# **Dynamics of a Brillouin fiber ring laser: Off-resonant case**

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The behavior of a nonresonant Brillouin laser is studied both theoretically and experimentally. Equations for the laser are revisited in order to derive the steady states. The stability of the stationary solutions is studied numerically in the case of a short length cavity. This study reveals unstable regimes and a modal bistability around the antiresonant frequency. Experimental results confirm these predictions.

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### **I. INTRODUCTION**

Since it was first observed in 1976  $[1]$ , Stimulated Brillouin scattering (SBS) in optical fiber resonators has been the object of numerous studies. These were mostly performed with fiber lengths of the order of a hundred meters. Just above the lasing threshold, instabilities such as periodic, quasi-periodic, solitonic, and chaotic regimes have been observed  $[2-5]$ . Steady states have been reached for pumping levels a few times greater than the lasing threshold  $[6]$ . In a previous paper  $[7]$ , we demonstrated that these behaviors can only be encountered in long cavities, for which many modes lie under the homogeneously broadened Brillouin gain curve. For cavities shorter than a critical length  $L_c$ , the emission involves only one longitudinal cavity mode, and its intensity remains stable. This result has been established theoretically when the maximum of the gain curve coincides with one cavity mode, hereafter called resonance. The role of a frequency detuning between the gain peak and the cavity mode (called Stokes detuning), has been investigated experimentally. The experiments, performed for a short cavity length, have shown that the laser is stable not only at the resonance, but also for a large range of Stokes detunings.

Up to now, theoretical studies focused on the resonant case, and the exact effect of Stokes detuning on the laser dynamics remains largely unknown. The present paper is devoted to an experimental and theoretical analysis of this effect. Our Brillouin ring laser has been modified in order to avoid the pump feedback. The behaviors then produced are roughly similar to those previously published, but their confrontation with theory is greatly simplified. From the theoretical point of view, the classical three-wave coherent model [8] is reformulated to describe a detuned Brillouin ring laser. We derive the steady states of the system and study numerically the stability of these solutions using parameters corresponding to the experimental conditions. Unstable regimes and modal bistability are observed around the antiresonance, for which the maximum of gain is halfway between the two cavity modes. These phenomena are also observed experimentally.

In Sec. II, we describe the off-resonant three-wave coherent model used in this study. The steady-state solutions are derived in Sec. III, and their stability is analyzed in Sec. IV. Finally, Sec. V is devoted to experimental results.

### **II. OFF-RESONANCE THREE-WAVE MODEL**

Let us now consider basic assumptions for a resonant SBS interaction. A pump field  $(\omega_p, k_p)$  is scattered by an acoustical wave ( $\omega_a^{\text{res}}, k_a^{\text{res}}$ ), thus generating a counterpropagating Stokes field ( $\omega_s^{\text{res}}, k_s^{\text{res}}$ ). For these three fields, the wave numbers satisfy the usual dispersion relations  $k_p = n\omega_p/c$ ,  $k_s^{\text{res}}$  $= n \omega_s^{\text{res}}/c$ , and  $k_a^{\text{res}} = \omega_a^{\text{res}}/v$  (where *n* is the refractive index,  $c$  the light velocity, and  $v$  the sound wave velocity). The interaction fulfills the conservation laws of the energy  $\omega_a^{\text{res}}$  $=\omega_p - \omega_s^{\text{res}}$  and momentum  $k_a^{\text{res}} = k_p + k_s^{\text{res}}$ . For a given value of the pump frequency  $\omega_p$ , all frequencies and momenta can be deduced from these relations.

This previous analysis implicitly implies that the fields and their interactions are described in frames rotating respectively at the frequencies  $\omega_p$ ,  $\omega_a^{\text{res}}$ , and  $\omega_s^{\text{res}}$ , leading to the standard three-wave coherent model [8]. In particular the Stokes field writes:

$$
S(z,t) = S'(z,t) \exp\left[-i\omega_s^{\text{res}}\left(t + \frac{n}{c}z\right)\right] + \text{c.c.}
$$
 (1)

Such rotating frames are well adapted to a description of the spontaneous Stokes emission which occurs at the frequency  $(\omega_s^{\text{res}})$  corresponding to the maximum of the gain curve.

