# Preparation of multiatom entangled states through dispersive atom–cavity-field interactions

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(Received 29 September 1995)

A technique based on a dispersive atom–cavity-field interaction is proposed for preparing two or more atoms in macroscopically separated entangled states. After suitably prepared atoms interact with the cavity field a subsequent measurement on this field projects the atoms onto the entangled states. Two-particle entangled states are discussed as well as a three-particle state of the type proposed by Greenberger, Horne, and Zeilinger.

PACS number(s): 03.65.Bz, 42.50.Dv

In the last ten years or so, there has been much progress in the experimental realization of two-particle entangled states. Essentially all of these experiments involve photons where entanglement arises from different photon polarizations [1] or from different paths taken by the photons [2]. In many of these experiments, Bell's inequalities have been violated, thus supporting quantum mechanics over local hidden-variable theories [3]. However, there are some drawbacks to the experiments performed with entangled photon states, particularly of those involving polarization states. As pointed out by Clauser and Horne [4], the lack of control of the directions of the photons emitted in the cascade transitions requires some supplementary assumption in order for Bell's inequalities to be violated. Furthermore, the photodetectors do not have high efficiency [4].

Several proposals to circumvent these drawbacks have been put forward in which the particles to be entangled are "two-level" Rydberg atoms directed through a micromaser cavity containing a resonant single-mode quantized electromagnetic field [5]. In the case of two or more particles the atoms can be directed through the cavity to achieve controllable spatial separation (see Fig. 1). Furthermore, measurement of the atomic state is nearly 100% efficient. The atomic inversion is measured by state selective ionization while the polarization can be measured by interaction with classical microwave fields followed by selective ionizations. Phoenix and Barnett [6], Kudryavtsev and Knight [7], and Cirac and Zoller [8] have proposed a method of generating entangled atomic states of the form

$$|\Psi\rangle=\frac{1}{\sqrt{2}}(|e\rangle_1|g\rangle_2\pm|g\rangle_1|e\rangle_2), \quad (1)$$

where $|e\rangle$ and $|g\rangle$ represent the excited and ground states and the subscripts 1 and 2 label the first and second atoms. Generation of such states requires each atom to be carefully velocity selected before entering the cavity. Furthermore, the cavity must be initially in the vacuum, atom 1 laser excited to state $|e\rangle$, and atom 2 in the ground state. After passage of the atoms, the cavity is left again in the vacuum. Such states as Eq. (1) are well known to violate Bell's inequalities. However, the states produced for *any* velocities generally constitute a mixture also capable of violating Bell's inequalities. Cirac and Zoller [8] have also shown that three-particle entangled states of the form

$$|\Psi\rangle_{\mathrm{GHZ}}=\frac{1}{\sqrt{2}}(|e\rangle_1|e\rangle_2|e\rangle_3-|g\rangle_1|g\rangle_2|g\rangle_3) \quad (2)$$

proposed by Greenberger, Horne, and Zeilinger (GHZ) [9] can also be produced. However, it is necessary to first engineer [10] a cavity field consisting of a superposition of the Fock states $|0\rangle$ and $|3\rangle$. All of these procedures for generating entangled atomic states assume that cavity damping and spontaneous decays are negligible during the time atomic measurements take place.

In this paper I propose an alternative method of generating entangled atomic states. Atoms are imagined to be directed through a cavity as in Fig. 1 except now the cavity is assumed to be prepared in a coherent state of large amplitude and it is assumed that the atom interacts with this cavity field in a highly *nonresonant dispersive* manner. The cavity field may be prepared by driving the cavity with a classical current [11]. The atom is prepared in a superposition of ground and excited states and then interacts with the field in a dispersive manner. The dispersive interaction gives rise to phase shifts of the initial coherent state. Atoms passing through the

