Fully relativistic distorted-wave Born procedure for electron-impact excitation

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A fully relativistic distorted-wave Born procedure for collision strength of highly charged ions is presented. Multiconfiguration Dirac-Fock wave functions and intermediate coupling are used to describe the target-ion states, and the relativistic continuum wave functions are calculated using a hybrid mesh point in the field of frozen target-ion charge distribution with the Dirac-Fock-Slater exchange potential. Also a normalization procedure of the continuum wave function is used. The factorization form of collision theory of Sampson and Zhang [Phys. Rev. A **45**, 1657 (1992)] is applied. The calculations of collision strengths of the ground state to singly excited states with an N-shell electron for nickel-like Gd and U ions are done and the results are compared with other work. There are several detailed considerations in our procedures, so the present results should be more reliable and accurate.

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I. INTRODUCTION

The purpose of present work is to develop a very efficient but very accurate program for evaluating relativistic distorted-wave electron impact collision strengths of highly charged ions. A main motivation for our work in this area is also the large amount of collision data needed for the modeling and diagnostics of high-temperature plasmas, such as occur in research to develop x-ray lasers, inertial confinement fusion (ICF), and in astrophysics [1–8]. A third objective is to provide atomic collision data for the development of soft-x-rays for the possibility of imaging biological specimens in the water window between K absorption edges of O (23.32 Å) and C (43.76 Å) [9] and for application to experiment, such as the electron beam ion trap (EBIT) experiment [10]. Relativistic effects in excitation of the ion (or atom) arise from two main sources: (a) relativistic effects in target-ion wave functions (b) the relativistic interaction of the free and bound electrons in the target ions. In addition to the usual nonrelativistic Coulomb interaction between the incident electron and the target ion, the electron undergoes a relativistic interaction with the target nucleus and accompanying bound electrons, such as spin-orbit, spin-spin, and spin-other-orbit interactions. The use of the Dirac Hamiltonian will adequately describe the relativistic interaction between the incident electron and the target nucleus. To describe relativistic interactions between the incident electrons and bound electrons, however, we must extract that part of the Möller scattering that corresponds to the exchange of a transverse photon [11]. In this study, we exclude Möller scattering because the incident energy ϵ_i ($<10^5$ eV) may not be high enough to make the Möller interaction scattering between the incident and bound electrons significant. Several elaborate fully relativistic distorted-wave Born (RDWB) programs now exist with results published in the literature [12–17]. A new, elaborate, rapid, and accurate program using relativistic Multiconfiguration Dirac-Fock (MCDF) and intermediate coupling treatments of the bound electrons and free electrons has now been completed. Our bound states are from Grant's code (also called GRASP, the acronym of general-purpose relativistic atomic structure program) [18]. GRASP has been tested several times and thought to be very reliable for atomic calculations of highly stripped ions. The continuum wave functions are calculated in the central potential using DFS exchange potential. A new WKB normalization of continuum orbital is used. The factorization method is used in collision strength calculation with partial wave expansion.

In Sec. II a brief description of bound wave functions and atomic structure is presented. Then in Sec. III a numerical solution and the normalization of the continuum orbital and continuum asymptotic are presented. In Sec. IV the theory of impact excitation with Bar-Shalom's factorization theory is outlined; also presented are the treatments of the difficulties of large-radial-distance and high-partial-wave contributions. Finally, the present results are compared with results calculated by others. These comparisons are made for collision strengths of nickel-like Gd and nickel-like U. For nickel-like Gd, resonance excitations to the N shell involving the 107-level MCDF configuration-expansion, and excitation to the N shell and O shell involving the 249-level MCDF configuration-expansion are calculated and discussed. As for nickel-like U, only later MCDF configuration-expansion is presented.

II. ATOMIC STRUCTURE AND WAVE FUNCTION

In outlining MCDF theory and intermediate coupling we will closely refer to Ref. [18] on the atomic structure program GRASP. In the GRASP program, treating an ion with N bound electrons, we use basis states $\Phi_\mu(1,2,...,N)$, which are single-configuration state functions (CSFs). These are the antisymmetric sum of products of N one-electron Dirac spinors $u_{n\kappa m}$ that are solutions of the Dirac equation for a central potential, [18,19]

$$u_{n\kappa m}=\frac{1}{r}\begin{bmatrix} P_{n\kappa}(r) & \chi_{\kappa m}(\mathbf{r}/r) \\ iQ_{n\kappa}(r) & \chi_{-\kappa m}(\mathbf{r}/r)\end{bmatrix}, \tag{1}$$

where $P_{n\kappa}$ and $Q_{n\kappa}$ are large and small component radial wave functions, respectively, and the function $\chi_{\kappa m}$ is the spinor spherical harmonics,

$$\chi_{\kappa m}(\mathbf{r}/r)=\sum_{\sigma=\pm 1/2}\langle lm-\sigma\ \tfrac{1}{2}\ \sigma|l\ \tfrac{1}{2}\ jm\rangle Y_l^{m-\sigma}(\mathbf{r}/r)\phi^{\sigma}, \tag{2}$$

where $\langle lm-\sigma(\frac{1}{2})\sigma|l(\frac{1}{2})jm\rangle$ is a Clebsch-Gordan coefficient, $Y_l^{m-\sigma}(\mathbf{r}/r)$ is a spherical harmonic, and ϕ^{σ} is a spinor basis function; κ is the relativistic angular quantum number, $\kappa=\pm(j+1/2)$ for $l=j\pm 1/2$, thus $j=|\kappa|-1/2$.

The ion atomic state functions (ASFs) are a linear combination of N_c CSFs sharing common values of parity, total target angular momentum

$$\Psi=\sum_{\mu=1}^{N_c} a_{\mu}\Phi_{\mu}; \tag{3}$$

the mixing coefficients a_{μ} are obtained by diagonalizing the Hamiltonian. Higher-order QED modifications due to transverse electromagnetic interaction and the radiative corrections are treated via perturbation theory [20–25].

All atomic structure data are obtained from the GRASP code. Bound-state orbitals and other required radial functions are transformed to the new hybrid grid using a well tested cubic spline interpolation which we find preserves accuracy to at least a one part per 10^6 accuracy. In addition to using a cubic spline for interpolation, we also use a cubic spline to evaluate the definite integrals of the dependable variable and to perform the derivatives required in the normalization section and other sections.