When the fiber is inserted in a cavity, the description remains valid, and an eventual mistuning between the Stokes resonant frequency ( $\omega_s^{\text{res}}$ ) and the passive cavity mode ( $\Omega_s$ ) carrying the laser oscillation (called the active mode) is taken into account through the boundary condition which then writes.

$$
S'(L,t) = r_s \exp\left(-i\Delta\omega_0 \frac{nL}{c}\right) S'(0,t). \tag{2}
$$

The reinjection rate of the field  $r<sub>s</sub>$  is real. *L* is the cavity length and  $\Delta \omega_0 = \Omega_s - \omega_s^{\text{res}}$  is the cavity detuning. Note that, *a priori*, the active mode can be any mode lying under the gain curve and not necessarily the closest to the maximum of  $gain$  (Fig. 1).

Owing to this condition, the stationary solutions of the three-wave model necessarily involve a time dependence of the phase of the complex field envelopes  $[7,9]$ . The resulting phase oscillation brings the Stokes frequency to a value  $\omega_L$ which depends on the medium gain profile and on the char-

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**TOKES** 

**SHIFT 20 GHZ** 

 $\Omega_{\rm S}^2$ 

 $\Omega_{\rm P}^0$ 

**PUMP** 

 $\omega_{\rm p}$   $\Omega_{\rm p}^1$ 



 $\Omega^1_{\mathrm{S}}$ 

**FREQUENCY** 

**SPONTANEOUS** 

**BRILLOUIN** 

**GAIN CURVE** 

 $\Omega_{\rm S}^0$   $\omega_{\rm L}$   $\omega_{\rm S}^{\rm Res}$ 

$$
P(z,t) = P(z,t) \exp\left[-i\omega_p \left(t - \frac{n}{c} z\right)\right] + \text{c.c.},\qquad(3a)
$$

$$
S(z,t) = S(z,t) \exp\left[-i\omega_L \left(t + \frac{n}{c} z\right)\right] + \text{c.c.},\qquad(3b)
$$

where the Stokes field envelope  $S(z,t)$  fulfills the boundary condition.

$$
S(L,t) = r_s S(0,t) \exp(-i\delta_s), \qquad (3c)
$$

with

$$
\delta_s = \frac{nL}{c} \left( \Omega_s - \omega_L \right).
$$

Through the electrostrictive effect, the interference between these two fields drives an acoustical wave  $M(z,t) = \rho(z,t) \exp(-i(\omega_a t - k_a z) + c.c.$  characterized by a frequency  $\omega_a = \omega_p - \omega_L$  and a wave number  $k_a = (\omega_p + \omega_L)(n/c)$ . Due to the Stokes detuning, this forced acoustical wave no longer respects the dispersion relation  $(\omega_a \neq k_a v)$ . Its growing is then frustrated, leading to a decrease of the gain. This effect is described by a destructive phase oscillation term, at the frequency  $\Delta\omega_1$ , appearing in the reduced Navier-Stokes equation of the acoustic wave  $[8]$ . The interaction of the three waves is then described by the following set of equations:

$$
\frac{\partial P}{\partial t} + \frac{c}{n} \frac{\partial P}{\partial z} = i \frac{\omega_p \gamma^e}{2 \rho_0 n^2} \rho S,
$$
  

$$
\frac{\partial S}{\partial t} - \frac{c}{n} \frac{\partial S}{\partial z} = i \frac{\omega_p \gamma^e}{2 \rho_0 n^2} \rho^* P,
$$
  

$$
\frac{\partial \rho}{\partial t} + (\gamma_a + i \Delta \omega_1) \rho = i \frac{2 \varepsilon_0 k_p^2 \gamma^e}{\omega_a} P S^*,
$$
 (4)

where  $\gamma^e$  is the electrostrictive coupling constant,  $\rho_0$  the mean density of the fiber core, and  $\gamma_a$  the damping rate of the

FIG. 1. Typical disposition of the pump laser, the maximum spontaneous gain, and the Stokes laser frequencies with respect to the passive cavity modes.

