Lasing without inversion in a V-type system: Transient and steady-state analysis

Yifu Zhu
Department of Physics, Florida International University, Miami, Florida 33199
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We analyze transient properties of light amplification without population inversion (LWI) in a closed three-level V-type system and study the system evolution from transient LWI into steady-state LWI. From the time evolution of the atomic coherence and population distribution with and without an incoherent pump field, we elucidate the light amplification mechanism and derive the conditions under which the V system exhibits LWI in any state basis. We derive analytical solution for the steady-state LWI and discuss other mechanisms of light amplification such as stimulated Raman scattering and population inversion in the dressed states.

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I. INTRODUCTION

There has been considerable interest recently in the study of light amplification and lasing without the requirement of population inversion (LWI). Many schemes for LWI have been proposed and the dependence of optical gain on various system parameters has been examined [1–9]. Experimental observations of inversionless gain and lasing have been reported by several groups [10–19]. Lasers based on inversionless systems may have interesting statistical properties, such as narrower intrinsic linewidths and amplitude squeezing [20–22]. From a practical point of view, the concept of lasing without population inversion may be useful in achieving laser actions in the spectral regions where lasing with population inversion is impractical with conventional pumping schemes. Among the proposed schemes of lasing without population inversion, most are based on the utilization of external coherent fields that induce atomic coherence and interference leading to optical gain in the absence of population inversion. However, a lingering question remains, which concerns whether population inversion occurs in a hidden state basis, or whether the gain is due to stimulated Raman scattering.

In a previous paper [9], we showed that LWI can be realized in a closed V-type, three-level atomic system driven by a strong-coupling laser on one transition while probed by a weak laser on another transition. We analyzed the steady-state spectral characteristics of the probe gain, derived the inversionless conditions, and discussed population distributions in the bare atomic states and the dressed states. Studies of LWI in similar V-type systems have also been carried out by several other groups [23,24]. In particular, Wilson *et al.* analyzed LWI in a V-type system with a coherent pump or an incoherent pump, and discussed the differences and similarities in the gain characteristics and population distributions for the two situations [25]. Here complementary to our earlier work [9], we present an analysis of the time evolution and the steady-state behavior of LWI in the closed V-type atomic system. We consider the situation in which there is an applied incoherent pump field as well as the situation in which there is no incoherent pump field. The steady-state responses of the system under the two situations are qualitatively different: with an incoherent pump field, the V system may exhibit steady-state LWI; without the incoherent field, the V system does not exhibit steady-state LWI. This is expected from the requirement of energy conservation. However, transient LWI can exist in the V system with or without the incoherent pump. From these analyses, we identify the physical mechanisms to which the gain may be attributed. We show that in the steady state, when the frequency of the coupling laser is near the atomic resonance frequency, there is no population inversion in any meaningful state basis, and LWI in the V system is not due to stimulated Raman gain. However, when the coupling laser is detuned sufficiently away from the atomic resonance, stimulated Raman gain and light amplification with inversion in the dressed states occur.

II. TRANSIENT ANALYSIS

We consider a closed V-type, three-level system with the ground state $|1\rangle$ and excited states $|2\rangle$ and $|3\rangle$ as illustrated in Fig. 1. The transition $|1\rangle\leftrightarrow\rangle|2\rangle$ of frequency ω_{21} is driven by a strong-coupling laser of frequency ω_1 with Rabi frequency 2Ω. The transition $|1\rangle\leftrightarrow|3\rangle$ of frequency ω_{31} is pumped with a rate Λ by an incoherent field (broadband excitation). γ_{31} (γ_{21}) is the spontaneous decay rate from state $|3\rangle$ ($|2\rangle$) to state $|1\rangle$. There is no direct coupling between states $|2\rangle$ and $|3\rangle$. A weak probe laser of frequency ω_p with Rabi frequency $2g$ is applied to the transition $|1\rangle\leftrightarrow|3\rangle$. Without loss of generality, Ω and g are chosen to be real. The semiclassical density matrix equations of motion under the electric-dipole and the rotating-wave approximations can be written as

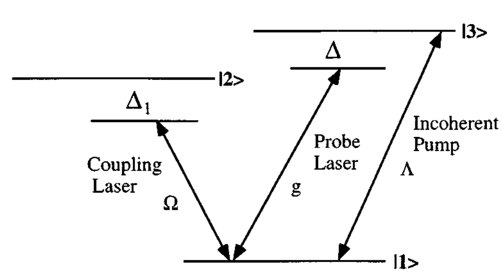

FIG. 1. V-type three-state system for lasing without population inversion.

