Relationship between field and atomic squeezing in the thermal Jaynes-Cummings model

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In this paper, the relationship between field and atomic squeezing in the thermal Jaynes-Cummings model with an initially coherent atom is examined.

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The squeezing of the radiation field, due to its potential application [1] in high-resolution spectroscopy, quantum nondemolition experiments, quantum communications, and low-light-level microscopy, has been extensively studied for the past fifteen years or so. It has been shown both theoretically [2] and experimentally [3] that the squeezed field can be generated by various physical processes. Recently, increased attention has also been paid to the squeezing of the fluctuations of the atomic dipole variables [4]. In addition, the field squeezing connected with the atomic squeezing has been discussed by some authors [5,6].

The thermal Jaynes-Cummings model (JCM) with an initially excited [7,8] or unexcited atom [9] can exhibit a sub-Poissonian field but not field squeezing for sufficiently small initial photon number. Very recently, Kozierowski, Poyatos, and Sanchez-Soto [10] have shown that the thermal JCM with an initially coherent atom can manifest the field squeezing only for small numbers of thermal photons.

In this paper, our aim is to show the relationship between the field and atomic squeezing in the thermal JCM with an initially coherent atom.

The Hamiltonian for the JCM in the rotating-wave approximation reads [11]

$$H=\Omega a^\dagger a+\omega S_z+G(a^\dagger S_-+aS_+), \tag{1}$$

where S_z and $S_\pm$ are operators of the atomic inversion and transition, respectively; ω is the atomic transition frequency; $a^\dagger$ and a are the creation and annihilation operators of the field mode with frequency Ω, respectively; and G is the atom-field coupling constant. Throughout we employ the unit with $\hbar=c=1$.

The density operator ρ of a thermal field can be written as $\rho=\Sigma_{n=0}^{\infty}P_n|n\rangle\langle n|$, and the photon-number distribution function P_n has the form $P_n=N_{\rm av}^n/(N_{\rm av}+1)^{n+1}$, where $N_{\rm av}=(e^{\Omega/K_BT}-1)^{-1}$ (K_B and T are the Boltzmann's constant and the temperature of the cavity, respectively) is the initial mean photon number.

If the system is initially prepared in the atomic coherent state $|\beta\rangle$ given below and the thermal field,

$$|\beta\rangle=\sin(\theta/2)e^{-i\phi/2}|-\tfrac{1}{2}\rangle+\cos(\theta/2)e^{i\phi/2}|\tfrac{1}{2}\rangle. \tag{2}$$

Then, the general time-dependent state $|\psi(t)\rangle$ of the system evolves as (under resonance)

$$\begin{aligned}|\psi(t)\rangle=&\sum_{n=0}^{\infty}\cos(\theta/2)f_ne^{-i[(n+1/2)\Omega t-\phi/2]}\cos(\sqrt{n+1}Gt)|\tfrac{1}{2},n\rangle-i\sum_{n=1}^{\infty}\sin(\theta/2)f_ne^{-i[(n-1/2)\Omega t+\phi/2]}\sin(\sqrt{n}Gt)|\tfrac{1}{2},n-1\rangle\\&+\sum_{n=0}^{\infty}\sin(\theta/2)f_ne^{-i[(n-1/2)\Omega t+\phi/2]}\cos(\sqrt{n}Gt)|-\tfrac{1}{2},n\rangle-i\sum_{n=0}^{\infty}\cos(\theta/2)f_ne^{-i[(n+1/2)\Omega t-\phi/2]}\sin(\sqrt{n+1}Gt)|-\tfrac{1}{2},n+1\rangle,\end{aligned} \tag{3}$$

where $|f_n|^2=P_n$.

