# Angular distributions of electrons following photoionization of spherically symmetric Rydberg states in alkaline-earth atoms

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Two theoretical treatments have been employed to investigate various angular distributions of ejected electrons following the photoionization of the $msns$ Rydberg states in the alkaline-earth atoms. The specific analytic expressions of angular distributions in terms of reduced dipole matrix elements have been carried out. The characteristics of angular distributions are interpreted in detail.

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## I. INTRODUCTION

For the past decade, extensive studies of autoionizing Rydberg series of the alkaline-earth atoms have been carried out, most of which have been focused on the line shapes of transitions and on the manifestations of the interaction of different autoionizing series in the excitation spectra [1–8]. Relatively speaking, less attention has been paid to energy and angular distributions of ejected electrons originating from the autoionization process.

Since the ions produced from autoionization process provide no information about the final states of ion core or of the ejected electrons, it is necessary to explore the properties of atoms further to reach a detailed understanding of the dynamics of autoionization and structure of atoms by studying the ejected electrons. The augular distribution of electrons depends very strongly on excitation amplitudes as well as continuum phases of the possible channels, while line shapes are determined only by the excitation amplitudes.

Up to now, with several exceptions [9,10], the experiments of angular distributions have been carried out by populating autoionizing states using the isolated-core excitation (ICE) scheme [11], in which an autoionizing state is reached by exciting a Rydberg atom. In the ICE approximation a photon only acts on the core electron due to the fact that the outer electron spends so little time near the nucleus that it cannot absorb a photon. As demonstrated recently [12,13], characterization of angular distributions of ejected electrons requires different numbers of the asymmetry parameters depending on whether the final excitation in the ICE is from an unpolarized or a polarized target. In this paper, we focus on angular distributions of electrons from the photoionization of an unpolarized target, or the $msns$ Rydberg states.

Measurements of angular distribution (AD) in analogous doubly excited autoionizing Rydberg status of Mg [12], Ca [4], Sr [5], and Ba [14–16] ($m=3$, 4, 5, and 6, respectively), have been performed previously. In these studies, the data were analyzed with the assistance of the angular-momentum transfer theory (AMTT) [17,18]. Here we work out the explicit expressions of AD in terms of specific dipole matrix elements using AMTT and an alternative approach, which was developed for the angular distributions of electrons from the photoionization of a polarized target using photoexcitation theory (PT).

In the following sections of the paper we outline the framework of AMTT and of PT, and present the explicit expressions of AD, and a discussion of the results, including the detailed comparisons between the two approaches.

## II. THEORETICAL FRAMEWORK

### A. PT method

Symmetry considerations and angular-momentum selection rules constrain AD to be of the form

$$\frac{d\sigma}{d\Omega}(\theta)=\frac{I_0}{4\pi}[1+\beta P_2(\cos\theta)], \tag{1}$$

or alternatively,

$$\frac{d\sigma}{d\Omega}(\theta)=\sum_k a_k P_k(\cos\theta), \quad \text{with } k=0,2, \tag{2}$$

where $P_k(\cos\theta)$ is the $k$th Legendre polynomial, and $\theta$ is the polar angle between the laser polarization and the direction of ejected electron; $I_0$ and $a_k$ are any real numbers, $I_0$ as a function of energy gives the total photoexcitation cross section, or the shapes of the final transition. The asymmetry parameter, $\beta$, varies as a function of energy, and manifests the angular distribution.