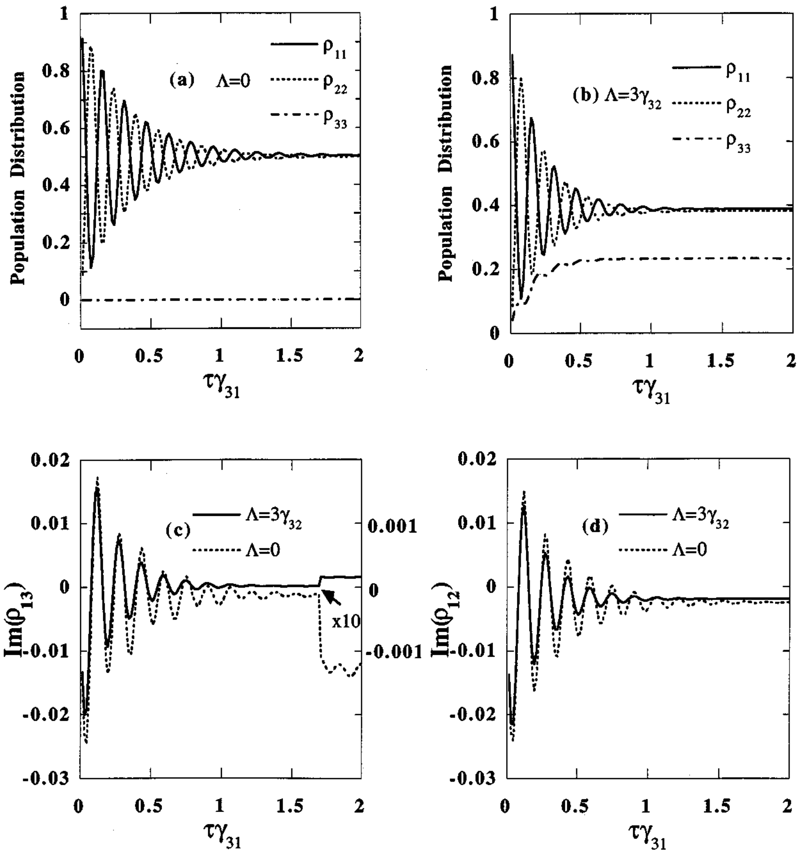

FIG. 2. Calculated time evolution of the atomic responses in the three-state V system. (a) Without the incoherent pump field ($\Lambda=0$), the population distribution ρ_{ii} $(i=1-3)$ versus the normalized time $\tau\gamma_{31}$. (b) With the incoherent pump field ($\Lambda=3\gamma_{31}$), the population distribution ρ_{ii} $(i=1-3)$ versus the normalized time $\tau\gamma_{31}$. (c) Im(ρ_{13}) (proportional to the probe gain or absorption coefficient) versus the normalized time $\tau\gamma_{31}$. In order to show the steady-state behavior clearly, the two curves have been multiplied 10 times at the end, as shown by the arrow. (d) Im(ρ_{12}) (proportional to the coupling-laser gain or absorption coefficient) versus the normalized time $\tau\gamma_{31}$. The chosen parameters are $\gamma_{21}=2\gamma_{31}$, $\Omega=20\gamma_{31}$, $g=0.1\gamma_{31}$, $\Delta_1=0$, and $\Delta=0$. The initial conditions are $\rho_{11}(0)=1$ and $\rho_{ij}(0)=0$ $(i,j=1$–$3)$.