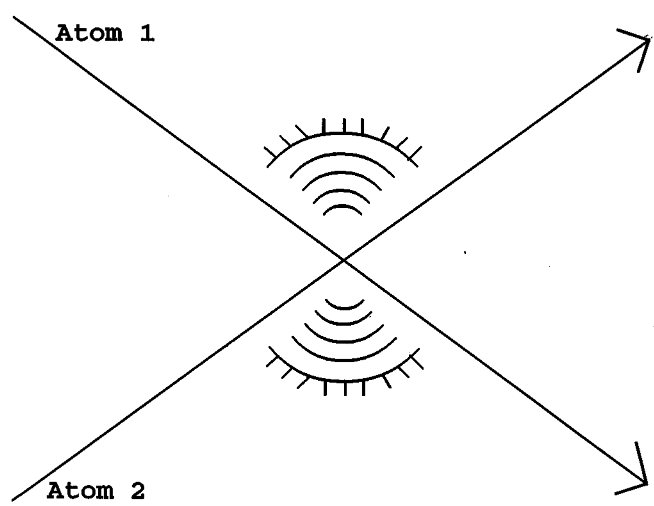


FIG. 1. Configuration of a cavity and atomic trajectories for the preparation of entangled atomic states. Both atoms enter the cavity in a superposition of ground and excited states and interact dispersively with the cavity field assumed initially to be in a coherent state.

 

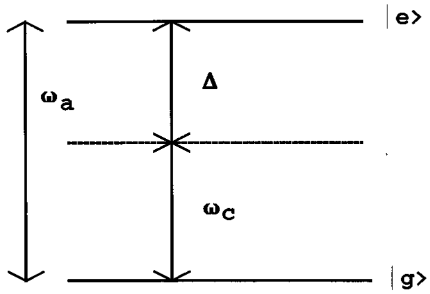


FIG. 2. Energy-level configuration of a two-level atom indicating a large detuning with the cavity field. The atomic transitions are virtual.

cavity create atom-field entangled states and selective measurements *on the cavity field* create the entangled atomic states. It is again assumed that measurements are made in a time short enough so that cavity dissipation effects can be ignored.

We assume that the atoms have the level structure as in Fig. 2 where $\omega_a$ is the atomic transition frequency and $\omega_c$ is the frequency of the cavity mode. We further assume that the detuning $|\Delta|=|\omega_a-\omega_c|$ is so large that only virtual transitions occur between levels $|e\rangle$ and $|g\rangle$. Let $a$ and $a^\dagger$ be the annihilation and creation operators for the cavity field. Then the effective interaction Hamiltonian for the $i$th atom interacting with the cavity field is

$$H_I^i=\hbar\,\eta a^\dagger a\sigma_3^i, \tag{3}$$

where $\sigma_3^i=|e\rangle_{ii}\langle e|-|g\rangle_{ii}\langle g|$, $\eta=\lambda^2/2\Delta$ and where $\lambda$ is the atomic dipole moment. The Hamiltonian is valid under the assumption that $\lambda^2 n\ll\Delta^2+\gamma$ where $n$ is a characteristic photon number and $\gamma$ is the spontaneous emission rate [12].

We further assume that before the $i$th atom enters the cavity, that by laser excitation and microwave manipulation, it is prepared in a superposition of the form

$$|\Psi_{\text{atom } i}\rangle=\frac{1}{\sqrt{2}}(|g\rangle_i+ie^{-i\theta_i}|e\rangle_i). \tag{4}$$

Furthermore, the cavity field is assumed initially to be in a coherent state:

$$|\alpha\rangle=e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle. \tag{5}$$

We now examine the case for two successive atoms passing through the cavity at angles that produce a macroscopic separation. After passage of the first atom, the atom-field state is

$$|\Psi_{\text{field-atom 1}}\rangle=\frac{1}{\sqrt{2}}[|g\rangle_1|\alpha e^{i\phi_1}\rangle+ie^{-i\theta_1}|e\rangle_1|\alpha e^{-i\phi_1}\rangle], \tag{6}$$

where $\phi_1=\eta t_1$, $t_1$ being the time of interaction. We have used the relation

$$e^{\pm i\beta a^\dagger a}|\alpha\rangle=|\alpha e^{\pm i\beta}\rangle. \tag{7}$$

After passage of the second atom, the atom-field state is

$$\begin{aligned}|\Psi_{\text{field-atoms 1+2}}\rangle=\frac{1}{2}[&|g\rangle_1|g\rangle_2|\alpha e^{i(\phi_1+\phi_2)}\rangle\\&-e^{i(\theta_1+\theta_2)}|e\rangle_1|e\rangle_2|\alpha e^{i(\phi_1+\phi_2)}\rangle\\&+ie^{-i\theta_1}|e\rangle_1|g\rangle_2|\alpha e^{i(\phi_1+\phi_2)}\rangle\\&+ie^{-i\theta_2}|g\rangle_1|e\rangle_2|\alpha e^{i(\phi_1+\phi_2)}\rangle],\end{aligned} \tag{8}$$