Equivalence of Eqs. (1) and (2) yields $I_0=4\pi a_0$ and $\beta=a_2/a_0$. In order to derive a general expression for the $\beta$ parameter, one must start from the most general formula for $d\sigma/d\Omega$ of an atom absorbing a linearly polarized photon and then ejecting an electron. This gives [13]

$$a_k=\frac{N^2}{4\pi}\sum_{\substack{\mathbf{J}_c,\mathbf{J}_{cs}\\ ll'JJ'}}(-1)^{\mathbf{J}_{cs}}[J][J'][l][l'][k]^2 CD_{\mathbf{J}_{cs}lJ}D^*_{\mathbf{J}_{cs}l'J'}, \tag{3}$$

where $N^2$ is a normalization constant. Since the factor $(N^2/4\pi)$ is common to all of the $a_k$ and cancels out in calculating the $\beta$ parameter, we will ignore it from now on. $\mathbf{J}_c$ is the total residual ion-core angular momentum. $\mathbf{J}_{cs}=\mathbf{J}_c+\mathbf{s}$, where $\mathbf{s}$ and $\boldsymbol{l}$ are the spin and the orbital angular momentum of the outer electron, respectively. $\mathbf{J}$ is the total angular momentum of the system, i.e., $\mathbf{J}=\mathbf{J}_{cs}+\boldsymbol{l}$. The symbol $[x]$ is

 

depicted as $[2x+1]^{1/2}$, where $x=J$, $J'$, $l$, $l'$, or $k$. $D_{\mathbf{J}_{cs}lJ}$ in Eq. (3) is an abbreviation of the reduced dipole matrix elements, i.e.,

$$D_{\mathbf{J_{cs}}lJ}=\langle (J_{cs}l)J_-\|r^{(1)}\|J_i\rangle, \tag{4}$$

where $r^{(1)}$ is a first-rank tensor, and $\mathbf{J}_i$ is the total angular momentum of the initial state (unpolarized target). The minus sign in the matrix element indicates that the wave function of the final state is normalized according to incoming-wave boundary conditions, namely, at large $r$ the wave function has the form of a plane wave plus incoming spherical waves. The coefficient $C$ in Eq. (3) is a product of several $3j$ and $6j$ symbols, i.e.,

$$C=\begin{bmatrix} J_i & 1 & J \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} J' & 1 & J_i \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} l' & 1 & k \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} k & J & J' \\ 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} k & J' & J \\ J_{cs} & l & l' \end{Bmatrix}. \tag{5}$$

Since $J=1$ is the only possible angular momentum for the autoionizing states in our case, the nonzero $6j$ symbol requires $k=0,2$ only. Thus to determine the $\beta$ parameter, it is necessary to evaluate $a_0$ and $a_2$.

### B. AMTT method

This method has been developed by Fano and Dill [17,18] and used by many authors for AD data analysis. The essence of AMTT, as applied to photoelectron spectroscopy, is that AD can be expressed as a sum of incoherent contributions corresponding to different magnitudes of angular momentum transferred to an unpolarized target. The asymmetry parameter $\beta$ can be conveniently expressed as the weighted average of the contributions from the parity-favored and parity-unfavored transitions, respectively. This can be done by introducing a transferred angular momentum $\mathbf{J_t}$, where

$$\mathbf{J_t}=\mathbf{J_{cs}}-\mathbf{J}_i. \tag{6}$$

Following AMTT, the process in which an atom is excited from $|i\rangle$ Rydberg state to an autoionizing state and then decays into a core and an electron can be devided into two main categories according to the parity of the $|i\rangle$ state and of the core, i.e.,

$$\pi_0\pi_c=\begin{cases} (-1)^{J_t} & \text{parity favored, no spin flip} \quad (7a) \\ -(-1)^{J_t} & \text{parity unfavored, spin flip} \quad (7b) \end{cases}$$

Obviously, in processes (7a) and (7b), the final states are singlet and triplet, respectively, since the atom is initially in a singlet state. Thus for the parity-favored transition we have [18]

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{fav}}=\frac{1}{4\pi}\left[\frac{\Sigma_{J_t}[\sigma(J_t)]_{\text{fav}}[\beta(J_t)]_{\text{fav}}}{\Sigma_{J_t}[\sigma(J_t)]_{\text{fav}}}\right]. \tag{8}$$