$$\frac{d\rho_{11}}{dt}=-\Lambda\rho_{11}+(\Lambda+\gamma_{31})\rho_{33}+\gamma_{21}\rho_{22}+i\Omega(\rho_{21}-\rho_{12})+ig(\rho_{31}-\rho_{13}),$$

$$\frac{d\rho_{22}}{dt}=-\gamma_{21}\rho_{22}+i\Omega(\rho_{12}-\rho_{21}),$$

$$\frac{d\rho_{33}}{dt}=\Lambda\rho_{11}-(\Lambda+\gamma_{31})\rho_{33}+ig(\rho_{13}-\rho_{31}), \tag{1}$$

$$\frac{d\rho_{12}}{dt}=\left(-\frac{\Lambda+\gamma_{21}}{2}-i\Delta_1\right)\rho_{12}+i\Omega(\rho_{22}-\rho_{11})+ig\rho_{32},$$

$$\frac{d\rho_{13}}{dt}=\left(-\frac{2\Lambda+\gamma_{31}}{2}-i\Delta\right)\rho_{13}+ig(\rho_{33}-\rho_{11})+i\Omega\rho_{23},$$

$$\frac{d\rho_{23}}{dt}=\left(-\frac{\Lambda+\gamma_{31}+\gamma_{21}}{2}-i(\Delta-\Delta_1)\right)\rho_{23}+i\Omega\rho_{13}-ig\rho_{21},$$

where $\Delta_1=\omega_{21}-\omega_1$ and $\Delta=\omega_{31}-\omega_p$ are the coupling laser and probe laser detunings, respectively. The closure of the system requires $\rho_{11}+\rho_{22}+\rho_{33}=1$. The gain or absorption coefficient for the probe laser (the coupling laser) coupled to the transition $|3\rangle\leftrightarrow|1\rangle$ $(|2\rangle\leftrightarrow|1\rangle)$ is proportional to Im(ρ_{13}) [Im(ρ_{12})]. If Im(ρ_{13})>0, the probe laser will be amplified. Similarly if Im(ρ_{12})>0, the coupling laser will be amplified. Taking $\Delta_1=\Delta=0$, we begin by examining time-dependent numerical solutions of Eq. (1) with and without the incoherent pump. The parameters for the numerical solutions are chosen such that the conditions for LWI in the steady state are satisfied [9]. Explicitly, the normalized parameters are $\Omega=20\gamma_{31}$, $\gamma_{21}=2\gamma_{31}$, $g=0.1\gamma_{31}$, and $\Lambda=3\gamma_{31}$ or 0.

With a resonant coupling laser and a resonant probe laser ($\Delta_1=\Delta=0$), we found that $\rho_{13}(t)=i\,\mathrm{Im}[\rho_{13}(t)]$, $\rho_{12}(t)=i\mathrm{Im}[\rho_{12}(t)]$, and $\rho_{23}(t)=\mathrm{Re}[\rho_{23}(t)]$. The dispersive response for the probe laser and the coupling laser vanishes, and the two-photon coherence ρ_{23} is real. The time evolution of atomic responses is plotted in Fig. 2. Figure 2(a) shows the time evolution of the population distribution in the V system without the incoherent pump ($\Lambda=0$). As expected, the atomic population oscillates back and forth between states $|1\rangle$ and $|2\rangle$ and reaches the steady-state values $\rho_{22}\approx\rho_{11}\approx0.5$. Since $g\ll\Omega$ and γ_{ij} $(i,j=1-3)$, the probability for the atoms being excited to state $|3\rangle$ is very small, and $\rho_{33}\approx0$. The atomic population evolution with $\Lambda=3\gamma_{31}$ is plotted in Fig. 2(b). It is seen that the ρ_{11} and ρ_{22} oscillation is similar to that in Fig. 2(a), but now ρ_{33} increases almost monotonically to its steady-state value. Note that the additional damping due to Λ causes a faster decay of the atomic response, and the atomic system reaches the steady state faster with $\Lambda=3\gamma_{31}$ than with $\Lambda=0$. The time evolution of Im(ρ_{13}) is plotted in Fig. 2(c). It shows similar oscillatory behavior versus time, i.e., the probe laser exhibits periodic amplification and absorption. The time evolution of Im (ρ_{12}) is plotted in Fig. 2(d). With and without the incoherent pump, the transient behavior of Im(ρ_{13}) and Im(ρ_{12}) is qualitatively the same. Comparing Figs. 2(c) and 2(d), it is