III. CONTINUUM WAVE FUNCTION

A. Numerics of continuum orbital

To determine the continuum wave function, we use $u_{\epsilon\kappa m}$ as the distorted-wave Dirac spinor for a free electron in the central potential $V(r)$ due to the target ion, which is analogous to Eq. (1),

$$u_{\epsilon\kappa m}=\frac{1}{r}\begin{bmatrix} P_{\epsilon\kappa}(r) & \chi_{\kappa m}(\mathbf{r}/r) \\ iQ_{\epsilon\kappa}(r) & \chi_{-\kappa m}(\mathbf{r}/r)\end{bmatrix}. \tag{4}$$

The large and small components $P_{\epsilon\kappa}$ and $Q_{\epsilon\kappa}$ satisfy the coupled Dirac equations,

$$\left[\frac{d}{dr}+\frac{\kappa}{r}\right]P_{\epsilon\kappa}(r)=\frac{1}{c}[\epsilon-V(r)+2c^2]Q_{\epsilon\kappa}(r), \tag{5}$$

$$\left[\frac{d}{dr}-\frac{\kappa}{r}\right]Q_{\epsilon\kappa}(r)=-\frac{1}{c}[\epsilon-V(r)]P_{\epsilon\kappa}(r). \tag{6}$$

These are like bound orbital Dirac equations, except that ϵ is positive and is the kinetic energy of the electron in a.u. when $r\to\infty$. $c\sim 137.036$ is the light speed in a.u. The relation between the relativistic wave quantum number k of the impact electron, the relativistic momentum p, and kinetic energy ϵ of the impact electron is,

$$k^2=\frac{p^2a_0^2}{\hbar^2}=\epsilon\left[2+\frac{\epsilon}{c^2}\right]. \tag{7}$$

The numerical solutions of the free Dirac equations are largely the same as those for Dirac-Fock wave functions for bound electrons, except for the modified orthogonality condition that is discussed in Sec. III B and the fact that the continuum orbitals do not vanish at large r. It is an inappropriate logarithm grid mesh for the continuum orbital because its one step may contain one or more oscillating cycles of the continuum orbital in the large-r region. Therefore the notable departure is that the logarithmic radial mesh popular in bound-state codes is unsuitable for continuum orbitals. In calculating continuum orbitals, Perger *et al.* [26,27] divided the whole r region into two parts, i.e., the inner region and the outer region, and used the logarithm grid $\rho=\ln(r)$ for the inner region and the linear grid $\rho=\alpha r$ for the outer region. This method would bring about some complexities of interpolation and extrapolation at the boundary of the two regions. Our approach is to work with a hybrid grid mesh of the form,

$$\rho=\alpha r+\beta\ \ln(r). \tag{8}$$

This kind of grid was first employed by Chernysheva *et al.* [28] in nonrelativistic continuum orbital calculations and was also used by Hagelstein and Jung [12] and Desclaux and co-workers [17,29] in their RDWB continuum orbital calculations. The hybrid mesh is more appropriate because it incorporates the characteristics of both the logarithm grid and the linear grid. Moreover, the hybrid grid approximates the logarithmic mesh in the inner region and approximates the linear mesh in the outer region. The parameters α and β can be determined as [30]

$$\alpha=\frac{n_p h k_{ini}}{2\pi}, \tag{9}$$

$$\beta=1-\frac{\alpha r_0}{\ln(r_0)}, \tag{10}$$

where n_p is the number of grid points per cycle of oscillation, and we choose $n_p=36$ for each continuum orbital. $h=\delta\rho$ is equidistant in the hybrid mesh, r_0 is the first grid point, and we choose $h=0.05$, $r_0=10^{-5}$ in all our present calculations. k_{ini} is the wave number of the continuum wave function for the initial state of each excitation by electron impact. So the mesh parameters α and β are different if k_{ini} in a certain excitation is different, but the parameters are the same for bound and free wave functions of initial and final states in a certain transition. This choice is the most appropriate.

The two first-order differential equations (5) and (6) are integrated using the five-point Adams method [31–33] which is chosen because of its fast, stable and accurate properties. The Adams method is a standard predictor-corrector method.

B. Normalization and continuum asymptotic

The radial functions for bound orbitals satisfy the orthonomality conditions,

$$\int_0^{\infty} dr[P_{n\kappa}(r)P_{n'\kappa}(r)+Q_{n\kappa}(r)Q_{n'\kappa}(r)]=\delta_{nn'}, \tag{11}$$

where $\delta_{nn'}$ is a Kronecker delta, and the radial functions for the continuum orbitals satisfy similar orthogonality conditions,

$$\int_0^\infty dr[P_{\epsilon\kappa}(r)P_{\epsilon'\kappa}(r)+Q_{\epsilon\kappa}(r)Q_{\epsilon'\kappa}(r)]=\pi\delta(\epsilon-\epsilon'). \tag{12}$$

For normalizing the continuum orbitals, we have adopted a method of matching wave functions to their asymptotic and relativistic WKB form. The method used presently makes some modifications to Hagelstein and Jung [12], and we shall outline them briefly. The large component of the continuum wave function is taken, in the WKB approximation, to be

$$P=A\left[\frac{\epsilon-V(r)+2mc^2}{2c\,\dfrac{d\phi}{dr}}\right]^{1/2}\sin\,\phi, \tag{13}$$

where A is the normalization constant (to be determined), and the expression $d\phi/dr$ is

$$\begin{aligned}-\left(\frac{d\phi}{dr}\right)^2+\frac{[\epsilon-V(r)+mc^2]^2-m^2c^4}{c^2}-\frac{\kappa(\kappa+1)}{r^2}\\ +\left(\frac{d\phi}{dr}\right)^{1/2}\frac{d^2}{dr^2}\left(\frac{d\phi}{dr}\right)^{-1/2}-[\epsilon-V(r)\\ +2mc^2]^{1/2}\frac{d^2}{dr^2}[\epsilon-V(r)+2mc^2]^{-1/2}\\ +\frac{\kappa}{r}\frac{dV(r)/dr}{[\epsilon-V(r)+2mc^2]}=0.\end{aligned} \tag{14}$$