In order to investigate the squeezing properties of the radiation field and the atom, we define the slowly varying Hermitian quadrature operates $a_1=\frac{1}{2}(ae^{i\Omega t}+a^\dagger e^{-i\Omega t})$, $a_2=(1/2i)(ae^{i\Omega t}-a^\dagger e^{-i\Omega t})$, $S_1=\frac{1}{2}(S_+e^{-i\omega t}+S_-e^{i\omega t})$, and $S_2=(1/2i)(S_+e^{-i\omega t}-S_-e^{i\omega t})$. The above operators obey the following commutation relations: $[a_1,a_2]=i\frac{1}{2}$ and $[S_1,S_2]=iS_z$. Correspondingly, the Heisenberg uncertainty relations are $(\Delta a_1)^2(\Delta a_2)^2\geqslant\frac{1}{16}$ and $(\Delta S_1)^2(\Delta S_2)^2\geqslant\frac{1}{4}\langle S_z\rangle^2$. It is convenient to define the following functions $h_i=(\Delta a_i)^2-\frac{1}{4}$ and $F_i=(\Delta S_i)^2-\frac{1}{2}|\langle S_z\rangle|$ $(i=1,2)$. Then, field squeezing [2] is defined if $h_i<0$, and so is atomic squeezing (dipole squeezing) [4] if $F_i<0$ $(i=1$ or $2)$.

Using (3), we arrived at

$$F_1=\tfrac{1}{4}-\tfrac{1}{4}\sin^2(\theta)\cos^2(\phi)A(t)-B(t), \tag{4}$$

$$F_2=\tfrac{1}{4}-\tfrac{1}{4}\sin^2(\theta)\sin^2(\phi)A(t)-B(t), \tag{5}$$

$$h_1=\frac{N_{\rm av}}{2}+C(t)-\tfrac{1}{4}\sin^2(\theta)\sin^2(\phi)D(t), \tag{6}$$

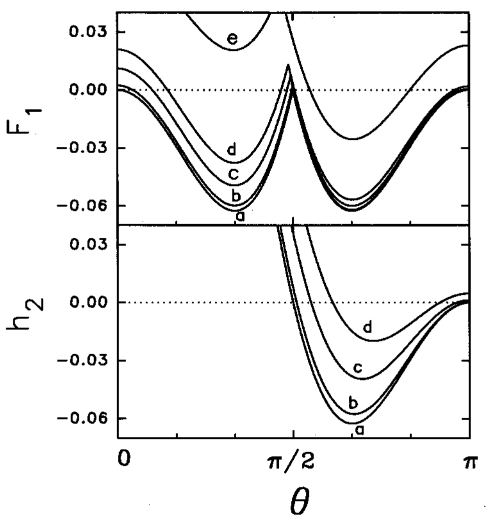

FIG. 1. $h_2(Gt=2.5\pi)$ and $F_1(Gt=3\pi)$ vs θ for $\phi=0$. (a) $N_{\rm av}=0$; (b) $N_{\rm av}=0.01$; (c) $N_{\rm av}=0.05$; (d) $N_{\rm av}=0.1$; (e) $N_{\rm av}=0.5$.

$$h_2=\frac{N_{\rm av}}{2}+C(t)-\tfrac{1}{4}\sin^2(\theta)\cos^2(\phi)D(t), \qquad (7)$$

where

$$A(t)=\left(\sum_{n=0}^{\infty}P_n\cos(\sqrt{n}Gt)\cos(\sqrt{n+1}Gt)\right)^2, \qquad (8)$$

$$B(t)=\frac{1}{2}\left|\frac{1}{2}\cos(\theta)-\left[\cos^2\left(\frac{\theta}{2}\right)-\frac{N_{\rm av}}{N_{\rm av}+1}\sin^2\left(\frac{\theta}{2}\right)\right]\sum_{n=0}^{\infty}P_n\sin^2(\sqrt{n+1}Gt)\right|, \qquad (9)$$

$$C(t)=\frac{1}{2}\left[\cos^2\left(\frac{\theta}{2}\right)-\frac{N_{\rm av}}{N_{\rm av}+1}\sin^2\left(\frac{\theta}{2}\right)\right]\sum_{n=0}^{\infty}P_n\sin^2(\sqrt{n+1}Gt), \qquad (10)$$

$$D(t)=\left(\sum_{n=0}^{\infty}P_n[\sqrt{n+1}\cos(\sqrt{n}Gt)\sin(\sqrt{n+1}Gt)-\sqrt{n}\sin(\sqrt{n}Gt)\cos(\sqrt{n+1}Gt)]\right)^2. \qquad (11)$$