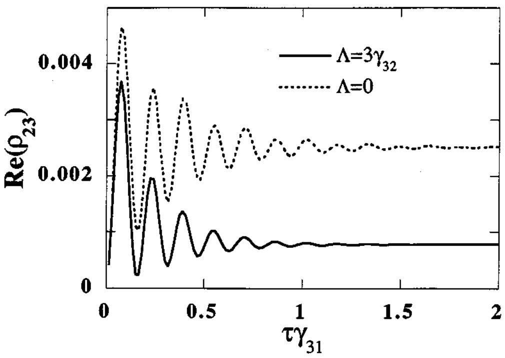

FIG. 3. Two-photon atomic coherence $\mathrm{Re}(\rho_{23})$ versus the normalized time $\tau\gamma_{31}$. The parameters are the same as those in Fig. 2. Note that with the incoherent pump field (the curve with $\Lambda=3\gamma_{31}$), the two-photon coherence is reduced in comparison with $\mathrm{Re}(\rho_{23})$ at $\Lambda=0$. The initial conditions are the same as that in Fig. 2.

seen that $\mathrm{Im}(\rho_{13})$ and $\mathrm{Im}(\rho_{12})$ are oscillating in phase with each other. The two lasers experience gain or absorption at the same time. There is a $\pi/2$ phase difference between Im (ρ_{13}) $[\mathrm{Im}(\rho_{12})]$ and ρ_{23}. The transient amplification of the probe laser and the coupling laser occurs after ρ_{22} reaches the maximum values. The amplification of the probe laser and the coupling laser occurs in the time interval in which $d\rho_{22}/dt<0$ and the peak amplification coincides with the time at which the change in the slope of ρ_{22} is steepest. The transient gain or absorption of the coupling laser is similar to the Rabi oscillation of a strongly driven two-state system. However, the origin of the probe laser amplification is quite different. Note from Figs. 2(a) and 2(b), one always has $\rho_{33}<\rho_{11}$ and $\rho_{33}<\rho_{22}$. As will be shown later, this is the necessary and sufficient condition of population noninversion in either the bare atomic states or the dressed states, and the transient probe amplification is induced by the oscillatory atomic coherence ρ_{23}. One may wonder if this transient light amplification is from the stimulated Raman scattering between states $|2\rangle$ and $|3\rangle$. Examination of Figs. 2(c) and 2(d) rules out such a possibility. If the probe laser is amplified by stimulated Raman scattering $|3\rangle\rightarrow|1\rangle\rightarrow|2\rangle$, the coupling laser has to be attenuated in the Raman scattering process ($|3\rangle\rightarrow|1\rangle$ corresponds to the emission of a probe-laser photon; $|1\rangle\rightarrow|2\rangle$ corresponds to the absorption of a coupling laser photon), and vice versa: i.e., $\mathrm{Im}(\rho_{13})$ and $\mathrm{Im}(\rho_{23})$ have to have a π phase difference between them. This is in contradiction with the in-phase solutions presented in Figs. 2(c) and 2(d). Furthermore, for the Raman gain to occur for the probe laser, a Raman inversion condition, $\rho_{33}>\rho_{22}$, must be satisfied. This is impossible for the resonantly coupled V system, as shown by Figs. 2(a) and 2(b): with or without the incoherent pump, ρ_{33} is always less than ρ_{22}, no Raman gain can be attributed to the transient probe amplification. The time evolution of the two-photon coherence ρ_{23} is plotted in Fig. 3. In the bare-state picture, ρ_{23} is responsible for LWI in the V system [9]. Note that $\mathrm{Re}(\rho_{23})$ is reduced with the addition of the incoherent pump field, yet the steady-state LWI occurs in the V system only with a sufficiently strong incoherent field. To understand this behavior, we write the steady-state ρ_{13} as

$$\rho_{13}=i\,\frac{g(\rho_{33}-\rho_{11})+\Omega\rho_{23}}{\Lambda+\gamma_{31}/2}. \tag{2}$$

The probe gain $[\propto\mathrm{Im}(\rho_{13})]$ is contributed to by two terms: the population difference, $\rho_{33}-\rho_{11}$ (<0) and the two-photon coherence ρ_{23} (>0). Without the incoherent pump $(\Lambda=0)$, the negative contribution from the first term is greater than the positive contribution from the ρ_{23} term, and the probe laser can only be attenuated. With a sufficiently strong incoherent field, even though the two-photon coherence ρ_{23} is reduced, the much greater increase of the population difference $\rho_{33}-\rho_{11}$ results in a positive value for Im (ρ_{13}). Therefore the steady-state LWI occurs in the V system as a combined effect of the increased population probability ρ_{33} (decreased ρ_{11}) and the residual two-photon coherence ρ_{23}.