acoustical wave field. We have neglected the convective term of the acoustical wave  $(\partial \rho/\partial z=0)$  [10] and the absorption of the fiber compared to the cavity losses. The length and time can be rescaled with  $(\gamma_a n/c)z \rightarrow z$  and  $\gamma_a t \rightarrow t$ , defining  $L' = (\gamma_a n/c) L$ , the normalized length of the cavity, and  $\Delta_i = \Delta \omega_i / \gamma_a$  (*j*=0 and 1), the normalized frequency detunings. We then introduce the following transformations:  $E_p = (K/\gamma_a)P$ ,  $E_s = (K/\gamma_a)S$ , and  $E_a = -i(K/\gamma_a \sigma)\rho$ , where  $K = \omega_p \gamma^e (\varepsilon_0/2nv c \rho_0)^{1/2}$  and  $\sigma^2 = 2\rho_0 n^3 \varepsilon_0/v c$ . Equations (4) then become

$$
\frac{\partial E_p}{\partial t} + \frac{\partial E_p}{\partial z} = -E_s E_a,
$$
  

$$
\frac{\partial E_s}{\partial t} - \frac{\partial E_s}{\partial z} = E_p E_a^*,
$$
  

$$
\frac{\partial E_a}{\partial t} + (1 + i\Delta_1) E_a = E_p E_s^*.
$$
 (5)

This set of equations must be completed by the boundary conditions imposed by the cavity for the Stokes field, which write

$$
E_s(L',t) = r_s E_s(0,t) \exp(-i\delta_s), \tag{6}
$$

and eventually for the pump wave. Compared to the usual model, written at resonance, the only difference lies in the term  $i\Delta_1$  which appears in the acoustic wave equation, and fully accounts for the unresonant nature of the interaction in a detuned cavity. Equations (5) were previously obtained by Chow and Pers [11] in order to model a Brillouin amplificator in which a detuning is imposed by the frequency of the seeding wave. However, let us emphasize that the choice of the appropriate rotating frames we recommend allows for a much simpler derivation of these equations.

### **III. STEADY STATES**

As the Stokes and pump fields  $E_s$  and  $E_p$  evolve in frames rotating at the lasing frequency  $\omega_L$  and the pump frequency  $\omega_p$ , respectively, the amplitudes and phases of the steadystate solutions must be time independent. We then transform the complex amplitudes to modulus-phase form by substituting the following expressions in Eqs.  $(5)$ :

By setting all the time derivatives to zero, we easily obtain the following equations for the steady state:

$$
\frac{dA_p}{dz} = -\frac{A_p A_s^2}{1 + \Delta_1^2},\tag{8a}
$$

$$
\frac{dA_s}{dz} = -\frac{A_p^2 A_s}{1 + \Delta_1^2},\tag{8b}
$$

$$
\frac{d\Phi_p}{dz} = \frac{\Delta_1}{1 + \Delta_1^2} A_s^2,\tag{8c}
$$

$$
\frac{d\Phi_s}{dz} = -\frac{\Delta_1}{1 + \Delta_1^2} A_p^2.
$$
 (8d)

As could be expected, the Stokes detuning entails a gain reduction by a factor  $1/(1+\Delta_1^2)$ , linked to the Lorentzian nature of the gain profile. The propagation of the acoustical wave being neglected, the field  $E_a$  is a slave variable of the pump and Stokes fields, and is fully defined by

$$
A_a = \frac{A_p A_s \exp(-i\Phi)}{1 + i\Delta_1},
$$
 (8e)

where  $\Phi = \Phi_a + \Phi_s - \Phi_p$  fulfills the relation tan  $\Phi = -\Delta_1$ .

A lossless energy exchange occurs  $[A_p^2(z) - A_s^2(z)]$  is constant], together with an action of each field on the other's phase. The integration of equations  $(8a)–(8d)$  leads to a longitudinal distribution of fields and phases similar to that previously given by Chow and Bers [11] and Botineau *et al.*  $[10]$  which writes

$$
A_p^2(z) = \frac{A_p^2(0)(1-F)}{1-F \exp[-2\eta(1-F)A_p^2(0)z]},
$$
 (9a)

$$
A_s^2(z) = \frac{A_p^2(0)(1 - F)F}{\exp[2\,\eta(1 - F)A_p^2(0)z] - F},\tag{9b}
$$

$$
\Phi_p(z) - \Phi_p(0) = -\frac{\Delta_1}{2} \ln \left( \frac{A_p^2(z)}{A_p^2(0)} \right),
$$
 (9c)

$$
\Phi_s(z) - \Phi_s(0) = \frac{\Delta_1}{2} \ln \left( \frac{A_s^2(z)}{A_s^2(0)} \right),
$$
\n(9d)

where  $F = A_s^2(0)/A_p^2(0)$  and  $\eta = 1/(1+\Delta_1^2)$ . All the fields are then fully described as functions of the pump and Stokes wave intensities at  $z=0$  and of the normalized frequency detuning  $\Delta_1$ .