Note that Eq. (13) differs from Eq. (4) of Ong and Russek [34] and Eq. (32) of Perger *et al.* [26]. Hence the amplitude of Eq. (13) approaches $(1/k)^{1/2}$ at large radial distance. Defining W as

$$W=A\left[\frac{\epsilon-V(r)+2mc^2}{2c\,\dfrac{d\phi}{dr}}\right]^{1/2}\cos\,\phi, \tag{15}$$

then taking the derivative of Eq. (13) with respect to r, W can be shown to be

$$\begin{aligned}W=\left(\frac{d\phi}{dr}\right)^{-1}\left[\frac{dP}{dr}+\frac{1}{2}\frac{dV(r)}{dr}[\epsilon-V(r)+2mc^2]^{-1}P\right.\\ \left.+\frac{1}{2}\frac{d^2\phi/dr^2}{d\phi/dr}P\right];\end{aligned} \tag{16}$$

combining Eqs. (13) and (15) to solve for A and ϕ yields

$$A=2c\left[\frac{1}{\epsilon-V(r)+2mc^2}\frac{d\phi}{dr}(P^2+W^2)\right]^{1/2}_{r_{max}}, \tag{17}$$

$$\phi(r_{max})=\tan^{-1}[P/W]_{r_{max}}, \tag{18}$$

where the radial point r_{max} for the evaluation of Eqs. (17) and (18) is chosen to be well beyond the point where the exchange potential is negligibly small. The expression for $d\phi/dr$ in Eq. (14) can easily be solved iteratively by approximating $d\phi/dr$ by the second and the third terms on the left-hand side of Eq. (14), then using that estimate in the next iteration for the fourth term on the left-hand side of Eq. (14). With only two or three iterations $d\phi/dr$ is stable to seven significant figures.

C. Potential choice

Our procedures have mainly two forms of potential for free electron orbitals. (a) The first candidate is the Dirac-Fock-Slater (DFS) potential, [37,13]

$$V(r)=V'(r)+V^{ex}_{DFS}(r), \tag{19}$$

where

$$V'(r)=-\frac{Z(r)}{r}+V_c(r), \tag{20}$$

$$V^{ex}_{DFS}(r)=-\left(\frac{3}{\pi}\rho(r)\right)^{1/3}, \tag{21}$$

$$V_c(r)=\sum_{n',\kappa'}\omega_{n',k'}\int_0^\infty\frac{1}{r_>}[P^2_{n'\kappa'}(r')+Q^2_{n'\kappa'}(r')]dr', \tag{22}$$

$$\rho(r)=\frac{1}{4\pi r^2}\sum_{n'\kappa'}\omega_{n'\kappa'}[P^2_{n'\kappa'}(r)+Q^2_{n'\kappa'}(r)]. \tag{23}$$

Here $\omega_{n'\kappa'}$ is the occupation number of subshell $n'\kappa'=n'l'j'$, $\omega_{n'\kappa'}$ is given by a fictitious occupation number, which is sometimes called a mean configuration. About half an electron is excited. To compare with data in Ref. [14], Eq. (30) of Ref. [13] is used in the present calculation. The $\omega_{n'\kappa'}$ here is used solely to determine the spherically averaged relativistic Dirac-Fock central field-free orbital potential. The summation is over all occupied subshell, $r_>=\max(r,r')$. Finite nuclear charge $Z(r)$, which differs from ordinary Z only for small $\mathbf{r}$, is chosen to be the Fermi charge distribution [35] and can be obtained from the GRASP code [18]. This choice of potential is used in Sec. V of the present paper for the purpose of making a comparison with Zhang *et al.* [14] because he used the similar form of DFS potential in his continuum-orbital calculation.

(b) The second candidate is one where the exchange potential given by Eq. (21) is replaced by a form based on the semiclassical exchange (SCE) approximation of Riley and Truhlar [36]. Following Zhang *et al.* [37], we refer to it as the Mann [38] potential $V_M(r)$ [in place of $V(r)$]; it is given by

$$V_M(r)=V'(r)+V^{ex}_M(r), \tag{24}$$

$$V^{ex}_M(r)=-\frac{1}{2}[V'(r)-\epsilon\ \text{(a.u.)}][(1+\beta^2)^{1/2}-1], \tag{25}$$

where

$$\beta^2=\frac{4\pi\rho(r)}{[V'(r)-\epsilon\ \text{(a.u.)}]^2} \tag{26}$$

and ϵ (a.u.) is the free-electron kinetic energy in atomic units. The potentials used in calculating impact and scattered electrons for a transition differed only by the free-electron energies used. r^2 in Eq. (27) of Ref. [37] should be omitted consistent with Refs. [36] and [38]. This potential has been proposed as an option in our code but it is not used in the present calculations. In above mentioned two choices of potential, since the orbitals of the free electron are not orthogonal to those of the bound electrons, it is necessary to replace the factor $r_<^\lambda/r_>^{\lambda+1}$ with $r_<^\lambda/r_>^{\lambda+1}-\delta_{\lambda 0}[V_c(r)/N]$ in the exchange matrix element E^λ [39].

IV. COLLISION THEORY

A. Formulas of collision strengths

It is convenient to express the relativistic cross section $\sigma_{if}(\epsilon)$ for the transition $i\rightarrow f$ in terms of collision strength $\Omega_{if}(\epsilon)$ by the relation

$$\sigma_{if}(\epsilon)=\frac{\pi a_0^2}{k_i^2 g_i}\,\Omega_{if}(\epsilon), \tag{27}$$

where the subscripts i and f refer to initial and final states, a_0 is the Bohr radius, k_i is the relativistic wave number of the impact electron, and $g_i=[J_i]=2J_i+1$ is the statistical weight of the initial state of the N-electron target ion. We further define the partial collision strength Ω^J from which the total transition collision strength is computed through evaluation of

$$\Omega_{if}(\epsilon)=\sum_J\ \Omega_{if}^J(\epsilon), \tag{28}$$

where the summation is over the total angular momentum J of the entire $(N+1)$-electron system of combined electron-plus-ion states. The partial collision strength is defined as

$$\Omega_{if}^J(\epsilon)=\frac{1}{2}\sum_{jj'}\ [J]|T^J(\Gamma_i J_i j,\Gamma_f J_f j')|^2, \tag{29}$$

where $[J]=2J+1$, $T^J(\Gamma_i J_i j,\Gamma_f J_f j')$ are the transition matrix elements, Γ_i and Γ_f refer to suppressed quantum numbers, J_i, J_f refer to the total angular momenta of the target ion in a certain transition, and j and j' refer to the continuum orbitals total angular momenta. For the highly charged ions of interest here, we express the **T** matrix in terms of reactance matrix **R**:

$$\mathbf{T}=\frac{-2i\mathbf{R}}{1-i\mathbf{R}}\approx -2i\mathbf{R}, \tag{30}$$

where the final approximation gives nonunitarized cross sections. This is a weak-coupling approximation and gives a very good treatment of the highly charged ions for which the elements of **R** are small. Then the relativistic distorted-wave expression for collision strength can be written as [14]

$$\Omega_{if}=8\sum_J\ [J]\sum_{\kappa,\kappa'}\left|\left\langle \Psi_i\left|\sum_{\substack{q,k\\ q<k}}^{N+1}\frac{1}{r_{qk}}\right|\Psi_f\right\rangle\right|^2, \tag{31}$$

where Ψ_i and Ψ_f in Eq. (31) are the initial and final antisymmetric wave functions for the total $(N+1)$-electron system consisting of the target ion plus the free electron, and they must match in parity and total angular momentum. κ and κ' are the initial and final relativistic quantum numbers for the free electrons.