III. STEADY-STATE ANALYSIS

It is easy to derive the analytical steady-state solutions to Eq. (1) with $\Delta_1=\Delta=0$. We found that the existence of LWI in the V system requires a strong-coupling laser, such that

$$\Omega>\left[\frac{\gamma_{31}\gamma_{21}(\Lambda+\gamma_{21})(\Lambda+\gamma_{31}+\gamma_{21})}{4(\Lambda(\gamma_{21}-\gamma_{31})-\gamma_{31}^2)}\right]^{1/2}. \tag{3}$$

Under normal conditions, $\Omega>>\gamma_{ij}$, Λ, and g is valid. Then, the steady-state solutions to Eq. (1) become very simple. The steady-state atomic polarizations ρ_{13} and ρ_{12} and the two-photon coherence ρ_{23} are

$$\rho_{13}=i\,\frac{g[\Lambda(\gamma_{21}-\gamma_{31})-\gamma_{31}^2]}{2\Omega^2(3\Lambda+2\gamma_{31})}, \tag{4}$$

$$\rho_{12}=-i\,\frac{\gamma_{21}(\Lambda+\gamma_{31})}{2\Omega(3\Lambda+2\gamma_{31})}, \tag{5}$$

and

$$\rho_{23}=\frac{g\gamma_{31}}{\Omega(3\Lambda+2\gamma_{31})}. \tag{6}$$

The steady-state population probabilities are given by

$$\rho_{11}=\rho_{22}=\frac{\Lambda+\gamma_{31}}{3\Lambda+2\gamma_{31}}, \tag{7}$$

and

$$\rho_{33}=\frac{\Lambda}{3\Lambda+2\gamma_{31}}. \tag{8}$$

These solutions are consistent with the numerical results presented in Fig. 2. If there is no incoherent pump, $\Lambda=0$, then $\mathrm{Im}(\rho_{13})<0$ and $\mathrm{Im}(\rho_{12})<0$; both the probe laser and the coupling laser are attenuated in the V system. With a sufficiently strong incoherent pump field, $[\Lambda>\gamma_{31}^2/(\gamma_{21}-\gamma_{31})$ and with $\gamma_{21}>\gamma_{31}]$, $\mathrm{Im}(\rho_{13})>0$, and the probe laser is amplified. With or without the incoherent pump field, one always has $\mathrm{Im}(\rho_{12})<0$, and the coupling laser is attenuated in the V system (with the incoherent pump field, ρ_{11} is reduced and the coupling laser is attenuated less than that without the incoherent pump field).

Next, we address the question of LWI in different state bases. The probe amplification occurs both in the transient regime and in the steady state. Since $\rho_{33}<\rho_{11}$ and $\rho_{33}<\rho_{22}$ are valid for any arbitrary time, there is no population inversion in the bare-state basis consisting of states $|1\rangle$, $|2\rangle$, and $|3\rangle$. The only other meaningful state basis is the dressed-state basis consisting of states $|3\rangle$, $|+\rangle$, and $|-\rangle$. For a resonant coupling laser, the semiclassical dressed states $|+\rangle$ and $|-\rangle$ are simply given by [26]: $|+\rangle=1/\sqrt{2}[|1\rangle+|2\rangle]$ and $|-\rangle=1/\sqrt{2}[|1\rangle-|2\rangle]$. Then the population distribution in the dressed states is given by

$$\rho_{++}=\rho_{--}=\frac{\rho_{11}+\rho_{22}}{2}=\rho_{11}=\rho_{22}. \tag{9}$$