In the following, taking the fluctuations in S_1 and a_2 as an example, we study the squeezing properties of the field and the atom in the thermal JCM.

(i) For the case of $N_{av}=0$. In the vacuum field JCM ($N_{\rm av}=0$), Wodkiewicz *et al.* [5] have examined the relation between the field and atomic squeezing. Now we give details in order to see the changes of squeezing due to the presence of thermal photons. Putting $N_{\rm av}=0$ into Eqs. (4) and (7), we have

$$h_2(t)=\frac{1}{2}\cos^2\left(\frac{\theta}{2}\right)\sin^2(Gt)-\sin^2\left(\frac{\theta}{2}\right)\cos^2\left(\frac{\theta}{2}\right)\sin^2(Gt)\cos^2(\phi), \qquad (12)$$

$$F_1(t)=\frac{1}{4}-\sin^2\left(\frac{\theta}{2}\right)\cos^2\left(\frac{\theta}{2}\right)\cos^2(Gt)\cos^2(\phi)-\frac{1}{2}\left|\cos^2\left(\frac{\theta}{2}\right)\cos^2(Gt)-\frac{1}{2}\right|. \qquad (13)$$

We have demonstrated that (1) for $0<\cos(\theta)<1$ and arbitrary ϕ, the fluctuations in S_1 can be squeezed but those in a_2 cannot; (2) for $-1<\cos(\theta)<-\tan^2(\phi)$ and $-1<\tan(\phi)<1$, the fluctuations in S_1 and a_2 can be squeezed almost all the time with identical squeeze duration ($GT_S=\pi$), squeeze period ($GT_P=\pi$), and height A_m of squeezing peak [A_m appears at $Gt=k\pi$ for atomic squeezing and at $Gt=(k+\frac{1}{2})\pi$ for field squeezing, $k=0$, integer]:

$$\begin{aligned}A_m&=|F_1<0|_{\max}\\&=|h_2<0|_{\max}\\&=\cos^2\left(\frac{\theta}{2}\right)\left[\sin^2\left(\frac{\theta}{2}\right)\cos^2(\phi)-\frac{1}{2}\right],\end{aligned} \qquad (14)$$

and there exists a symmetry between the field and atomic squeezing (SFAS). In case (1), it is obvious that the squeezed atom cannot radiate a squeezed field. In case (2), the squeezed atom can radiate a squeezed field [except a special time $Gt=(k+\frac{1}{2})\pi$, at which the field squeezing appears but the atomic squeezing cannot]. Moreover, we have also shown that the maximum squeezing $A_m=0.0625$ can be obtained for $\phi=0$ and $\theta=2\pi/3$ (or $\theta=\pi/3$ only for the atomic squeezing).

According to the above analysis, it might appear surprising that the fluctuation in a_2 (a_1) is related to those in S_1 (S_2) rather than S_2 (S_1). In fact, the origin of such a relationship is connected with the form of the interaction Hamiltonian $H_I=G(aS_++a^\dagger S_-)$ [5]. As for the fact that the field squeezing can be obtained only under the parameter condition $-1<\cos(\theta)<-\tan^2(\phi)$ and $-1<\tan(\phi)<1$, it is a result of the fact (which has been demonstrated by Wodkiewicz *et al.* [5]) that in order to obtain field squeezing, the initial atomic superposition state has to satisfy two conditions: one of the atomic dipole operators should be squeezed and the expectation value of the commutator $[S_+,S_-]=2S_z$ should be negative. Otherwise, the field squeezing cannot appear.