We follow the work of Sampson and Zhang [39], who make use of the key and important research of Bar-Shalom *et al.* [40]. The main features are that, in general, the various 6-j and 9-j factors entering the exchange and direct scattering matrix elements can be arranged so that they contain a common factor that can be factored out and summed over total angular momenta J. This leads to a large reduction in the angular part of the calculation. Following the notation of Ref. [39], the collision strength then factors into the form

$$\Omega_{if}=8\sum_{\substack{S,S'\\ S_1,S_1'}}\ \sum_\lambda\ B^\lambda(i,SS_1;f,S'S_1')Q^\lambda(n_a l_a j_a,n_a' l_a' j_a';n_{a1} l_{a1} j_{a1},n_{a1}' l_{a1}' j_{a1}'). \tag{32}$$

Here $n_a l_a j_a, n_{a1} l_{a1} j_{a1}$ indicate initial orbital of the active electron in the pure jj-coupled states S and S_1 that contribute to the initial level i. An analogous statement applies for corresponding primed quantities contributing to the final level f. Ω_{if} is given by sums over S, S_1, S', S_1', and λ, which is the order of the tensor products in the angular parts. The factors B^λ and Q^λ are give by Eqs. (30) and (31) of Ref. [39].

B. Large radial contribution in the direct matrix element

Since continuum orbitals do not quickly reach asymptotic sinusoidal behavior, some direct transition matrix elements that constitute Ω depend very much on the large-r behavior of continuum orbitals. Therefore it is necessary to calculate the continuum radial function to very large kr. This is a difficult problem and it seems that Zhang *et al.* did not consider this problem [14] sufficiently. They only used 1800 mesh points [see Eq. (16) of Ref. [14]]. In most cases 1800 mesh points is enough, but in a few cases, this is obviously not sufficient because of the unduly oscillatory behavior of the continuum wave function. For our hybrid mesh point, 6000 mesh points have been tabulated in the present paper. This approach, though somewhat slow in computation time, is more stable and efficient. Belling's method [41] extended to the relativistic case has been coded as done by Hagelstein [12] and compared with the present results. We will continue

the research of this aspect, namely asymptotic expressions for affected matrix elements, by extending the method of Sil *et al.* [42] which transforms the integration contours into the complex plane to the relativistic case [43]. Sil's method has solved this problem perfectly in the nonrelativistic case.

C. High-partial-wave contribution

Calculating large partial waves is another very difficult problem. The large number of partial waves have to be computed, especially for high impact-electron energies and optically allowed transition with $\Delta n=0$. Hence, a dependable method must be found to estimate contributions from higher partial waves. We find that results from distorted, Coulomb and plane partial waves are sufficiently different only for small partial waves. Kim and Desclaux [17] used the plane-wave method to compensate the very high partial wave contributions in addition to the detailed RDWB calculations of low partial waves. This approach was elaborate and accurate, but it was very time consuming because it also needed to calculate the low-partial-wave contribution using the plane-wave method. Another approach is (a) for dipole-allowed transitions the very rapid and simpler Coulomb-Bethe approximation was used for high partial waves, which involve a function given in the paper of Burgess *et al.* [44],

$$I(k_1,l_1,k_2,l_2;\lambda)=\int_0^\infty F(k_1l_1|\rho)F(k_2l_2|\rho)\rho^{-\lambda+1}d\rho, \tag{33}$$

where we have tried to remain true to the notation of that paper and (b) for transitions that occur only through exchange, the contribution from large partial waves is insignificant. For other non-optically-allowed transitions, the ratio of partial-wave contributions for successive partial waves becomes nearly constant for large partial waves, so this ratio could be used to estimate the contribution for high partial waves [45–47]. This approach was used by Zhang *et al.* [14]. To obtain the results presented in the next section we used the Shanks method for accelerating the summation of a series to extend the summation to large partial waves [48,49]. This method is very efficient and accurate and has been used by Hagelstein's RDWB code [12] and IMPACT code [50], a close-coupling code developed at University College, London.

V. NUMERICAL RESULTS AND DISCUSSIONS

In Table I we present the collision strengths for nickel-like gadolinium, evaluating in 107 levels and 249 levels the MCDF configuration-expansion. The excitations to the $n=4$ shell only contain the configurations $3s^23p^63d^94l$, $3s^23p^53d^{10}4l$, $3s3p^63d^{10}4l,l=s,p,d,f$, which involve 107 levels, and these were done by Hagelstein [51] and Zhang *et al.* [14]. We calculate the collision strengths in this case in order to compare them with the results of Hagelstein and Zhang *et al.* in the same MCDF configuration-expansion. However, as noted by Goldstein *et al.* [52], the $n=4$ shell with holes in the $3s$ and $3p$ subshells overlaps in energy the $n=5$ shell with holes in the $3d$ subshells, i.e., $3s^23p^63d^95l,l=s,p,d,f,g$ in addition to the $n=4$ shell involving 181 levels in their calculation. Thus, one must include mixing with the latter in order to obtain accurate results for excitations to some of the $n=4$ levels. So, we calculate the collision strength using excitations to all $n=4$ levels and $n=5$ levels (i.e., including $3s^23p^63d^95l$, $3s^23p^53d^{10}5l$, $3s3p^63d^{10}5l,l=s,p,d,f,g$), which involve 249 levels. Comparison of our results between the 107 levels and the 249 levels, does show some differences. The largest difference is about 9% due to an additional O-shell level approach. The present results using the 249-level MCDF configuration-expansion we believe to be more accurate than those of previous calculations. Here we only present the results for excitations of a $3d_{3/2}$ or $3d_{5/2}$ electron because of limited space, and the values are listed in increasing orders of energy in Tables I and II.

Table II presents the results of the nickel-like ion with $Z=92$, using the 249-level MCDF configuration-expansion. To our knowledge there have been no other collision strengths of this ion presented, except those by Zhang *et al.*, so we only compare with their result. In most cases, the agreement is fairly good. But in a few cases, e.g., excitation to $(3d_{3/2}4f_{5/2})_3$, the difference is about 6% for $Z=92$; in the case of excitation to $(3d_{3/2}4s_{1/2})_2$, the difference is about 8% for $Z=64$. Because the values are almost unchanged by an additional O-shell level approach in the two transitions, implies that detailed numerical considerations result in two differences: e.g., the detailed treatments of Sec. IV B and IV C, etc., in the RDWB calculating. In conclusion, both additional O-shell levels and the detailed numerical approaches are necessary for the calculations in some transitions.

