation in frame S' is no longer H, the two end clocks remain equidistant from the center clock. This is consistent with the interpretation given in special relativity of the initial and final clock configurations [11].

The same procedure can be carried out under the assumption that the clocks maintain their proper separation during the acceleration. However, the conclusion that the clocks preserve their synchronization in frame S', does not change.

III. CIRCULAR MOTION

Now consider a light clock which follows a circular trajectory of radius R with constant angular velocity. A clock comparison is made with an identical clock in an inertial frame at the center of the circle.

As an operational scheme for such a comparison, let a spherical electromagnetic wave be emitted from the inertial clock. The period of this wave is determined by the period of the inertial clock. The issue to be addressed is how the period of this wave compares with that of the period of the rotating clock. If the periods coincide then, for an atomic or nuclear clock, this result manifests itself in the absorption of the electromagnetic wave. Since absorption is frame independent the result does not depend on which frame the calculation is performed in.

The clock comparison is now made in the frame of the inertial clock. Let the light clocks be constructed as described above. The period of the inertial clock is $\Delta T = 2L_0/c$. To calculate the period of the rotating clock a spatial orientation for the rotating light clock must be chosen. Let the source be a distance R away from the axis of rotation and the mirror be aligned along a line extending from the center of the circle to the source, a distance $R-L_0$ away from the center of the circle, as shown in Fig. 1.

Consider consecutive spherical pulses emitted from the inertial clock at the center of the circle. When the first pulse reaches the source in the rotating clock, the rotating clock emits its clock pulse which reflects from the mirror and returns to the source. The next pulse from the inertial clock arrives at the source in a time $\Delta T = 2L_0/c$. The time of arrival of the pulse which was emitted from the rotating clock back to the source is now shown to be different from that of the arrival time of the second pulse from the inertial clock.

However, an issue arises with regard to the angle of reflection of a light ray from the rotating mirror. By considering this mirror to be a transponder which emits a spherical pulse upon reception of a pulse, the detailed knowledge of the reflection mechanism can be avoided.

The period ΔT_{rot} of the rotating clock, as seen in the inertial frame, is determined using Fig. 1. During the time ΔT_{rot} the source travels an arc length $v\Delta T_{\mathrm{rot}} = R\theta$. However, the light pulse travels a distance $2x$, where x is given by

$$x^2 = (R-L_0)^2 + R^2 - 2R(R-L_0)\cos(\theta/2). \tag{11}$$

In the time ΔT_{rot} this pulse travels a distance $2x$. Using this relation to eliminate x from Eq. (11) and using the above relation for θ in terms of ΔT_{rot} yields a nonlinear equation in ΔT_{rot}. This can be reduced to a quadratic equation with the approximation that $v/c \ll 1$. The solution of this equation is

$$\Delta T_{\mathrm{rot}} \approx \Delta T\left[1 + \frac{v^2}{2c^2} - \frac{aL_0}{2c^2}\right], \tag{12}$$

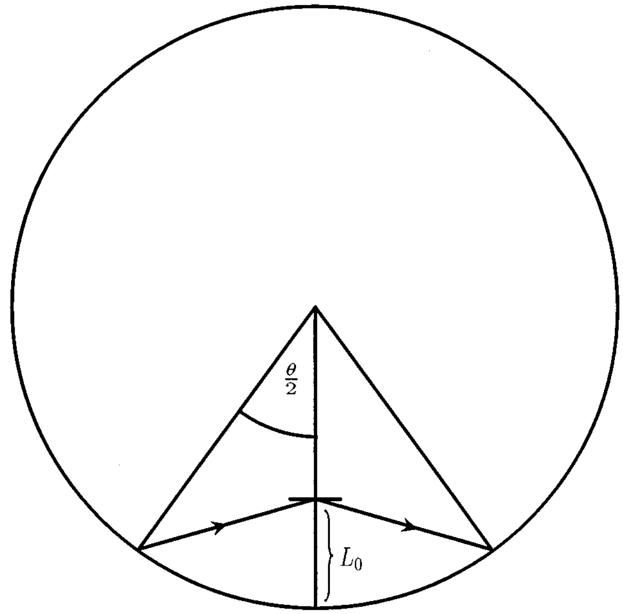

FIG. 1. A light clock moving in a circle as viewed from an inertial frame. The trajectory of the pulse of light is shown by the arrows.

where the centripetal acceleration is given by $a = v^2/R$.

The first two terms of this result correspond to the transverse Doppler shift. Note that the final term does depend on the dimensions of the clock.

IV. EXPERIMENTAL VERIFICATION

Experimental consequences of the above predictions fall easily into two types. One involves the decay rate for moving particles while the other encompasses Doppler shift measurements.

Consider the experimental evidence with regard to the predictions of Sec. II B. Here the largest modification to the standard Doppler shift is given by $a/2c$. In Mössbauer experiments, the frequency of the clock is typically 4×10^{18} Hz with a half maximum full resonance width of about 3×10^{6} Hz. In most applications of this technique, any observable frequency shift is a small fraction, typically 1/200, of the resonance width. This requires a linear acceleration greater than 10^{13} m/s^2 to detect this new term.