Similarly, for the parity-unfavored transition,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unf}}=\frac{\lambda^2}{4\pi(2J_i+1)}\sum_{J_i}(2J_t+1)|S_0(J_t)|^2 \times[1-P_2(\cos\theta)], \tag{9}$$

where

$$[\sigma(J_t)]_{\text{fav}}=\frac{(2J_t+1)\lambda^2}{4\pi(2J_i+1)}[|S_+(J_t)|^2+|S_-(J_t)|^2] \tag{10}$$

and

$$[\sigma(J_t)]_{\text{unf}}=\frac{(2J_t+1)\lambda^2}{4\pi(2J_i+1)}|S_0(J_t)|^2. \tag{11}$$

The $\beta$ parameters for the two transitions are

$$[\beta(J_t)]_{\text{fav}}=\frac{(J_t+2)|S_+(J_t)|^2+(J_t-1)|S_-(J_t)|^2-6[J_t(J_t+1)]^{1/2}\text{Re}[S_+(J_t)S_-^*(J_t)]}{(2J_t+1)[|S_+(J_t)|^2+|S_-(J_t)|^2]} \tag{12}$$

and

$$[\beta(J_t)]_{\text{unf}}\equiv -1, \tag{13}$$

where $\lambda$ is the wavelength of the incident photon, $S_\pm(J_t)$ denotes the photoionization amplitude $S_l(J_t)$ for a given $J_t$ and for $l=J_t\pm 1$, and $S_0(J_t)$ is for the value of $J_t=1$.

Therefore, the partial photoionization cross section of a given ion-core state is

$$\sigma_c=\sum_{J_t}\sigma(J_t). \tag{14}$$

The asymmetry parameter $\beta$ for a given ion-core state is

$$\beta_c=\frac{\sum_{J_t}[\sigma(J_t)\beta(J_t)]_{\rm fav}-\sum_{J_t}[\sigma(J_t)]_{\rm unf}}{\sum_{J_t}\sigma(J_t)}. \quad (15)$$

It is worthwhile to note that $S_l(J_t)$ is a scattering matrix element, which represents the photoionization amplitude. In order to characterize the photoexcitation from the initial state to the final state, $S_l(J_t)$ must be related to the reduced transition dipole matrix element $D_{J_{cs}lJ}$ defined in Eq. (4). For the special case under consideration we obtain

$$S_l(J_t)=\left(\frac{2\alpha\hbar\omega^3}{3c^2}\right)^{1/2}(-1)^{J_i-J-1}[J]\begin{Bmatrix} J_{cs} & 1 & J \\ 1 & J_i & J_t\end{Bmatrix}D_{J_{cs}lJ}. \quad (16)$$

Here, $\alpha$ is the fine-structure constant and $\omega$ the frequency of incident photon.

The energy-dependent dipole matrix element $D_{J_{cs}lJ}$ can be evaluated according to the well-documented procedures [13]. It depends on the specific theoretical model corresponding to different atoms or particular states. Usually, it is derived from multichannel quantum-defect theory [19,20] and will not be discussed here.

Now we are in a position to obtain the specific expressions of the $\beta$ parameter for every possible ion-core state using the two approaches described above and make comparisons between them.

## III. RESULTS AND DISCUSSION

When a photon excites an atom to the $mp_{3/2}ns$, $J=1$ autoionizing state, the $ns$ electron is then scattered from the $mp_{3/2}$ excited core. The inner-core electron's energy, when transferred to the outer electron, is enough to ionize the atom. The ejected electrons have different energies depending on where the ions go. For instance, when the process produces the $mp^+_{1/2}$ ions, the corresponding electrons should have a much smaller energy than when the process produces the $ms^+_{1/2}$ ions. Since AD is dependent on the electrons' energy it is necessary to carry out their expressions for each ionic core.