where $\phi_2=\eta t_2$. The phase shifts $\phi_1$ and $\phi_2$ can be controlled by velocity selection on the atoms. Let us suppose that this can be done such that $\phi_1=\phi_2=\pi/2$ so that we have

$$\begin{aligned}|\Psi_{\text{field-atoms 1+2}}\rangle=\frac{1}{2}[&(|g\rangle_1|g\rangle_2-e^{i(\theta_1+\theta_2)}|e\rangle_1|e\rangle_2)|-\alpha\rangle\\&+ie^{-i\theta_1}(|e\rangle_1|g\rangle_2\\&+e^{i(\theta_1-\theta_2)}|g\rangle_1|e\rangle_2)|\alpha\rangle].\end{aligned} \tag{9}$$

Now for large $|\alpha|$ the states $|\alpha\rangle$ and $|-\alpha\rangle$ are orthogonal, i.e., $\langle-\alpha|\alpha\rangle=0$. Thus if the cavity field is measured and found to be in state $|\alpha\rangle$, the atoms are in the entangled state

$$|\Psi\rangle_\alpha=\frac{1}{\sqrt{2}}(|e\rangle_1|g\rangle_2+e^{i(\theta_1-\theta_2)}|g\rangle_1|e\rangle_2) \tag{10}$$

or if in $|-\alpha\rangle$,

$$|\Psi\rangle_{-\alpha}=\frac{1}{\sqrt{2}}(|g\rangle_1|g\rangle_2-e^{-i(\theta_1+\theta_2)}|e\rangle_1|e\rangle_2). \tag{11}$$

It is well known that these states violate Bell's inequalities. This may be shown by following Cirac and Zoller [8] in using a form due to Clauser *et al.* [3], which states that in order to be consistent with local hidden-variable theories observations must satisfy the inequality

$$|P(a,b)-P(a,b')|+|P(a',b')+P(a',b)|\leqslant 2, \tag{12}$$

where

$$P(a,b)=\langle\vec{\sigma}_1\cdot\vec{a}\,\vec{\sigma}_2\cdot\vec{b}\rangle. \tag{13}$$

The angles between the unit vectors $\vec{a}$, $\vec{b}$, $\vec{a}\,'$, and $\vec{b}\,'$ are controlled by microwave fields applied to the atoms prior to selective ionization, equivalent to rotating a Stern-Gerlach magnet as in the Bohm formulation [13] of the Einstein-Podolsky-Rosen paradox [14]. For example, for the state $|\Psi\rangle_{-\alpha}$ of Eq. (11) we obtain

$$\begin{aligned}P(a,b)=&a_zb_z-(a_xb_x-a_yb_y)\cos(\theta_1+\theta_2)\\&-(a_yb_x+a_xb_y)\sin(\theta_1+\theta_2).\end{aligned} \tag{14}$$

A violation of Bell's inequality may be obtained as follows: Setting $\theta_1+\theta_2=\pi$, $a_y=a_y'=b_y=b_y'=0$, $a_x=\sin\delta$, $a_z=\cos\delta$, $b_x=\sin\beta$, $b_z=\cos\beta$, $a_x'=\sin\beta'$, $a_z'=\cos\beta'$, $b_x'=\sin\beta'$, $b_z=\cos\beta'$ with $\delta=0$, $\beta'=\pi/2$ we obtain from Eqs. (12)–(14),

$$|\cos\beta-\cos\beta'|+|\sin\beta+\sin\beta'|\leqslant 2. \tag{15}$$

The left-hand side is maximized for $\cos\beta' = -\cos\beta$, $\sin\beta = \sin\beta'$, which for $\beta = \pi/4$ and $\beta' = 3\pi/4$ yield $2\sqrt{2} < 2$ and thus a violation of Bell's inequality.