Thus, in the dressed-state basis, one has $\rho_{33}<\rho_{++}$ and $\rho_{33}<\rho_{--}$: no population inversion in the dressed states either. Furthermore, since $\rho_{33}<\rho_{22}$, there is no Raman inversion, and the probe amplification cannot be attributed to the stimulated Raman gain $|3\rangle\rightarrow|1\rangle\rightarrow|2\rangle$. The light amplification is due to the coherence-induced interference. The coupling laser generates a pair of dressed states $|+\rangle$ and $|-\rangle$ separated by the Rabi frequency 2Ω. In the dressed-state picture, the atoms will not absorbed the probe laser at $\Delta=0$ since the probability amplitudes for transitions of $|+\rangle\rightarrow|3\rangle$ and $|-\rangle\rightarrow|3\rangle$ interfere destructively. However, for the stimulated emission $|3\rangle\rightarrow|+\rangle$ and $|3\rangle\rightarrow|-\rangle$, the final state is not a single state. The probability amplitudes add at $\Delta=0$ and result in the probe amplification [27,28]. In the bare-state picture, the two-photon coherence ρ_{23} is induced between the two excited states $|2\rangle$ and $|3\rangle$ by the coupling laser and the probe laser, which results in light amplification without population inversion [9].

From the above analysis, it is seen that the criterion of LWI in any state basis is $\rho_{11}\geqslant\rho_{22}\geqslant\rho_{33}$. It is obvious that the population distribution in the V system depends on the coupling laser detuning Δ_1. With a weak probe laser ($g\ll\Omega$, γ_{ij}, and Λ), the steady-state population probabilities with an arbitrary detuning Δ_1 in the bare states are given by

$$\rho_{22}=\frac{\Omega^2(\Lambda+\gamma_{21})(\Lambda+\gamma_{31})}{\gamma_{21}(\Lambda+\gamma_{21})^2(2\Lambda+\gamma_{31})/4+\gamma_{21}(2\Lambda+\gamma_{31})\Delta_1^2+\Omega^2(\Lambda+\gamma_{21})(3\Lambda+2\gamma_{31})}, \tag{10}$$

$$\rho_{11}=\frac{\Lambda+\gamma_{31}}{2\Lambda+\gamma_{31}}(1-\rho_{22}), \tag{11}$$

and

$$\rho_{33}=\frac{\Lambda}{2\Lambda+\gamma_{31}}(1-\rho_{22}). \tag{12}$$

Setting $\rho_{33}-\rho_{22}=0$, we obtain a critical value of $|\Delta_1|$:

$$\Delta_{c1}=\left(\frac{\Omega^2\gamma_{31}(\Lambda+\gamma_{21})}{\Lambda\gamma_{21}}-\frac{(\Lambda+\gamma_{21})^2}{4}\right)^{1/2}\approx\left(\frac{\gamma_{31}(\Lambda+\gamma_{21})}{\Lambda\gamma_{21}}\right)^{1/2}\Omega. \tag{13}$$

When $|\Delta_1|<\Delta_{c1}$, the population distribution satisfies $\rho_{11}\geqslant\rho_{22}>\rho_{33}$. The V system exhibits LWI in any state basis as discussed before. When $|\Delta_1|>\Delta_{c1}$, the population distribution satisfies $\rho_{11}>\rho_{33}>\rho_{22}$, and population inversion for the Raman transition $|3\rangle\rightarrow|1\rangle\rightarrow|2\rangle$ occurs. The probe amplification may be viewed as due to the stimulated Raman scattering $|3\rangle\rightarrow|1\rangle\rightarrow|2\rangle$ in which the atoms are incoherently pumped into state $|3\rangle$, then emit a probe-laser photon, absorb a coupling-laser photon, and end up in state $|2\rangle$. In the dressed-state picture, the corresponding population probabilities are given by

$$\rho_{++}=|a_+|^2\rho_{11}+|b_+|^2\rho_{22}, \tag{14}$$

$$\rho_{--}=|a_-|^2\rho_{11}+|b_-|^2\rho_{22}, \tag{15}$$

where

$$a_\pm=\frac{\Omega}{\left[\frac{\Delta_1^2}{2}+2\Omega^2\pm\frac{\Delta_1}{2}(\Delta_1^2+4\Omega^2)^{1/2}\right]^{1/2}},$$

$$b_\pm=\frac{\Delta_1\pm(\Delta_1^2+4\Omega^2)^{1/2}}{2\left(\frac{\Delta_1^2}{2}+2\Omega^2\pm\frac{\Delta_1}{2}(\Delta_1^2+4\Omega^2)^{1/2}\right)^{1/2}}.$$