The boundary conditions to apply on the modulus and phase of the Stokes field are then

$$
A_s(L') = r_s A_s(0), \t(10a)
$$

$$
\Phi_s(L') - \Phi_s(0) = -\delta_s. \tag{10b}
$$

Coupled with Eqs.  $(9b)$  and  $(9d)$ , expressed at the boundaries, these two relations fix the value of the unknown parameters *F* and  $\Delta_1$  which are given by

$$
(1 - F) = r_s^2 \{ \exp[2 \eta (1 - F) A_p^2(0) L'] - F \}, \quad (11a)
$$

$$
\Delta_1 = \frac{\Delta_0}{1 + \frac{\kappa}{\gamma_a}} = \frac{\Delta_0}{1 + \frac{\Delta \nu^{PC}}{\Delta \nu^B}},
$$
\n(11b)

where  $\kappa = c |Lnr_s|/nL$  is the Stokes field damping rate out of the cavity, and  $\Delta \nu^{PC} = \kappa/\pi$  and  $\Delta \nu^{B} = \gamma_a/\pi$  are the full width at half maximum of a passive cavity mode and of the spontaneous Brillouin gain curve, respectively.

Equation  $(11b)$  is the classical mode pulling formula obtained for lasers with a population inversion  $[12]$ , and already shown on a Brillouin laser at the lasing threshold  $[13]$ . In fact, this equation is valid whatever the pump power, and gives the exact lasing frequency, which depends only on the Stokes detuning and on the ratio of the cavity and medium damping rates. The implicit equation  $(11a)$  can be solved numerically. For a given detuning  $\Delta_0$ , the parameter *F* and thus all the stationary fields depend only on the pump intensity in the plane  $z=0$  [ $A_p^2(0)$ ]. These results are valid whatever the choice of the boundary condition imposed on the pump field. The general expression of this condition writes

$$
E_p(0) = \mu + r_p \exp(-i\delta_p) E_p(L'). \tag{12}
$$

 $\mu$  is the normalized pump field injected in the cavity which is assumed to be real, and then defines a phase reference for the intracavity pump field,  $r_p$  is the reinjection rate of the pump field, and  $\delta_p = (nL/c)(\dot{\Omega}_p - \omega_p)$  is the accumulated phase difference of the pump field per round trip.  $\Omega_p$  is the frequency of the passive cavity mode which is the closest to the pump frequency. In order to avoid any confusion with the Stokes detuning previously defined, the quantity  $(\Omega_p - \omega_p)$ will be called the pump detuning hereafter. For a given value of  $A_p^2(0)$ , which fully defines the stationary solution [see Eq. (9)], the input pump intensity  $\mu^2$  is directly given by Eq.  $(12)$ . *A*  $_p^2(0)$  can be adjusted by an iterative method in order to obtain a particular value of  $\mu^2$ .

The intracavity pump field at the lasing threshold is obtained by setting  $F$  to zero in Eq.  $(11a)$ . The required pump intensity, effectively launched into the fiber, then deduced from Eq.  $(12)$ , is

$$
\mu_{\text{thres}}^2 = \frac{\kappa}{\gamma_a} \left( 1 + \Delta_1^2 \right) \left[ 1 + r_p^2 - 2r_p \cos(\delta_p) \right]. \tag{13}
$$

This relation gives the ring Brillouin laser threshold in the general case.  $\kappa/\gamma_a$  is the threshold in the absence of Stokes detuning and pump feedback [9]. The two factors  $(1+\Delta_1^2)$ and  $[1 + r_p^2 - 2r_p \cos(\delta_p)]$  stand, respectively, for the increase of the threshold with the detuning, directly linked to the gain reduction, and its decrease with the pump feedback efficiency. The distributed losses of the fiber, neglected here, might however, be included in the boundary conditions as a slight modification of the reinjection rates  $r_s$  and  $r_p$ .