### A. $ms_{1/2}$ core

In this case, $J_t=J_{cs}=0$, 1, and $l=l'=1$ is the only possible value for the electron ejected from the $mpns$ state as $J=J'=1$. It is straightforward to determine $a_k$ by applying the PT method described previously, i.e.,

$$a_0=\sum_{J_{cs}}(-1)^{J_{cs}}C|D_{J_{cs}11}|^2, \quad (17a)$$

with

$$C=\frac{1}{9}\begin{Bmatrix} 0 & 1 & 1 \\ J_{cs} & 1 & 1\end{Bmatrix}, \quad (17b)$$

or explicitly,

$$a_0=\tfrac{1}{3}[|D_{011}|^2+|D_{111}|^2]. \quad (18a)$$

Similarly, we have

$$a_2=\tfrac{1}{3}[2|D_{011}|^2-|D_{111}|^2], \quad (18b)$$

which yields the $\beta$ parameter and the total cross section of photoabsorption for the $ms_{1/2}$ core as

$$\beta_{ms_{1/2}}=\frac{2|D_{011}|^2-|D_{111}|^2}{|D_{011}|^2+|D_{111}|^2} \quad (19a)$$

and

$$\sigma_{ms_{1/2}}=\tfrac{1}{3}[|D_{011}|^2+|D_{111}|^2]. \quad (19b)$$

Alternatively, one may apply the AMTT method to the same problem, keeping in mind that $J_t=0$ and 1 correspond parity-favored and parity-unfavored transitions, respectively. One thus obtains

$$\beta_{ms_{1/2}}=\frac{2|S_1(0)|^2-3|S_1(1)|^2}{|S_1(0)|^2+3|S_1(1)|^2} \quad (20a)$$

and

$$\sigma_{15s_{1/2}}=[|S_1(0)|^2+3|S_1(1)|^2]. \quad (20b)$$

Substituting $S_1(0)=D_{011}$ and $S_1(1)=-(1/\sqrt{3})D_{111}$ into Eq. (20) one also obtains the same expression as Eq. (19), which verifies that the two methods are equivalent to each other. Since the $msns$ target is unpolarized, AD is less complicated, which is characterized by a single asymmetry parameter $\beta$. It shows simple patterns determined by the second-order Legendre polynomial $P_2(\cos\theta)$.

Theoretically, the $\beta$ parameter depends only on dipole matrix elements of single $J$ values, since $J=1$ is the only possible final state of the transition. This puts the $\beta$ parameter in a simple form so that one may draw some conclusions from its expression. According to Eq. (19a) electrons will eject isotropically when $|D_{111}|^2=2|D_{011}|^2$, indicating a spherically symmetric AD. When $D_{111}=0$ the $\beta$ parameter reaches its maximum value, $\beta_{\rm max}=2$, indicating that the final state of the transition is a parity-favored singlet. When $D_{011}=0$, the $\beta$ parameter will take its minimum value, $\beta_{\rm min}=-1$, representing a parity-unfavored triplet. Obviously $\beta=2$ and $-1$ represent $\sin^2\theta$ and $\cos^2\theta$ angular distributions, respectively.

Experimentally, AD is measured by varying the polar angle $\theta$ between the direction of polarization of light used in excitation and that of electron detector. Note that in this particular experiment only the polarization of light for the $msns\rightarrow mpns$ transition is required to control, while the polarization of light used to prepare the $msns$ Rydberg atoms remains unchanged during the experiment. This certainly simplifies the experiment substantially. Basically, electron signals that are measured at two different $\theta$ angles may uniquely determine the $\beta$ parameter, or AD. To achieve an istropical AD all one has to do is set $\theta=\theta_m=54.7°$, at which $P_2(\cos\theta_m)=0$. At this "magic angle" $\theta_m$, the electron spectrum should be identical to the ion spectrum. In another words, the differential cross section $d\sigma/d\Omega(\theta_m)$ is proportional to the total cross section, which provides a convenient check in the experiment.