The semiclassical dressed states $|+\rangle$ and $|-\rangle$ are given by [26]

$$|+\rangle=a_+|1\rangle+b_+|2\rangle, \tag{16}$$

$$|-\rangle=a_-|1\rangle+b_-|2\rangle. \tag{17}$$

Let $\rho_{33}-\rho_{++}=0$ (or $\rho_{33}-\rho_{--}=0$), one obtains the second critical value Δ_{c2} ($>\Delta_{c1}$) for $|\Delta_1|$. When $|\Delta_1|<\Delta_{c2}$, one obtains $\rho_{33}<\rho_{++}$ and $\rho_{33}<\rho_{--}$: there is no population inversion in the dressed states. However, when $|\Delta_1|>\Delta_{c2}$, the population distribution satisfies $\rho_{33}>\rho_{++}$ (for $\Delta_1>0$) or $\rho_{33}>\rho_{--}$ (for $\Delta_1<0$), and a population inversion in the

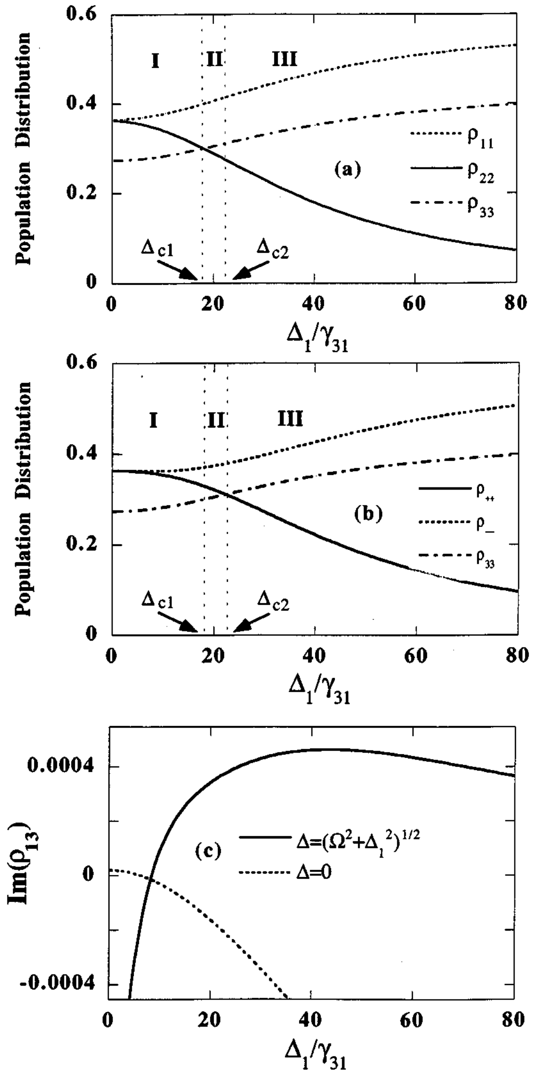

FIG. 4. (a) Steady-state atomic population distribution in the bare states versus the coupling laser detuning Δ_1/γ_{31}. (b) The steady-state atomic population distribution in the dressed states versus the coupling laser detuning Δ_1/γ_{31}. Three regions of population distributions in (a) and (b) are marked by I, II, and III. (c) The steady-state Im(ρ_{13}) at $\Delta=0$ (dashed line) and $\Delta=(\Omega^2+\Delta_1^2)^{1/2}$ (solid line) versus Δ_1/γ_{31}. The relevant parameters are $\gamma_{21}=2\gamma_{31}$, $\Omega=20\gamma_{31}$, $g=0.1\gamma_{31}$, i.e., the same as those in Fig. 2.