The question remains as to how to collapse the state of Eq. (9) onto $|\alpha\rangle$ or $|-\alpha\rangle$. One way would be to adapt a method proposed by Brune *et al.* [15] for the detection of Schrödinger cat states in cavities. After the passage of the two atoms, the cavity can again be driven by classical currents to produce a reference field $|\alpha_r\rangle$ such that the total cavity field state is now $|-\alpha+\alpha_r\rangle$ or $|\alpha+\alpha_r\rangle$. Obviously, if $\alpha_r = \alpha$ the state $|\Psi\rangle_{-\alpha}$ of Eq. (11) is correlated with the cavity being in the vacuum state $|0\rangle$. A stream of ground-state two-level atoms *resonant* with the cavity can be injected and selectively ionized upon emergence. The lack of atoms in the excited state would indicate that the cavity field was indeed in the vacuum state, there being a very high probability of absorbing a photon if the cavity is in state $|2\alpha\rangle$. Alternatively, one could use the dispersive interaction in the following way. Suppose that the cavity field contains a definite number state $|n\rangle$. Further, suppose we prepare the atom in a superposition state

$$|\Psi_{\text{atom}}\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) \tag{16}$$

by using a classical microwave field so that the initial state is $|\Psi_{\text{atom}}\rangle|n\rangle$. Now when the atom exits the cavity the state is then

$$\frac{1}{\sqrt{2}} e^{-i\eta n t}(|e\rangle + e^{2i\eta n t}|g\rangle)|n\rangle. \tag{17}$$

A second classical microwave field can be used to cause the transitions

$$|e\rangle \to \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle), \quad |g\rangle \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle) \tag{18}$$

such that the above state becomes

$$\frac{1}{2} e^{i\eta n t}[|e\rangle(1 - e^{2i\eta n t}) + |g\rangle(1 + e^{2i\eta n t})]|n\rangle. \tag{19}$$

The probabilities of finding the atom in the ground or excited state are

$$P_g(t) = \sin^2(\eta n t), \quad P_e(t) = \cos^2(\eta n t). \tag{20}$$

Obviously, for the vacuum, $n=0$, the atom will never be found in the ground state. Also this result will occur for all atomic speeds since the probabilities in the case of $n=0$ are independent of time.

Likewise, if $\alpha_r = -\alpha$, a cavity vacuum state is correlated with the atomic state $|\Psi\rangle_\alpha$ of Eq. (10).

On the other hand, we now suppose that the atoms are velocity selected so that $\phi_1 = \phi_2 = \pi$. In this case all of the field states in Eq. (8) return to the initial coherent state $|\alpha\rangle$:

$$|\Psi_{\text{field-atoms }1+2}\rangle = \frac{1}{2}[|g\rangle_1|g\rangle_2 - e^{-i(\theta_1+\theta_2)}|e\rangle_1|e\rangle_2 + ie^{-i\theta_1}|e\rangle_1|g\rangle_2 + ie^{-i\theta_2}|g\rangle_1|e\rangle_2]|\alpha\rangle. \tag{21}$$

At first sight it would seem that the atomic states, which are now disentangled from the field, are themselves entangled states of the two atoms and therefore capable of violating Bell's inequality. Unfortunately this turns out not to be the case. For example, consider the two-atom state for $\theta_1 = \theta_2 = 0$:

$$|\Psi_{\text{atoms}}\rangle = \frac{1}{\sqrt{2}}[|g\rangle_1|g\rangle_2 - |e\rangle_1|e\rangle_2 + i(|e\rangle_1|g\rangle_2 + |g\rangle_1|e\rangle_2)]. \tag{22}$$

This is actually a special case of a state discussed by Kudryavtsev and Knight [7] who stated that because it is ''never a product state'' that Bell's inequality is violated as follows from a paper by Gisin [16]. However, it can be shown that Eq. (22) is not truly an entangled state [17]. If we define new bases

$$|g\rangle_k' = \frac{1}{\sqrt{2}}|e\rangle_k + \frac{i}{\sqrt{2}}|g\rangle_k,$$
$$|e\rangle_k' = \frac{i}{\sqrt{2}}|e\rangle_k + \frac{1}{\sqrt{2}}|g\rangle_k, \quad k=1,2, \tag{23}$$

then Eq. (22) becomes

$$|\Psi_{\text{atoms}}\rangle = |e\rangle_1'|e\rangle_2', \tag{24}$$

clearly a factorized state. Thus Bell's inequality is not violated. This can be shown to be generally true for all $\theta_1$ and $\theta_2$.