### B. $(m-1)d_{3/2,5/2}$ cores

When an atom is ionized to the $(m-1)d_{3/2}$ ion and a $p$-electron or an $f$ electron, $l=1,3$ and $l'=1,3$ are possible, and $J_{cs}=J_t=1,2$. In this case, the PT method gives

$$a_0=(\tfrac{1}{3})[|D_{011}|^2+|D_{211}|^2+|D_{231}|^2] \tag{21a}$$

and

$$a_2=(\tfrac{1}{15})[|D_{211}|^2+4|D_{231}|^2-5|D_{111}|^2-6\sqrt{6}\,\mathrm{Re}(D_{21}D^*_{231})]. \tag{21b}$$

Therefore, we have

$$\beta_{(m-1)d_{3/2}}=\frac{|D_{211}|^2+4|D_{231}|^2-5|D_{111}|^2-6\sqrt{6}\,\mathrm{Re}(D_{211}D^*_{231})}{5[|D_{211}|^2+|D_{231}|^2+|D_{111}|^2]} \tag{22a}$$

and

$$\sigma_{(m-1)d_{3/2}}=a_0. \tag{22b}$$

However, according to the AMTT method, $J_t=1$ and 2 represent parity-unfavored and parity-favored transitions, respectively. Thus one easily obtains

$$\beta_{(m-1)d_{3/2}}=\frac{|S_1(2)|^2+4|S_2(2)|^2-3|S_1(1)|^2-6\sqrt{6}\,\mathrm{Re}[S_3(3)S_1^*(2)]}{5[|S_1(2)|^2+3|S_1(1)|^2+|S_3(2)|^2]} \tag{23a}$$

and

$$\sigma_{(m-1)d_{3/2}}=3|S_1(1)|^2+5|S_1(2)|^2+5|S_3(2)|^2. \tag{23b}$$

One may show that $S_2(2)=0$, $S_1(1)=-(1/\sqrt{3})D_{111}$, $S_1(2)=(1/\sqrt{5})D_{211}$, and $S_3(2)=(1/\sqrt{5})D_{231}$. Substituting them into Eq. (23), one may obtain exactly the same results as Eq. (22).

For the $(m-1)d_{5/2}$ core, the situation is similar to the case of the $(m-1)d_{3/2}$ core. But in this case the autoionization process makes $l=1,3$ and $l'=1,3$ possible. Also $J_{cs}=J_t=2,3$, respectively. Following a similar procedure one obtains

$$a_0=(\tfrac{1}{3})[|D_{211}|^2+|D_{231}|^2+|D_{331}|^2] \tag{24a}$$

and

$$a_2=(\tfrac{1}{15})[|D_{211}|^2+4|D_{231}|^2-5|D_{331}|^2-6\sqrt{6}\,\mathrm{Re}(D_{211}D^*_{231})]. \tag{24b}$$

Thus,

$$\beta_{(m-1)d_{5/2}}=\frac{|D_{211}|^2+4|D_{231}|^2-5|D_{331}|^2-6\sqrt{6}\,\mathrm{Re}(D_{211}D^*_{231})}{5[|D_{211}|^2+|D_{231}|^2+|D_{331}|^2]} \tag{25a}$$

and

$$\sigma_{(m-1)d_{5/2}}=a_0. \tag{25b}$$

Obviously $J_t=2$ represents the parity-favored transition, while $J_t=3$ corresponds to the parity-unfavored transition. They are used to carry out the expressions of $\beta$ and $\sigma$ in terms of the scattering matrix elements, i.e.,

$$\beta_{(m-1)d_{5/2}}=\frac{|S_1(2)|^2+4|S_3(2)|^2-7|S_3(3)|^2-6\sqrt{6}\,\mathrm{Re}[S_3(2)S_1^*(2)]}{5[|S_1(2)|^2+5|S_3(2)|^2+7|S_3(3)|^2]} \tag{26a}$$

and

$$\sigma_{(m-1)d_{5/2}}=5|S_1(2)|^2+5|S_3(2)|^2+7|S_3(3)|^2. \tag{26b}$$

In this case, the only $S_l(J_t)$ value that needs to be determined is $S_3(3)$; the rest of them were already evaluated previously. Substitution of $S_3(3)=-(1/\sqrt{7})D_{331}$ into Eq. (26) will give the same result as Eq. (25).