Commercial devices can have accelerations of the order 10 m/s^2 [12]. Lattice vibrations on the other hand generate periodic motion of the nuclear clock with accelerations of order 10^{17} m/sec^2. However, since the lifetime of the excited state in this case is long compared with the period of oscillation, effects linear in velocity and acceleration cancel [13].

A more promising technique is that used by Vessot *et al.* [14] in which an atomic clock on the surface of the earth is compared with one in free fall. The maser clock frequency is of the order 10^9 Hz with a bandwidth of the order 1 Hz [15]. The precision of the experiment allows a measurement of a frequency shift $\Delta\nu/\nu \approx 2\times10^{-15}$. The modified term in the Doppler shift accounts for a shift $\Delta\nu/\nu = a/(2c\nu) \approx 10^{-17}$.

However, since such an experiment is done in curved space time, any extrapolation of the results to flat space time requires additional assumptions. Nevertheless, the advantage of using a smaller clock frequency to detect this term is apparent.

The second type of experiment involves particle decay. In a linearly accelerating system this has not, to my knowledge, been directly tested. However, consider the time dilation of muons as they fall toward the earth. Again, additional assumptions are necessary to apply the flat space-time result of Eq. (4) to the curved space time of this example. Independent of such assumptions, the precision of experiments which measure this muon decay is still insufficient to verify the acceleration dependent term in Eq. (4).

Next consider the predictions of Eq. (12) for circular motion. The $aL_0/2c^2$ term depends on the interpretation given to L_0. For an atomic or nuclear clock, let L_0 be of order the size of the atom or nucleus.

The transverse Doppler shift has been measured [16] for circular motion using the Mössbauer effect to a precision of 4×10^{-2} for $\beta=v/c\approx10^{-6}$. The $aL_0/2c^2$ term, however, contributes an effect given by $\beta^2L_0/R\approx10^{-28}$.

Time dilation has been measured for muons in a circular orbit [17] to a precision of 2×10^{-3}. Here, the $aL_0/2c^2$ term contributes an effect given approximately by $L_0/R\approx10^{-16}$.

Therefore the available experimental evidence is insufficient to rule out the acceleration dependent terms in Eqs. (4), (9), (10), and (12).

V. DISCUSSION

It is interesting to note the similarities between the results obtained in the above calculations and related work on accelerating systems. The a/c and $a^2/(\nu_0c^2)$ terms in the Doppler shift, given in Eqs. (9) and (10), are a consequence of both the ACP and the light-clock model. Terms proportional to these have been calculated for the frequency of a light wave as seen by an accelerating *receiver* using the ACP [18].

However, it should be pointed out that this shift, for light reflecting from an accelerating mirror, is an issue different from that of the Doppler shift discussed above. The former frequency shift is determined by the transit time of a wavecrest to and from the moving mirror [19].

Also, in calculating the period of the accelerating light clock, the constraint $2aL_0/c^2<1$ is necessary to obtain a real value for ΔT [see Eq. (2)]. It is interesting to note that the proper length of an accelerating rod is limited by a similar constraint [20]. This constraint limits the period of the clock, given by $\Delta T\approx2L_0/c$, to $\Delta T<c/a$.

The focus of this work has been on the effect of acceleration on light clocks. In special relativity such a study is fundamental to the theory. Light clocks, which are kinematic in nature, predict a time structure in which all clocks, independent of mechanism, exhibit time dilation.

Yet this connection is severed in the ACP. The kinematic consequences described here for light clocks are not those allowed for a clock in the ACP. If clocks, governed by dynamics such as atomic and nuclear clocks, do obey the ACP then there must be at least two times: dynamic and kinematic (or light-clock) time.

However since the period of the vertical light clock, given in Eq. (4), differs from that predicted by the ACP in the term which is quadratic in the period, one might argue that by choosing a light clock whose size is arbitrarily small, the results of the ACP can be reproduced.

Nevertheless, using the above approximations, two effects, which differ from the predictions of the ACP, remain. First, the Doppler shift in Eq. (10) has a term in it, $av/2c^2$, which is different from the corresponding term in the Doppler shift prediction of the ACP, given in Eq. (9), by a factor of 1/2. This term is independent of clock size. Second, the ACP predicts that spatially separated clocks, synchronized in one inertial frame, do not remain synchronized when they are accelerated into a new inertial frame. It was shown in Sec. II C that spatially separated light clocks, which are synchronized in one inertial frame, maintain their synchronization after accelerating into another inertial frame. The consequences of this result have been considered before [21].

The spatial extent of the light clock, whose acceleration is along the direction of the light ray, also influences the rate at which it ticks. This is true for both for the horizontal light clock [see Eq. (7)] and the light clock in circular motion [see Eq. (12)]. Although a small clock (e.g., nuclear) can again be chosen so that these effects are negligible, the implication is that a clock of larger size (e.g., biological) would age at a different rate. This also contradicts the ACP, which predicts that the clock rate is independent of the clock size. These results then cast doubt on the original premise of this work which was that the study of light clocks would lead to a unique structure of time in accelerating systems.

The relationship between kinematics and dynamics in accelerating systems will be resolved only when predictions, such as those above, are tested with clocks whose mechanism is determined by dynamics (with the stipulation given in the introduction regarding the effects of acceleration). Such experiments will influence both our understanding of the nature of time in accelerating systems and of the theory of general relativity via the equivalence principle.

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