It can further be shown that if no measurement is made on the cavity field, then Eq. (8) does not lead to a violation of Bell's inequalities for any values of $\phi_1$ and $\phi_2$. Generally the atoms are entangled with the field so are themselves in a mixed state and such states exhibit nonlocality in a much weaker form. If we allow for arbitrary phase shifts $\phi_1$ and $\phi_2$ then using Eq. (8) we find that (with $a_y = b_y = 0$, $\theta_1 = \theta_2 = 0$)

$$P(a,b) = a_x b_x \frac{1}{2}\{e^{-|\alpha|^2[1-\cos 2(\phi_1-\phi_2)]} \times \cos[|\alpha|^2 \sin 2(\phi_1-\phi_2)] + e^{-|\alpha|^2[1-\cos 2(\phi_1+\phi_2)]} \times \cos[|\alpha|^2 \sin 2(\phi_1+\phi_2)]\}, \tag{25}$$

where we have used the result

$$\langle\alpha|\beta\rangle = \exp\left[-\frac{1}{2}(|\alpha|^2 + |\beta|^2) + \alpha^*\beta\right]. \tag{26}$$

The term in brackets in Eq. (24) is maximal for $\phi_1 = \phi_2 = \pi/2$ such that $P(a,b) = a_x b_x$. In this case Eq. (12) becomes

$$|a_x(b_x-b_x')|+|a_x'(b_x'+b_x)|\leq 2 \tag{27}$$

or

$$|\sin\delta(\sin\beta-\sin\beta')|+|\sin\delta'(\sin\beta+\sin\beta')|\leq 2, \tag{28}$$

which is always satisfied. Thus in cases where no measurement of the cavity field is made, Bell's inequalities are not violated.

Finally, we consider the case when three atoms are directed through the cavity at different angles. We follow the same procedure as before and assuming the atoms' velocity selected so that the phase shifts on the coherent states satisfy $\phi_1=\phi_2=\phi_3=\pi/3$ we arive at

$$|\Psi_{\text{field-3 atoms}}\rangle=\frac{1}{2\sqrt{2}}[(|g\rangle_1|g\rangle_2|g\rangle_3-e^{-i(\theta_1+\theta_2+\theta_3)}|e\rangle_1|e\rangle_2|e\rangle_3)|-\alpha\rangle+i(e^{-i\theta_1}|e\rangle_1|g\rangle_2|g\rangle_3+(e^{-i\theta_2}|g\rangle_1|e\rangle_2|g\rangle_3$$
$$+e^{-i\theta_3}|g\rangle_1|g\rangle_2|e\rangle_3)|-\alpha e^{i\pi/3}\rangle-(e^{-i(\theta_1+\theta_2)}|e\rangle_1|e\rangle_2|g\rangle_3+e^{-i(\theta_1+\theta_3)}|e\rangle_1|g\rangle_2|e\rangle_3$$
$$+e^{-i(\theta_2+\theta_3)}|g\rangle_1|e\rangle_2|e\rangle_3)|\alpha e^{-i\pi/3}\rangle]. \tag{29}$$

Now for large $|\alpha|$ the states $|-\alpha\rangle$ and $|\alpha e^{\pm i\pi/3}\rangle$ are orthogonal. Thus a detection of the cavity field in the state $|-\alpha\rangle$ using the procedure discussed above produces the GHZ [9] atomic state

$$|\Psi\rangle_{\text{GHZ}}=\frac{1}{\sqrt{2}}[|g\rangle_1|g\rangle_2|g\rangle_3-e^{-i(\theta_1+\theta_2+\theta_3)}|e\rangle_1|e\rangle_2|e\rangle_3]. \tag{30}$$

Clearly, $n$-atom generalizations of this type of state showing extreme entanglement as discussed by Mermin [18] are possible if the field state $|-\alpha\rangle$ is nearly orthogonal to the other phase-shifted coherent states that appear. This appears to be possible for fields of large amplitude $|\alpha|$.

Now it must be admitted that generation of two atom entangled states by the method proposed in this paper is perhaps more involved than in the proposal of Cirac and Zoller [8]. In their case, with careful atom velocity selection and with the cavity initially in the vacuum, the two-atom entangled state appears factored from the vacuum field after passage of the atoms. However, to produce the three-atom GHZ state, as we have said, the cavity must first be engineered into superposition of the number states $|0\rangle$ and $|3\rangle$, a procedure that could be rather problematic. In the present work, however, all that is required is the determination that the cavity is in the vacuum state—a procedure that should be much easier than engineering a special initial cavity state. Thus we believe that the present method could be advantageous for generating GHZ-type states for three or more atoms.

The author gratefully acknowledges clarifying discussions with P. K. Aravind.

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