Unlike the AD of electrons corresponding to the $ms^+_{1/2}$ ions, the ADs corresponding to the $(m-1)d_{3/2,5/2}$ ions are much more complicated for several reasons; first, the $\beta$ parameters in current cases contain a cross term of dipole matrix elements corresponding to the $p$-wave and $f$-wave electrons, which introduces various effects due to their

interactions. Since $D_{I_{csLJ}}$ contains radial and angular integrals and all information about phases, such as the Coulumb phase $\Delta_l=\arg\Gamma(l+1+i/k)$, where $k^2/2$ is the kinetic energy of the free electron, the phase shift between the $p$-wave and $f$-wave electrons may play an important role in AD; second, the complexity is due to the small fine-structure splitting between the $(m-1)d_{3/2}$ and $(m-1)d_{3/2}$ states (60, 280, and 800 $cm^{-1}$) for the analogous levels of $Ca^+$, $Sr^+$, and $Ba^+$, respectively (note that the Mg atom cannot autoionize to a lower-energy $d$ state of the ion). Since the $(m-1)d_j^+$ states are closer to the $mp^+$ state than the $ms$ state is, the energy resolution is limited for the two $(m-1)^+d_j$ states. In fact, so far only a few experiments have successfully resolved the electron signals from the two $(m-1)d_j^+$ states. When an experiment does not resolve them, the AD measured represents an average of two $(m-1)d_j^+$ states, i.e.,

$$\beta_{(m-1)d}=\frac{\sigma_{d_{3/2}}\beta_{d_{3/2}}+\sigma_{d_{5/2}}\beta_{d_{5/2}}}{\sigma_{d_{5/2}}+\sigma_{d_{5/2}}}, \tag{27}$$

where all the quantities involved are expressed in terms of reduced dipole matrix elements [see Eqs. (23) and (25)]. This certainly is not an ideal situation for AD study for either experiment or theory.

Note that the ejection of pure $p$-wave electrons requires $D_{231}=D_{331}=0$. In this case, from Eq. (22) we obtain

$$\beta_{(m-1)d_{3/2}}=\frac{|D_{211}|^2-5|D_{111}|^2}{5[|D_{211}|^2+|D_{111}|^2]}\leqslant\frac{|D_{211}|^2}{5[|D_{211}|^2+|D_{111}|^2]}$$
$$\leqslant 0.2 \tag{28a}$$

and

$$\sigma_{(m-1)d_{3/2}}=\tfrac{1}{3}[|D_{111}|^2+|D_{211}|^2]. \tag{28b}$$

Similarly, from Eq. (25) we have

$$\beta_{(m-1)d_{5/2}}=\frac{|D_{211}|^2}{5|D_{211}|^2}=0.2 \tag{29a}$$

and

$$\sigma_{(m-1)d_{5/2}}=\tfrac{1}{3}|D_{211}|^2. \tag{29b}$$

A comparison of the Eqs. (28) and (29) immediately tells us that

$$\beta_{(m-1)d_{5/2}}>\beta_{(m-1)d_{3/2}},\quad \sigma_{(m-1)d_{5/2}}<\sigma_{(m-1)d_{5/2}}, \tag{30}$$

which means that the amount of ejected electrons from the $(m-1)d_{3/2}$ state is greater than that from the $(m-1)d_{5/2}$ state.