dressed states occurs. As shown by Eqs. (10)–(12), ρ_{11} and ρ_{33} increase with increasing $|\Delta_1|$, and ρ_{22} decreases with increasing $|\Delta_1|$. When $|\Delta_1|$ becomes greater than Δ_{1c}, the population distribution satisfies $\rho_{11}>\rho_{33}>\rho_{22}$, and Raman inversion takes place. Further increases of $|\Delta_1|$ above the second critical value Δ_{c2} will bring the population distribution to $\rho_{--}>\rho_{33}>\rho_{++}$ $(\Delta_1>0)$ or $\rho_{++}>\rho_{33}>\rho_{--}$ $(\Delta_1<0)$, i.e., besides the Raman inversion, population inversion also occurs in the dressed states. To show graphically the three regions of population distributions and the associated probe gain, we have calculated numerically the steady-state atomic response versus Δ_1 and the results are plotted in Fig. 4. The relevant parameters are the same as those chosen in Fig. 2. Figure 4(a) shows the atomic population distribution versus Δ_1 in the bare-state basis and Fig. 4(b) shows the atomic population distribution versus Δ_1 in the dressed-state basis. There are three regions of population distributions. In region I $(|\Delta_1|\leqslant\Delta_{c1})$, population distributions satisfy $\rho_{11}\geqslant\rho_{22}\geqslant\rho_{33}$ in the bare states and $\rho_{--}\geqslant\rho_{++}>\rho_{33}$ in the dressed states. LWI in any state basis occurs, and it cannot be attributed to the stimulated Raman scattering gain. In region II $(\Delta_{c1}<|\Delta_1|\leqslant\Delta_{c2})$, the population distributions satisfy $\rho_{11}>\rho_{33}>\rho_{22}$ in the bare states and $\rho_{--}>\rho_{++}\geqslant\rho_{33}$ in the dressed states. Although no population inversion exists for the probe transition in either the bare states or the dressed states, LWI may be viewed as due to the stimulated Raman scattering process $|3\rangle\rightarrow|1\rangle\rightarrow|2\rangle$ from Raman inversion $\rho_{33}>\rho_{22}$. In region III $(|\Delta_1|>\Delta_{c2})$, the population distributions become $\rho_{11}>\rho_{33}>\rho_{22}$ in the bare states and $\rho_{--}>\rho_{33}>\rho_{++}$ in the dressed states. LWI is valid only in the bare states, and the light amplification can be attributed to the population inversion in the dressed states or the stimulated Raman gain in the bare states. Depending on Δ_1, the frequencies at which the probe laser experiences amplification are shifted as shown in Fig. 4(c). When Δ_1 is much smaller than Ω, the probe gain occurs near $\Delta=0$, the resonance frequency of the bare-state transition $|3\rangle\leftrightarrow|1\rangle$. As Δ_1 increases, the gain frequency is gradually shifted from $\Delta=0$ to $\Delta=(\Omega^2+\Delta_1^2)^{1/2}$, which corresponds to one of the Aulter-Townes doublet transitions.

IV. CONCLUSION

In summary, we have presented an analysis of the time evolution of LWI in a three-state V-type system. We have shown that transient LWI can be observed in the V system with or without the incoherent pump field, and that steady-state LWI can occur only with a sufficiently strong incoherent pump field. We have identified three regions of population distributions determined by the coupling-laser detuning Δ_1, and discussed the corresponding light amplification mechanisms. Specifically, when $|\Delta_1|\leqslant\Delta_{c1}$, LWI in any state basis can be observed in the V system, which cannot be attributed to the stimulated Raman gain; when $\Delta_{c1}<|\Delta_1|\leqslant\Delta_{c2}$, the probe amplification may be attributed to the stimulated Raman gain in the bare states; when $|\Delta_1|>\Delta_{c2}$, the probe amplification may be viewed as due to population inversion in the dressed states, or equivalently, the stimulated Raman gain in the bare states. For comparison, we also carried out similar calculations in a three-level, Λ-type atomic system [1], and found that the dependence of light amplification mechanisms on the coupling laser detuning is very similar to that of the V system presented. In LWI experiments employing vapor cells as gain media, the coupling laser detuning Δ_1 will be different for atoms moving at different velocities because of the Doppler shift. The experimentally measured gain and/or lasing will inevitably involve the statistical Doppler average for a wide range of the detunings Δ_1. Depending on the Rabi frequency of the coupling laser, different gain mechanisms may simultaneously be present. Therefore, care has to be taken in interpreting experimental measurements in terms of specific physical mechanisms.

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