One sees that our results are good agreement with those of Zhang or Hagelstein. However, as noted by Zhang *et al.* [14] the collision strength of excitation to the $(3s_{1/2}4p_{1/2})_1$ level in his calculation is a value about 1.5 times that of Hagelstein, in high energies. In our RDWB calculations, the cross section for this excitation is 1.336×10^{-22} cm^2 in $\epsilon_f=2500$ eV, whereas the value of Hagelstein [51] is 6.719×10^{-23} cm^2; our value is about 2.0 times that of Hagelstein. So our collision strength for this excitation is closer to the value of Zhang *et al.* than that of Hagelstein. As noted before, our results may be more accurate and reliable. We will continue this research to consider the Möller interaction in future work and make the calculations more accurate at high impact energies.

VI. SUMMARY

A very rapid, extremely accurate, fully RDWB procedure and corresponding computer code have been developed in the present work for the calculation of electron-impact excitation of highly charged ions. This work addresses an acute need for population kinetics modeling in the research of x-ray lasers and ICF development. The features of the collision theory are somewhat followed by Hagelstein, Sampson, and Desclaux, but there are several differences. The merits of the numerical techniques of the above mentioned three elaborate fully relativistic distorted-wave programs are selected and utilized in our RDWB electron impact excitation code; e.g., with the GRASP atomic structure code, the factorization technique is used so that various 6-j and 9-j symbols entering the direct and exchange matrix elements can be arranged and contain a common factor that can be factored out and summed over the total angular momentum of the com-

TABLE I. Comparison of collision strengths for nickel-like gadolinium. Abbreviations like Eq. (28) of Ref. [13] are used in designating the upper level as done by Zhang *et al.* in Ref. [14]. The first and second entries for each transition are calculated by the present approach using 107- and 249-level MCDF configuration-expansions, respectively. The third and the fourth entries are fully RDWB results by Zhang *et al.* (Ref. [14]) and Hagelstein (Ref. [51]), respectively. ΔE (eV) is the transition energy in eV. The numbers in square brackets are powers of 10 by which adjacent entries should be multiplied.

Upper level	ΔE (eV)	ϵ_f (eV) 160	ϵ_f (eV) 2500	Upper level	ΔE (eV)	ϵ_f (eV) 160	ϵ_f (eV) 2500
$(3d_{5/2}4s_{1/2})_3$	1027.02	8.81[−4]	3.11[−4]	$(3d_{5/2}4s_{1/2})_2$	1028.51	2.40[−3]	3.07[−3]
	1027.49	8.53[−4]	3.05[−4]		1028.99	2.41[−3]	3.07[−3]
	1027	8.45[−4]	3.05[−4]		1028	2.24[−3]	2.85[−3]
	1029	8.47[−4]	3.05[−4]		1030	2.32[−3]	2.99[−3]
$(3d_{3/2}4s_{1/2})_1$	1059.31	3.99[−4]	1.36[−4]	$(3d_{3/2}4s_{1/2})_2$	1060.34	1.81[−3]	2.17[−3]
	1059.77	4.01[−4]	1.36[−4]		1060.80	1.82[−3]	2.16[−3]
	1060	3.80[−4]	1.32[−4]		1061	1.69[−3]	2.01[−3]
	1062	3.80[−4]	1.32[−4]		1063	1.74[−3]	2.11[−3]
$(3d_{5/2}4p_{1/2})_2$	1100.89	8.76[−4]	3.35[−4]	$(3d_{5/2}4p_{1/2})_3$	1101.97	1.44[−3]	1.18[−3]
	1101.37	8.43[−4]	3.29[−4]		1102.47	1.44[−3]	1.18[−3]
	1101	8.68[−4]	3.30[−4]		1102	1.37[−3]	1.12[−3]
	1101	8.37[−4]	3.30[−4]		1102	1.41[−3]	1.16[−3]
$(3d_{3/2}4p_{1/2})_2$	1133.32	6.95[−4]	2.30[−4]	$(3d_{3/2}4p_{1/2})_2$	1135.87	2.26[−3]	4.92[−3]
	1133.80	6.86[−4]	2.30[−4]		1136.33	2.32[−3]	4.98[−3]
	1134	6.75[−4]	2.26[−4]		1136	2.16[−3]	4.59[−3]
	1134	6.80[−4]	2.32[−4]		1137	2.13[−3]	4.69[−3]
$(3d_{5/2}4p_{3/2})_4$	1148.35	1.22[−3]	4.12[−4]	$(3d_{5/2}4p_{3/2})_2$	1150.10	6.58[−4]	2.37[−4]
	1148.84	1.19[−3]	4.12[−4]		1150.58	6.36[−4]	2.33[−4]
	1149	1.18[−3]	4.05[−4]		1150	6.57[−4]	2.33[−4]
	1149	1.20[−3]	4.22[−4]		1151	6.46[−4]	2.39[−4]
$(3d_{5/2}4p_{3/2})_2$	1150.91	4.32[−3]	1.08[−2]	$(3d_{5/2}4p_{3/2})_3$	1152.50	8.56[−4]	7.23[−4]
	1151.35	4.50[−3]	1.09[−2]		1152.98	8.69[−4]	7.23[−4]
	1151	4.21[−3]	1.04[−2]		1153	8.22[−4]	6.76[−4]
	1152	4.46[−3]	1.23[−2]		1153	8.29[−4]	6.92[−4]
$(3d_{3/2}4p_{3/2})_0$	1177.86	2.45[−4]	9.84[−5]	$(3d_{3/2}4p_{3/2})_2$	1181.80	8.51[−4]	1.35[−3]
	1178.31	2.44[−4]	9.90[−5]		1182.25	8.60[−4]	1.35[−3]
	1179	2.45[−4]	9.70[−5]		1183	8.35[−4]	1.27[−3]
	1179	2.40[−4]	9.93[−5]		1183	8.33[−4]	1.33[−3]
$(3d_{3/2}4p_{3/2})_3$	1181.87	1.27[−3]	1.07[−3]	$(3d_{3/2}4p_{3/2})_2$	1184.08	4.26[−4]	1.40[−4]
	1182.34	1.30[−3]	1.07[−3]		1184.54	4.20[−4]	1.39[−4]
	1183	1.21[−3]	1.00[−3]		1185	4.23[−4]	1.38[−4]
	1184	1.25[−3]	1.05[−3]		1186	4.20[−4]	1.42[−4]
$(3d_{5/2}4d_{3/2})_3$	1261.17	2.02[−3]	6.92[−4]	$(3d_{5/2}4d_{3/2})_4$	1265.46	2.00[−3]	9.65[−4]
	1261.64	2.00[−3]	6.84[−4]		1265.98	1.98[−3]	9.46[−4]
	1262	1.95[−3]	6.66[−4]		1266	1.89[−3]	9.13[−4]
	1262	1.95[−3]	6.86[−4]		1267	1.99[−3]	9.81[−4]
$(3d_{5/2}4d_{3/2})_2$	1266.47	1.87[−3]	1.02[−3]	$(3d_{5/2}4d_{3/2})_3$	1268.35	1.35[−3]	3.72[−4]
	1266.98	1.86[−3]	1.01[−3]		1268.85	1.34[−3]	3.66[−4]
	1267	1.81[−3]	9.82[−4]		1269	1.31[−3]	3.58[−4]
	1267	1.89[−3]	1.09[−3]		1269	1.36[−3]	3.84[−4]
$(3d_{5/2}4d_{5/2})_1$	1273.30	1.32[−3]	4.10[−4]	$(3d_{5/2}4d_{5/2})_5$	1274.40	2.67[−3]	7.78[−4]
	1273.79	1.30[−3]	4.06[−4]		1274.91	2.63[−3]	7.50[−4]
	1274	1.28[−3]	4.00[−4]		1275	2.57[−3]	7.50[−4]
	1275	1.23[−3]	3.92[−4]		1276	2.55[−3]	7.54[−4]
$(3d_{5/2}4d_{5/2})_3$	1277.48	1.74[−3]	5.03[−4]	$(3d_{5/2}4d_{5/2})_2$	1278.42	3.59[−3]	4.45[−3]
	1277.99	1.72[−3]	4.94[−4]		1278.92	3.61[−3]	4.44[−3]
	1278	1.69[−3]	4.87[−4]		1279	3.47[−3]	4.26[−3]
	1279	1.65[−3]	4.84[−4]		1280	3.77[−3]	4.82[−3]
$(3d_{5/2}4d_{5/2})_4$	1278.99	1.36[−3]	5.04[−4]	$(3d_{5/2}4d_{5/2})_0$	1286.04	3.67[−3]	3.65[−3]
	1279.50	1.36[−3]	5.01[−4]		1286.45	3.71[−3]	3.67[−3]