However, the ejection of pure $f$-wave electrons requires $D_{111}=D_{211}=0$, which yields

$$\beta_{(m-1)d_{3/2}}=\frac{4|D_{231}|^2}{5|D_{231}|^2}=0.8 \tag{31a}$$

and

$$\sigma_{(m-1)d_{3/2}}=\tfrac{1}{3}|D_{231}|^2. \tag{31b}$$

Similarly,

$$\beta_{(m-1)d_{5/2}}=\frac{4|D_{231}|^2-5|D_{331}|^2}{5[|D_{231}|^2+|D_{331}|^2]}\leqslant 0.8 \tag{32a}$$

and

$$\sigma_{(m-1)d_{5/2}}=\tfrac{1}{3}[|D_{231}|^2+|D_{331}|^2]. \tag{32b}$$

A comparison of Eqs. (31) and (32) reveals that

$$\beta_{(m-1)d_{3/2}}>\beta_{(m-1)d_{5/2}},\quad \sigma_{(m-1)d_{3/2}}<\sigma_{(m-1)d_{5/2}}. \tag{33}$$

Thus, the conclusion is exactly reverse for $p$-wave and $f$-wave ejections. Therefore, under the circumstance of unresolved $(m-1)d_j^+$ states, we have (i) for $p$-wave ejection

$$\beta_{(m-1)d}\leqslant 0.2 \text{ or } [\beta_{(m-1)}]_{\max}=0.2, \tag{34}$$

(ii) for $f$-wave ejection

$$\beta_{(m-1)d}\leqslant 0.8 \text{ or } [\beta_{(m-1)}]_{\max}=0.8. \tag{35}$$

As mentioned previously, the $\beta_{(m-1)d_{3/2}}$ and $\beta_{(m-1)d_{5/2}}$ are affected by interference effects caused by the cross term $6\sqrt{6}\mathrm{Re}(D_{211}D_{231}^*)$. More explicitly, $\beta_{(m-1)d}$ could be larger than 0.8 if the interferences from the $p$-wave and $f$-wave electrons are constructive; $\beta_{(m-1)d}$ could be a negative value due to destructive interferences. For these reasons the determination of the extremes of $\beta_{(m-1)d}$ values is not as easy as that for $\beta_{ms_{1/2}}$ values, and requires the Monte Carlo calculation as done in AD of Mg $3pnd$ states [13].

Turning now to the partial cross section $\sigma_{(m-1)d}$, although the $mpns$ levels decay preferentially to the $(m-1)d\varepsilon p$ continua, the $f$ electrons contribute significantly to the $\beta_{(m-1)d}$ which can be seen from the above analysis. Since for $p$-wave ejection $[\sigma_{(m-1)d}]_p=\frac{1}{3}[|D_{111}|^2+2|D_{211}|^2]$, whereas for $f$-wave ejection $[\sigma_{(m-1)d}]_p=\frac{1}{3}[|D_{331}|^2+2|D_{231}|^2]$, and due to the fact that $D_{J_{csll}}$ is greater when $J=l$ than that when $J\neq l$, we conclude

$$[\sigma_{(m-1)d}]_p>[\sigma_{(m-1)d}]_f. \tag{36}$$

### C. $mp_{1/2}$ core

For the $mp_{1/2}^+$ ions, the ejected electron could be $s$ and $d$ electrons, and $l=0,2$ and $l'=0,2$ are possible. Also $J_{cs}=J_t=0,1$, respectively. It is straightforward to show that

$$a_0=(\tfrac{1}{3})[|D_{101}|^2+|D_{121}|^2] \tag{37a}$$

and

$$a_2=(\tfrac{1}{3})[|D_{121}|^2-2\sqrt{2}\mathrm{Re}(D_{101}D_{121}^*)], \tag{37b}$$

which yields

$$\beta_{mp_{1/2}}=\frac{|D_{121}|^2-2\sqrt{2}(D_{101}D_{121}^*)}{|D_{101}|^2+|D_{121}|^2} \tag{38a}$$

and

$$\sigma_{mp_{1/2}}=a_0. \tag{38b}$$

Obviously, $J_{cs}=J_t=0$ represents the parity-unfavored transition, and $J_{cs}=J_t=1$ represents the parity-favored transition. Therefore we have from the AMTT method