FIG. 2. Domains of stability of the two adjacent modes 0 and 1 with respect to the pump intensity  $\mu^2$ , and the Stokes frequency detuning (expressed in FSR units). Parameters are the following:  $r_s = 0.36$ ,  $r_p = 0$ ,  $K = 83$  ms<sup>-1</sup> V<sup>-1</sup>, *L* = 12 m, and  $\Delta \nu^B = 60$  MHz.

#### **IV. STABILITY OF THE STEADY STATE**

For short enough cavity lengths, we have previously shown that the Brillouin laser emission is stable at the resonance  $[7]$ . This emission is then expected to be carried by the mode which coincides with the gain peak. In the presence of a Stokes detuning, the Brillouin gain bandwidth usually being larger than the cavity free spectral range (FSR), more than one mode is susceptible to carrying the laser oscillation, and a mode-mode competition is not excluded, especially around the antiresonance. First, let us emphasize that the variation of the Stokes detuning can be monitored not only by a cavity length sweeping, but also by a pump frequency sweeping, the Stokes shift ( $\omega_p - \omega_s^{\text{res}}$ ) then remaining constant. In both cases, the Stokes and pump detuning are simultaneously swept, and their contributions to the laser dynamics are hardly distinguishable.

For the sake of simplicity, we first consider the simpler case for which the pump feedback is avoided. We have analyzed the stability of the steady states corresponding to two adjacent cavity modes, respectively labeled  $0$  and  $1$  (see Fig. 1), as a function of the pump intensity and the Stokes detuning of the mode 0. For each value of the intensity and detuning, the set of Eq.  $(5)$ , completed by Eq.  $(11b)$  and by the limit conditions  $(6)$  and  $(12)$ , is numerically integrated for each cavity mode. The initial conditions are obtained by adding a small perturbation to the stationary solution. The evolution of this perturbation is observed over a large number of cavity round trips. At the stability boundaries of a particular active mode  $[Fig. 2]$ , two behaviors are observed.

 $(i)$  An unstable regime, characterized by a periodic oscillation of the Stokes intensity at a frequency close to the FSR.

(ii) A change of the active mode, shown by a rotation of the Stokes field's phase at about the FSR frequency, the rotating frame used being no more adapted to a side-mode emission.

Figure 2 depicts stability domains obtained with parameters characterizing the experiment described in Sec. V. This map shows that the steady states are stable over a wide range of intensity and detuning, the laser emission then being monomode. Nevertheless, instabilities are observed in a small domain located around the antiresonance  $(A - B$  domain). In this region, the gain peak is halfway between the two modes which nearly experience the same gain. As the input pump power remains low, so does the laser emission. Thus the pump depletion and its phase rotation [see Eq.  $(9c)$ ] over one round trip induced by one oscillating mode are too small to prevent an emission on the other mode. The two modes can oscillate simultaneously, leading to a periodic instability at the beating frequency. Note that in the vicinity of point *A*, the frequency of this beating is exactly the quantity FSR/ $[1+(\Delta \nu^{PC}/\Delta \nu^B)]$ . This points out that the two modes oscillate independently, and that their characteristics are those of the steady state previously obtained. In particular, they both undergo the frequency pulling given by Eq.  $(11b)$ . From *A* to *B*, the frequency increases by a few percent, denoting a reduction of the pulling effect in a multimode oscillation regime. At the right side of point *B*, the stability domains of the two modes overlap around the antiresonance. The Brillouin gain is then quite high, and the oscillating mode strongly depletes the pump. In this interaction, the phase rotation of the pump field over one round trip is no more negligible, and changes of sign from one mode to the other [see Eq.  $(9c)$ ]. The coexistence of the two modes is then impossible, and, moreover, an active mode can prevent the other from being active, even if this last one is closer to the gain peak. The resulting domain of bistability becomes larger and larger as the pump intensity increases, even leading to a multistability with the next sidemodes for much higher pump power (not shown on this map).