TABLE I. (*Continued*).

Upper level	ΔE (eV)	ϵ_f (eV) 160	ϵ_f (eV) 2500	Upper level	ΔE (eV)	ϵ_f (eV) 160	ϵ_f (eV) 2500
	1280	1.30[−3]	4.77[−4]		1287	3.72[−3]	3.71[−3]
	1280	1.27[−3]	4.89[−4]		1287	3.51[−3]	3.60[−3]
$(3d_{3/2}4d_{3/2})_1$	1296.95	1.17[−3]	3.69[−4]	$(3d_{3/2}4d_{3/2})_3$	1297.35	1.54[−3]	4.52[−4]
	1297.43	1.18[−3]	3.71[−4]		1297.85	1.52[−3]	4.49[−4]
	1298	1.14[−3]	3.56[−4]		1299	1.48[−3]	4.36[−4]
	1299	1.09[−3]	3.50[−4]		1299	1.47[−3]	4.39[−4]
$(3d_{3/2}4d_{3/2})_2$	1301.91	2.35[−3]	2.89[−3]	$(3d_{3/2}4d_{5/2})_1$	1305.61	1.35[−3]	4.35[−4]
	1302.39	2.35[−3]	2.88[−3]		1306.08	1.36[−3]	4.37[−4]
	1303	2.26[−3]	2.76[−3]		1307	1.31[−3]	4.21[−4]
	1304	2.36[−3]	2.94[−3]		1308	1.33[−3]	4.41[−4]
$(3d_{3/2}4d_{5/2})_4$	1308.02	1.89[−3]	8.71[−4]	$(3d_{3/2}4d_{5/2})_2$	1309.39	2.00[−3]	1.50[−3]
	1308.51	1.92[−3]	8.80[−4]		1309.87	1.99[−3]	1.49[−3]
	1310	1.78[−3]	8.21[−4]		1311	1.93[−3]	1.44[−3]
	1310	1.89[−3]	8.88[−4]		1311	1.97[−3]	1.49[−3]
$(3d_{3/2}4d_{5/2})_3$	1310.93	1.21[−3]	3.24[−4]	$(3d_{3/2}4d_{3/2})_0$	1338.77	6.17[−2]	6.56[−2]
	1311.42	1.25[−3]	3.22[−4]		1337.19	5.69[−2]	6.03[−2]
	1313	1.17[−3]	3.12[−4]		1340	5.98[−2]	6.34[−2]
	1313	1.22[−3]	3.36[−4]		1340	5.64[−2]	6.17[−2]
$(3d_{5/2}4f_{5/2})_0$	1392.77	1.14[−3]	3.39[−4]	$(3d_{5/2}4f_{5/2})_1$	1394.78	3.22[−3]	1.44[−3]
	1393.23	1.16[−3]	3.45[−4]		1395.24	3.27[−3]	1.46[−3]
	1394	1.11[−3]	3.31[−4]		1396	3.13[−3]	1.40[−3]
	1394	1.15[−3]	3.44[−4]		1396	3.22[−3]	1.40[−3]
$(3d_{5/2}4f_{5/2})_2$	1398.13	3.36[−3]	9.22[−4]	$(3d_{5/2}4f_{5/2})_5$	1398.42	1.88[−3]	8.08[−4]
	1398.61	3.41[−3]	9.34[−4]		1398.86	1.88[−3]	8.06[−4]
	1399	3.22[−3]	8.84[−4]		1400	1.80[−3]	7.56[−4]
	1400	3.37[−3]	9.20[−4]		1400	1.95[−3]	7.90[−4]
$(3d_{5/2}4f_{7/2})_6$	1399.41	3.07[−3]	7.64[−4]	$(3d_{5/2}4f_{5/2})_3$	1401.13	2.20[−3]	5.80[−4]
	1399.84	3.10[−3]	7.69[−4]		1401.62	2.23[−3]	5.86[−4]
	1401	3.03[−3]	7.58[−4]		1402	2.17[−3]	5.77[−4]
	1401	3.27[−3]	7.99[−4]		1403	2.33[−3]	6.08[−4]
$(3d_{5/2}4f_{7/2})_2$	1401.15	1.82[−3]	4.26[−4]	$(3d_{5/2}4f_{5/2})_4$	1402.24	1.71[−3]	3.81[−4]
	1401.64	1.84[−3]	4.29[−4]		1402.72	1.72[−3]	3.83[−4]
	1402	1.82[−3]	4.32[−4]		1403	1.66[−3]	3.69[−4]
	1403	1.96[−3]	4.55[−4]		1404	1.79[−3]	3.90[−4]
$(3d_{5/2}4f_{7/2})_4$	1403.78	2.12[−3]	4.99[−4]	$(3d_{5/2}4f_{7/2})_5$	1404.89	1.25[−3]	3.46[−4]
	1404.26	2.14[−3]	5.04[−4]		1405.36	1.25[−3]	3.44[−4]
	1405	2.11[−3]	5.01[−4]		1406	1.20[−3]	3.25[−4]
	1406	2.26[−3]	5.24[−4]		1407	1.32[−3]	3.45[−4]
$(3d_{5/2}4f_{7/2})_3$	1405.53	3.96[−3]	4.69[−3]	$(3d_{5/2}4f_{7.2})_1$	1412.61	3.01[−2]	4.91[−2]
	1406.01	3.95[−3]	4.68[−3]		1412.93	2.99[−2]	4.89[−2]
	1407	3.66[−3]	4.40[−3]		1414	2.99[−2]	4.82[−2]
	1407	3.76[−3]	4.50[−3]		1414	3.00[−2]	4.95[−2]
$(3d_{3/2}4f_{7/2})_2$	1430.41	3.23[−3]	8.97[−4]	$(3d_{3/2}4f_{5/2})_4$	1430.50	1.90[−3]	4.66[−4]
	1430.87	3.28[−3]	9.09[−4]		1430.93	1.90[−3]	4.62[−4]
	1432	3.14[−3]	8.75[−4]		1433	1.87[−3]	4.62[−4]
	1433	3.32[−3]	9.09[−4]		1433	2.03[−3]	4.90[−4]
$(3d_{3/2}4f_{5/2})_2$	1433.09	1.69[−3]	4.09[−4]	$(3d_{3/2}4d_{7/2})_5$	1433.62	1.62[−3]	6.91[−4]
	1433.56	1.71[−3]	4.13[−4]		1434.06	1.63[−3]	6.93[−4]
	1435	1.67[−3]	4.08[−4]		1436	1.54[−3]	6.44[−4]
	1435	1.79[−3]	4.28[−4]		1436	1.67[−3]	6.71[−4]
$(3d_{3/2}4f_{5/2})_3$	1435.62	1.43[−3]	3.26[−4]	$(3d_{3/2}4d_{7/2})_3$	1436.41	3.49[−3]	3.94[−3]
	1436.09	1.44[−3]	3.29[−4]		1436.87	3.50[−3]	3.92[−3]
	1438	1.36[−3]	3.50[−4]		1438	3.27[−3]	3.65[−3]
	1438	1.46[−3]	3.82[−4]		1439	3.38[−3]	3.72[−3]
$(3d_{3/2}4d_{7/2})_4$	1437.12	1.43[−3]	3.03[−4]	$(3d_{3/2}4f_{5/2})_1$	1454.93	1.71[−1]	2.87[−1]
	1437.59	1.44[−3]	3.04[−4]		1454.26	1.59[−1]	2.68[−1]
	1439	1.40[−3]	2.98[−4]		1457	1.68[−1]	2.80[−1]
	1440	1.52[−3]	3.16[−4]		1458	1.65[−1]	2.80[−1]