Hu and Aravind [6] have reported that for $\phi=0$ and arbitrary θ, there exists the SFAS. However, detailed results here have shown that the SFAS can exist only for a special interval of θ, and it can also appear for arbitrary ϕ at fixed θ.

In the following, taking the case of $\phi=0$ as an example, we show the relationship between the field and atomic squeezing.

In Fig. 1(a), it is seen that both field and atomic squeezing appear for $\pi/2<\theta<\pi$, and only atomic squeezing appears

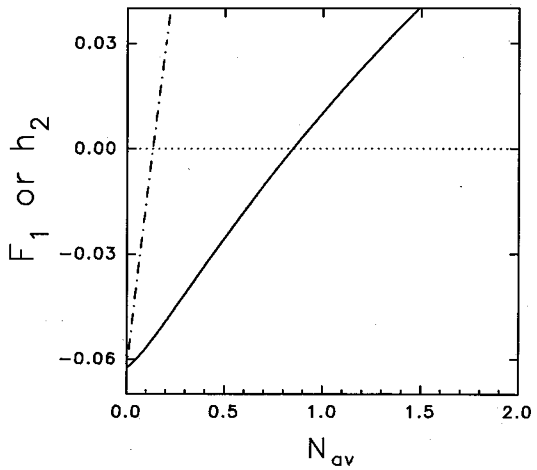

FIG. 2. $h_2(Gt=2.5\pi$, the dot-dashed line) and $F_1(Gt=3\pi$, the solid line) vs $N_{\rm av}$ for $\theta=2\pi/3$ and $\phi=0$.

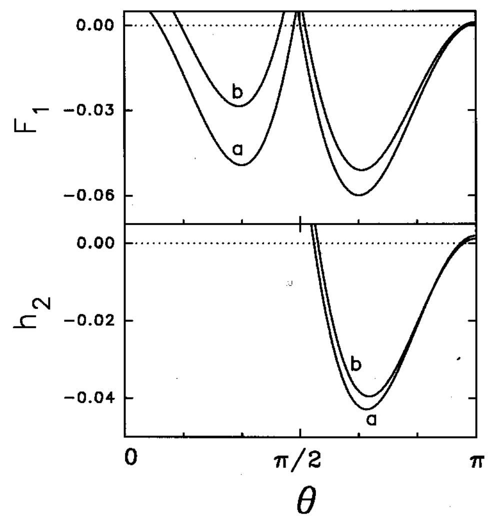

FIG. 3. h_2 (line a at $Gt=2.5\pi$ and line b at $Gt=6.5\pi$) and F_1 (line a at $Gt=3\pi$ and line b at $Gt=6\pi$) vs θ for $\phi=0$ and $N_{\rm av}=0.05$.

for $0<\theta<\pi/2$. Also, we notice that the maximum squeezing $A_m=0.0625$ is obtained for $\theta=2\pi/3$ (or $\theta=\pi/3$ only for the atomic squeezing).

In Fig. 4(a), we show the time evolution of F_1 and h_2 for $\theta=2\pi/3$. It is seen that field and atomic squeezing appear almost all the time, and the SFAS is shown clearly, where $GT_S=GT_P=\pi$, and $A_m=0.0625$ appearing at $Gt=k\pi$ for atomic squeezing and at $Gt=(k+\frac{1}{2})\pi$ for field squeezing.

(ii) For the cases of $N_{av}\neq 0$. The JCM dynamics is exactly solvable in the rotating-wave approximation when the field is in a number state, showing the quantum Rabi oscillations. For the thermal field, the dynamics is not so simple and the main problem is the appearance of infinite sums over n in Eqs. (8)–(11). Here we examine the relationship between the field and atomic squeezing with the help of numerical calculations. From the experimental point of view, the result of large time is not practical, so we only restrict our attention to the small times.