In order to illustrate these results, let us consider a quasistatic sweeping of the Stokes detuning, whereas the input pump intensity is held constant. Figure  $3(a)$  shows the evo-



FIG. 3. Stokes (solid line) and transmitted pump (dashed line) steady-state intensities while sweeping the Stokes detuning over two passive cavity FSR's. (a)  $\mu^2$ =0.17 (the brackets delimitate the unstable regions), (b)  $\mu^2$ =0.33. Other parameters are those of Fig. 2.

lution of stationary solutions for the pump and Stokes intensities at  $z = L'$  and  $z = 0$ , respectively, during a sweeping of mode 0 detuning over two FSR's. Departing from the resonant condition, the intensity of the laser, emitting on mode 0, decreases. Around antiresonance, the laser is unstable (see map 2), and a multimode oscillation regime emerges. Beyond this unstable region, the emission restabilizes on mode 1 and the process recurs with the next sidemode. For a higher pump power [Fig.  $3(b)$ ], mode 0 destabilizes well after the antiresonance. In this case, the two adjacent modes cannot coexist, and the emission abruptly changes from mode 0 to mode 1. At this point, mode 1 has a higher gain than mode 0, and the field intensities undergo a discontinuous jump. Back and forth sweeping around the antiresonance shows a butterfly-shaped hysteresis cycle, symmetric with respect to the antiresonance. The width of this cycle increases with the emission strength.

The effect of the Stokes detuning on the system's dynamics being characterized, we can investigate the case of a common cavity with a pump feedback. In this configuration, the sweeping of the pump detuning induces strong variations of the intracavity pump intensity, and then of the Stokes emission. As previously shown, the emission strength is a determinant factor of the mode hop positions, so that the relative position of the pump and Stokes resonances will strongly



FIG. 4. Stokes (solid line) and transmitted pump (dashed line) steady-state intensities while simultaneously sweeping the pump and Stokes detuning over two passive cavity FSR's. (a)  $\mu^2$ =0.17. (b)  $\mu^2 = 0.33$ . Other parameters:  $r_s = r_p = 0.36$ ,  $K = 83$  ms<sup>-1</sup> V<sup>-1</sup>,  $L=12$  m, and  $\Delta \nu^B = 60$  MHz.

influence the system dynamics. However, this last parameter is fixed by the pump frequency, the Stokes shift, and the exact cavity length, and may then have any value. We then choose a value of this parameter which is as representative as possible of all the encountered behaviors. The steady-state solutions plotted in Fig. 4 are calculated in conditions such that the pump frequency is resonant with a cavity mode when the Stokes detuning is equal to 0.4. The strong intensity modulation, evident on these curves, is then obviously linked to the pump intracavity intensity changes with the pump detuning. At the origin of sweeping  $4(a)$ , the intracavity pump field is just high enough to initiate a laser oscillation. Departing from this frequency, the intracavity pump field increases, and so does the intensity of the laser emitting on mode 0, which reaches its maximum when the pump frequency coincides with a cavity mode. As in the previous case, the mode hop, characterized by the discontinuous jump, does not occur at the exact antiresonance of the Stokes wave, but a bit further. After the mode hop, mode 1 oscillates alone, its intensity decreasing with the pump intracavity field down to the threshold, and reappears as the pump frequency approaches the next cavity resonance; then the process recurs. For a higher pumping level [Fig.  $4(b)$ ], the intracavity field is above the threshold for any frequency. The mode hop is displaced further again from the antiresonance point. Note that,



FIG. 5. Schematic setup of the experimental arrangement.

in this configuration, due to the frequency difference between pump and Stokes resonances, a back and forth frequency sweeping shows an asymmetric cycle of hysteresis. However as a whole, the main effect of the pump feedback on the laser stands in strong variations of the intracavity fields with the detuning, which do not reveal new dynamical features.

The map of Fig. 2, calculated in particular conditions, is in fact generic of the general behavior of most of the Brillouin lasers. The reinjection rate  $r_s$  mainly acts on the width *A*-*B* of the domain of instability, so that *A*-*B* vanishes when  $r<sub>s</sub>$  approaches 1. As the cavity length increases, the FSR decreases, and the simultaneous emission of two adjacent modes is favored, leading to an expansion of the unstable domain toward the resonances. For a cavity length longer than  $L_c$ , the laser may, within a finite range of pump power, be unstable for any Stokes detuning [7]. Note that the fluctuations of the cavity length can induce erratic passages from stable to unstable domains, leading to a bursting phenomena [2]. As the pump power increases and approaches the value corresponding to point  $B$ , the stable domains enlarge and the burst time spacing is expected to increase, whereas their duration decreases  $[5]$ . Above *B*, only mode hops can be observed.