TABLE II. Comparison of collision strengths for nickel-like uranium. The notation is the same as in Table I, but in this case only Zhang *et al.*'s results (Ref. [14]) can be obtained. For brevity, the results of our calculations using 107 level MCDF configuration-expansion are not presented.

Upper level	ΔE (eV)	ϵ_f (eV) 400	ϵ_f (eV) 10000	Upper Level	ΔE (eV)	ϵ_f (eV) 400	ϵ_f (eV) 10000
$(3d_{5/2}4s_{3/2})_3$	2688.14	3.32[−4]	8.07[−5]	$(3d_{5/2}4s_{1/2})_2$	2690.85	1.15[−3]	1.60[−3]
	2683	3.28[−4]	7.95[−5]		2685	1.10[−3]	1.53[−3]
$(3d_{5/2}4p_{1/2})_2$	2833.29	3.38[−4]	8.11[−5]	$(3d_{5/2}4p_{1/2})_3$	2834.57	6.44[−4]	5.44[−4]
	2831	3.32[−4]	7.98[−5]		2833	6.22[−4]	5.26[−4]
$(3d_{3/2}4s_{1/2})_1$	2876.72	1.63[−4]	3.61[−5]	$(3d_{3/2}4s_{1/2})_2$	2878.28	7.10[−4]	8.84[−4]
	2875	1.58[−4]	3.53[−5]		2876	6.82[−4]	8.47[−4]
$(3d_{3/2}4p_{1/2})_2$	3021.48	2.66[−4]	5.79[−5]	$(3d_{3/2}4p_{1/2})_1$	3026.59	1.59[−3]	4.01[−3]
	3023	2.62[−4]	5.69[−5]		3028	1.57[−3]	3.88[−3]
$(3d_{5/2}4p_{3/2})_4$	3160.39	4.19[−4]	1.00[−4]	$(3d_{5/2}4p_{3/2})_2$	3162.76	2.89[−4]	6.48[−5]
	3162	4.16[−4]	9.85[−5]		3164	2.85[−4]	6.41[−5]
$(3d_{5/2}4p_{3/2})_1$	3164.30	1.49[−3]	4.37[−3]	$(3d_{3/2}4p_{3/2})_3$	3167.48	3.29[−4]	2.74[−4]
	3166	1.44[−3]	4.19[−3]		3169	3.14[−4]	2.62[−4]
$(3d_{3/2}4p_{3/2})_0$	3344.96	9.24[−5]	2.44[−5]	$(3d_{3/2}4p_{3/2})_1$	3350.13	2.36[−4]	1.99[−4]
	3350	9.19[−5]	2.38[−5]		3355	2.37[−4]	1.75[−4]
$(3d_{3/2}4p_{3/2})_3$	3350.92	4.38[−4]	3.59[−4]	$(3d_{3/2}4p_{3/2})_2$	3354.05	1.79[−4]	3.78[−5]
	3356	4.14[−4]	3.41[−4]		3359	1.80[−4]	3.81[−5]
$(3d_{5/2}4d_{3/2})_1$	3362.37	6.46[−4]	1.47[−4]	$(3d_{5/2}4d_{3/2})_4$	3369.10	7.26[−4]	3.39[−4]
	3364	6.30[−4]	1.43[−4]		3372	7.02[−4]	3.31[−4]
$(3d_{5/2}4d_{3/2})_2$	3371.09	8.57[−4]	6.72[−4]	$(3d_{5/2}4d_{3/2})_3$	3374.60	4.47[−4]	7.45[−5]
	3373	8.39[−4]	6.57[−4]		3377	4.38[−4]	7.42[−5]
$(3d_{5/2}4d_{5/2})_1$	3432.33	5.30[−4]	1.15[−4]	$(3d_{5/2}4d_{5/2})_5$	3434.60	9.48[−4]	1.79[−4]
	3436	5.24[−4]	1.14[−4]		3439	9.34[−4]	1.77[−4]
$(3d_{5/2}4d_{5/2})_3$	3440.33	6.27[−4]	1.18[−4]	$(3d_{5/2}4d_{5/2})_2$	3442.10	1.45[−3]	1.96[−3]
	3444	6.17[−4]	1.16[−4]		3445	1.40[−3]	1.90[−3]
$(3d_{5/2}4d_{5/2})_4$	3442.89	4.88[−4]	1.61[−4]	$(3d_{5/2}4d_{5/2})_0$	3473.90	7.97[−3]	8.58[−3]
	3447	4.67[−4]	1.55[−4]		3478	8.12[−3]	8.79[−3]
$(3d_{3/2}4d_{3/2})_3$	3556.73	5.40[−4]	1.06[−4]	$(3d_{3/2}4d_{3/2})_2$	3558.37	3.30[−4]	6.95[−5]
	3563	5.30[−4]	1.05[−4]		3564	3.38[−4]	7.17[−5]
$(3d_{3/2}4d_{3/2})_2$	3566.01	4.86[−4]	5.18[−4]	$(3d_{3/2}4d_{3/2})_0$	3608.53	1.88[−2]	2.04[−2]
	3572	5.01[−4]	5.45[−4]		3615	1.94[−2]	2.11[−2]