Figures 1(a)–1(b) show how the squeezing versus θ changes in the presence of the thermal photons. Figure 1(a), absence of the thermal photons, has been discussed in case (i). With the increase of $N_{\rm av}$, we find that both field and atomic squeezing start to disappear. And, we see that the width of the θ interval in which squeezing appears varies with $N_{\rm av}$. Clearly, it is difficult to obtain the field and/or atomic squeezing near $\theta=0$, $\pi/2,\pi$. In detail, it is seen that for sufficiently small number of thermal photons, as shown in Fig. 1(b) and 1(c), the atomic squeezing for $0<\theta<\pi/2$ and the field squeezing are much more sensitive to the presence of thermal photons than the atomic squeezing for $\pi/2<\theta<\pi$. When the initial photon number is up to 0.5, field squeezing disappears, and only the atomic squeezing for $\pi/2<\theta<\pi$ can appear. When $N_{\rm av}$ is bigger than 0.9, not only the field squeezing but also the atomic squeezing [except the case at/near $Gt=0$, details are given in case (iii)] disappears, which can be seen from Fig. 2.

In Fig. 3, we show the relationship between F_1 (and h_2) and θ for different times. It is seen that the width of the θ interval in which squeezing appears varies in time. This result is the same as that of Kozierowski, Poyatos, and Sanchez-Soto [10].

Figure 4 presents the time evolution of the function F_1 and h_2 for $\theta=2\pi/3$. Figure 4(a) is the result of $N_{\rm av}=0$, which has been discussed in case (i). With the increase of $N_{\rm av}$, it is seen that the position of the squeeze peak changes slightly, while the squeeze duration ($GT_S=\pi$) is decreased greatly. Details are as follows. For sufficiently smaller $N_{\rm av}$, at the time $0\leqslant Gt\leqslant 4\pi$, the dynamics of the squeezing can

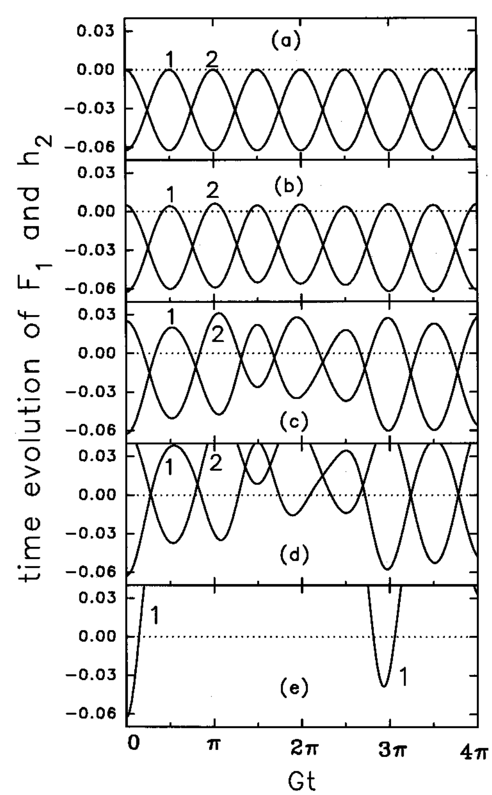

FIG. 4. Time evolution of h_2 (the line 2) and F_1 (the line 1) for $\theta=2\pi/3$ and $\phi=0$. (a) $N_{\rm av}=0$; (b) $N_{\rm av}=0.01$; (c) $N_{\rm av}=0.05$; (d) $N_{\rm av}=0.1$; (e) $N_{\rm av}=0.5$.

be described by that of $N_{av}=0$. When N_{av} is up to 0.01, we notice from Fig. 4(b) that the SFAS begins to be destroyed due to the presence of thermal photons. When N_{av} is increased a bit more, as shown in Fig. 4(c), the field and atomic squeezing can appear, but the SFAS is destroyed greatly, where mostly the squeezed atom cannot radiate a squeezed field. When N_{av} is increased up to 0.1, it is seen from Fig. 4(d) that although the field and atomic squeezing can appear at some time regions, the squeezed atom cannot radiate a squeezed field any longer. When N_{av} is increased further, as shown in Fig. 4(e), we find that only two peaks of the atomic squeezing appear near $Gt=0$, 3π, and the SFAS is destroyed thoroughly.