$$\beta_{mp_{1/2}}=\frac{|S_2(1)|^2-2\sqrt{2}\mathrm{Re}[S_2(1)S_0^*(1)]}{|S_2(1)|^2+|S_0(1)|^2} \tag{39a}$$

and

$$\sigma_{mp_{1/2}}=\frac{3\pi c^2}{\omega}[|S_2(1)|^2+|S_0(1)|^2]. \tag{39b}$$

Substituting $S_0(1)=(1/\sqrt{3})D_{101}$ and $S_2(1)=(1/\sqrt{3})D_{121}$ into Eq. (39) we easily obtain the same results with Eq. (38). From Eq. (38a) it is clear that only parity-favored transfers ($J_{cs}=1$) are involved with the ejection of $s$ and $d$ electrons.

For the AD of electrons corresponding to the $mp_{1/2}^+$ ions, the $\beta$ parameter depends only on a small number of reduced dipole matrix elements. However, it is similar to the case of $(m-1)d^+$ ions in that it also depends on a cross term. In another words, the interference between the $s$ and $d$ waves exists in this case.

As mentioned above, only the parity-favored transfers (singlet final states) are involved with $s$ and $d$ wave ejections. From Eq. (38a) it is clear that $D_{121}=0$ for the $s$-wave ejection, which yields $\beta_{mp_{1/2}}=0$, indicating a spherically symmetric distribution; similarly, $D_{101}=0$ for the $d$-wave ejection, which yields $\beta_{mp_{1/2}}=1$, indicating a $\sin^2\theta$ distribution. The $\beta$ value may be greater than 1.0 if the constructive interferences between $D_{121}$ and $D_{101}$ terms occur, otherwise it may be negative due to destructive interferences. For example, in the case of the Sr atom, $\beta_{5p_{1/2}}\sim 1.6$ [5], indicating that the interferences are always constructive.

However, the cross sections $\sigma_{mp_{1/2}}=\frac{1}{3}|D_{101}|$ for a $s$-wave ejection and $\sigma_{mp_{1/2}}=\frac{1}{3}|D_{121}|^2$ for a $d$-wave ejection. This tells us that the amount of ejected $d$ electrons is greater than that of ejected $s$ electrons because $D_{121}>D_{101}$. However, the $s$ wave contributes to the $\beta_{mp_{1/2}}$ significantly.

Note that the energy gap between the $mp_j^+$ fine-structure levels is various (92, 223, 801, and 1691 $\mathrm{cm}^{-1}$ for the anologous levels of $Mg^+$, $Ca^+$, $Sr^+$, and $Ba^+$), and there have been a few measurements of AD to determine the asymmetry parameter $\beta_{mp_{1/2}}$ [5,14–16,21].

Above all, for the $msns$ unpolarized target the two approaches described in Sec. I are equivalent. In the AMTT, introduction of an intermediate quantity $S_l(J_t)$ helps one to make a direct connection of autoionization with the scattering process. However, one has to transform $S_l(J_t)$ to the reduced dipole matrix elements $D_{J_{csl}J}$ to describe the photoexcitation process. Note that since the target is unaligned, the AD depends only on one asymmetry parameter. In contrast, the angular distribution of ejected electrons from the photoionization of a polarized target depends on three asymmetry parameters, and shows much more complicated patterns [13].

## IV. CONCLUSION

We have investigated angular distribution of ejected electrons following the photoionization of spherically symmetric Rydberg states for alkaline-earth atoms employing two different methods. The equivalence of angular-momentum transfer theory with general photoexcitation theory is verified. Expressions for the asymmetry parameter and the partial cross section for photoionization resulting in a particular final ionic state are given in terms of reduced dipole matrix elements. Characteristics of different angular distributions are discussed in detail to explore the interference effects involved, and several possible ejections ($s$-, $p$-, $d$-, and $f$-wave electrons) are investigated to determine the properties of the asymmetry parameter and partial cross section. A comparison among them is also given.

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