$(3d_{3/2}4d_{5/2})_1$	3621.62	5.37[−4]	1.22[−4]	$(3d_{3/2}4d_{5/2})_4$	3625.35	6.58[−4]	2.80[−4]
	3628	5.20[−4]	1.19[−4]		3633	6.48[−4]	2.66[−4]
$(3d_{3/2}4d_{5/2})_2$	3628.15	7.25[−4]	5.30[−4]	$(3d_{3/2}4d_{5/2})_3$	3630.57	4.42[−4]	7.50[−5]
	3635	7.04[−4]	5.14[−4]		3638	4.31[−4]	7.37[−5]
$(3d_{5/2}4f_{5/2})_0$	3639.89	4.70[−4]	9.57[−5]	$(3d_{5/2}4f_{5/2})_1$	3644.84	1.09[−3]	2.34[−4]
	3643	4.52[−4]	9.29[−5]		3649	1.04[−3]	2.22[−4]
$(3d_{5/2}4f_{5/2})_5$	3649.28	7.42[−4]	3.23[−4]	$(3d_{5/2}4f_{5/2})_5$	3651.82	8.90[−4]	1.43[−4]
	3655	7.21[−4]	3.06[−4]		3656	8.56[−4]	1.39[−4]
$(3d_{5/2}4f_{5/2})_3$	3655.91	1.00[−3]	5.24[−4]	$(3d_{5/2}4f_{5/2})_4$	3657.03	5.49[−4]	6.99[−5]
	3660	9.68[−4]	5.05[−4]		3661	5.38[−4]	6.95[−5]
$(3d_{5/2}4f_{7/2})_6$	3667.52	1.19[−3]	1.91[−4]	$(3d_{5/2}4f_{7/2})_2$	3669.72	1.05[−3]	1.79[−4]
	3674	1.18[−3]	1.92[−4]		3675	1.02[−3]	1.75[−4]
$(3d_{5/2}4f_{7/2})_4$	3676.01	9.05[−4]	1.40[−4]	$(3d_{5/2}4f_{7/2})_5$	3678.15	4.55[−4]	1.22[−4]
	3681	8.81[−4]	1.38[−4]		3684	4.37[−4]	1.15[−4]
$(3d_{5/2}4f_{7/2})_3$	3679.03	1.70[−3]	2.06[−3]	$(3d_{5/2}4f_{7/2})_1$	3704.11	4.03[−2]	7.43[−2]
	3684	1.60[−3]	1.97[−3]		3710	4.03[−2]	7.35[−2]
$(3d_{3/2}4f_{5/2})_4$	3837.49	7.34[−4]	1.18[−4]	$(3d_{3/2}4f_{5/2})_2$	3841.02	7.58[−4]	1.26[−4]
	3846	7.27[−4]	1.20[−4]		3849	7.33[−4]	1.24[−4]
$(3d_{3/2}4f_{3/2})_3$	3848.11	8.70[−4]	9.79[−4]	$(3d_{3/2}4d_{7/2})_2$	3855.99	1.23[−3]	2.30[−4]
	3856	8.16[−4]	9.38[−4]		3864	1.18[−3]	2.25[−4]
$(3d_{3/2}4f_{7/2})_5$	3859.90	6.18[−4]	2.67[−4]	$(3d_{3/2}4d_{7/2})_3$	3864.57	1.06[−3]	7.25[−4]
	3869	5.86[−4]	2.50[−4]		3873	1.00[−3]	6.84[−4]
$(3d_{3/2}4d_{7/2})_4$	3866.38	5.41[−4]	7.24[−5]	$(3d_{3/2}4f_{5/2})_1$	3872.77	5.60[−2]	1.04[−1]
	3875	5.26[−4]	7.14[−5]		3882	5.66[−2]	1.04[−5]

bined system, using the mean configuration to obtain the central potential, etc. A code for the calculating free-electron radial function has been created. The WKB method and the cubic spline method are used to obtain the normalization factor for the free-electron wave function. Thus, a fully RDWB code for evaluating the electron-impact excitation cross sections of highly ionized ions is completed. The collision strengths of resonance excitations of nickel-like highly ionized ions have been compared with others and are in good agreement with them. Most of the differences are within a very small percentage. Because there are several detailed considerations in our RDWB procedures, the present results should be more reliable and accurate for highly stripped ions.

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