(iii) For the case at or near Gt=0. For $Gt=0$, according to Eqs. (4) and (7), we can easily find that the fluctuations in a_2 cannot be squeezed, while the atomic squeezing can appear for any N_{av} under the following parameter condition: $0<\cos(\theta)<1$ and arbitrary ϕ, or, $-1<\cos(\theta)<-\tan^2(\phi)$ and $-1<\tan(\phi)<1$, where the height of squeeze peak $A_m=\frac{1}{4}\sin^2(\theta)\cos^2(\phi)+\frac{1}{4}|\cos(\theta)|-\frac{1}{4}$.

Now we reexamine the squeezing in Fig. 4 during the time interval $0<Gt\leqslant 1$. Near the time $Gt=0$, we find that only the atomic squeezing can appear for any N_{av}. Although the field squeezing can appear near $Gt=0$ for a sufficiently small number of thermal photons, it is too weak to be observed.

(iv) Conclusions. We draw main conclusions as follows: (1) At and near $Gt=0$, the atomic squeezing can appear for any initial photon number. At $Gt=0$, the field squeezing cannot appear. Although the field squeezing can appear near $Gt=0$ for sufficiently small initial photon number, it is too weak to be seen. (2) The width of the θ interval, where squeezing appears, varies with time or N_{av}. (3) For sufficiently small initial photon number ($N_{av}<0.01$), the dynamics of the field and atomic squeezing can be approximately described by that of $N_{av}=0$. In detail, the field and atomic squeezing can appear almost all the time and there exists a SFAS under the parameter condition $-1<\cos(\theta)<-\tan^2(\phi)$ and $-1<\tan(\phi)<1$; atomic squeezing can appear but the field squeezing cannot under the parameter condition $0<\cos(\theta)<1$ and arbitrary ϕ. (4) The squeeze duration of the field squeezing or atomic squeezing is decreased with the increase of N_{av}. The field squeezing disappears entirely when $N_{av}>0.1$, while the atomic squeezing except that at or near $Gt=0$ also disappears entirely when $N_{av}>0.9$. (5) The field squeezing is much more sensitive to the presence of thermal photons than the atomic squeezing. (6) For small photon number ($0.1\leqslant N_{av}<0.5$), both the field and atomic squeezing can appear, but the squeezed atom cannot radiate a squeezed field at all. (7) The SFAS begins to be destroyed when N_{av} is up to 0.01, and it is destroyed thoroughly when N_{av} is up to 0.5.

Recent experiments with the one-atom maser [12], where Rydberg atoms with a very large principal quantum number were used and the quality factor is high enough for a periodic energy exchange between atom and cavity field to be observed, have demonstrated that it is possible to study the interaction of a single two-level atom with a single quantized mode of a radiation field. Taking the transition $63P_{3/2}\leftrightarrow 61D_{3/2}$ ($\omega=135$ GHz) of the Rydberg atom ^{85}Rb [12] as an example, we have obtained cavity temperature $T=0.223$, 0.339, 0.43, 0.94, and 1.38 K for $N_{av}=0.01$, 0.05, 0.1, 0.5, and 0.9, respectively. Very recently, Rempe, Schmidt-Kaler, and Walther have cooled a very high-Q (10^7–10^{10}) superconducting niobium maser cavity down to a temperatures of 0.5 and 0.15 K by means of a ^{3}He cryostat. In light of this point, our results are significant in any micromaser experiment.

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