#### **V. EXPERIMENTS**

The experimental setup is presented in Fig. 5. The cw emission of a Ti-Sa laser, operating at 800 nm, is used as a pump source. This laser is characterized by a 500-KHz linewidth, and its frequency can be linearly swept over a range adjustable from 10 MHz to 30 GHz. The Brillouin ring laser comprises a 12-m-long polarization maintaining optical fiber.





FIG. 6. Experimental records of the backscattered Stokes and the transmitted intensity obtained while sweeping the pump laser frequency in the presence of an intracavity isolator: (a)  $P_{in}$ =70 mW and (b)  $P_{\text{in}} = 140 \text{ mW}$ .

FIG. 7. Experimental records of the backscattered Stokes and transmitted pump intensities obtained while sweeping the pump la $ser$  frequency in the presence of a pump cavity feedback:  $(a)$  $P_{\text{in}}$ =70 mW and (b)  $P_{\text{in}}$ =140 mW.

The fiber core diameter is 2.75  $\mu$ m, and the cutoff wavelength for monomode propagation is 630 nm. The cavity is closed through a beam splitter, and the power reinjection rate  $(r<sub>s</sub><sup>2</sup>)$  is about 13%. A Faraday isolator is inserted in the cavity to avoid the pump feedback. A second beam splitter is inserted to extract the two counterpropagative beams out of the cavity for detection. With this setup, a pump frequency sweeping is easier to perform than a cavity length sweeping, and is then used to drive the Stokes detuning. The recordings of the pump and Stokes signals, obtained while sweeping the input pump frequency over two FSR's are presented in Figs. 6 and 7 (with and without the intracavity isolator, respectively). The sweeping rate is sufficiently slow  $(<100$  Hz) to avoid dynamical effects on the location of the bifurcation points. The accordance with the theoretical results of Figs. 3 and 4 is evident. In this experiment, the exact positions of the passive cavity resonances were unknown, because of the drift of the cavity length. The origin of the frequency scale on each recording was suggested by the field variations, and does not correspond to any measurement. The record of Fig.  $6(a)$ , obtained for an input power just above the lasing threshold, shows a domain of instability around the Stokes antiresonance frequency. These instabilities are periodic oscillations at a frequency close to the FSR, and result from the beating of a multimode oscillation. Note that the weak modulation of the intensities is only due to the step-index reflections at both ends of the fiber, which add a Perot-Fabry cavity effect. At a higher input power [Fig.  $6(b)$ ], the discontinuous jump of the Stokes intensity clearly reveals the presence of a bistability between two modes close to the antiresonances. Of course the laser cannot switch abruptly from one mode to the next, and a small transient regime can still be observed at each mode hop. In the presence of a pump  $f$ eedback  $(Fig. 7)$ , the strong variations of the field intensities with the detuning do not change the global dynamics of the

laser. In particular, discontinuous jumps are observed which point out the existence of a bistability between two adjacent modes, in good agreement with theoretical results.

### **VI. CONCLUSION**

In this paper, we have underlined the importance of the detuning on the dynamics of a Brillouin laser. As a whole, the features of this kind of laser are not different from those of classical lasers with population inversion  $[12]$ . With an appropriate formulation of the Brillouin fiber ring laser model, we have derived an expression of the steady state in the presence of a pump and Stokes frequency mismatch. We have observed and interpreted a bistability of the system, which can emit on the two cavity modes lying closest to the maximum of the spontaneous gain. The frequency domain over which the laser emission on one mode is stable increases with the emission strength. If care is taken to avoid the second-order Stokes emission  $[14,15]$ , it would then be possible to increase the cavity quality and the pumping level so much that the mode hops do not happen on each FSR, but on every two FSR's or more. The intensity of such a laser would then be much more stable with respect to the cavity length fluctuations. Moreover, cavity length control, could be used to obtain a tunable laser over at least one FSR, with a very high coherence of the emission which characterizes the Brillouin laser.

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