

## BRIEF REPORTS

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## Quantum teleportation without irreversible detection

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We show that the teleportation of an unknown quantum state can be achieved without the irreversible amplification of an intermediate detector, as required in the original scheme. This allows us to show how the quantum information is “hidden” within the correlations between the system and the environment while being wholly absent from any of the individual subsystems. This revival of correlations from the environment is quite surprising since it seems to go against the usual intuition of the environment irreversibly destroying information. By developing a description of quantum teleportation at the amplitude level we can see why the relevant information is robust to such irreversible actions of the environment.

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The “teleportation” of an unknown quantum state [1] is a surprising theoretical demonstration of the apparent nonlocal character of quantum information. Indeed, it is also likely to be demonstrated soon experimentally [2–6]. Teleportation is effected by making a joint measurement on a particle (labeled particle  $a$ ) in an unknown state and particle  $b$  which is one-half of a maximally entangled pair of particles ( $b$  and  $c$ ). If the result of the joint measurement is transmitted to the location of particle  $c$  then a suitable unitary rotation of that particle turns its state into that of the fiducial particle  $a$ . This transmission of the state from particle  $a$  to particle  $c$  in a disembodied way completes the teleportation.

In this Brief Report we go a long way to resolving the mysteries of teleportation, in particular the question of where the information is hidden and why it is so robust. We develop a reversible description of the detection apparatus involved in the teleportation scheme. We do this by expressing the teleportation scheme in terms of unitary evolution with *no* collapse of the wave packet. The usual description of teleportation involves only a classical channel and a completely entangled pair of particles without entanglement with any environment. This description cannot be unitary because unitary evolution does not allow a general state to be mapped into a product of two states, neither of which contains any information about the original state. Thus, in order to obtain a unitary description we must treat the detection apparatus as a fully quantum mechanical object. As a consequence, the usually classical signals required to complete the teleportation procedure are in our scheme orthogonal states of a quan-

tum system. We start by giving a brief review of the teleportation scheme and then proceed to include the detection apparatus explicitly.

The mathematics behind the teleportation effect is quite simple and may be summarized by the decomposition

$$|\phi\rangle_a |\Psi^{(0)}\rangle_{bc} = \sum_{J=0}^{N^2-1} |\Psi^{(J)}\rangle_{ab} \hat{U}_c^{(J)\dagger} |\phi\rangle_c / N. \quad (1)$$

Here particles  $a$ ,  $b$ , and  $c$  each correspond to  $N$ -state systems and  $\{|\Psi^{(J)}\rangle_{ab}\}$  is the complete orthonormal set of maximally entangled states given by [1]

$$|\Psi^{(J)}\rangle_{ab} \equiv \sum_{\ell=0}^{N-1} \exp(2\pi i \ell n / N) |\ell\rangle_a \times |(\ell+m) \bmod N\rangle_b / \sqrt{N}, \quad (2)$$

where on the right-hand-side the pair  $(n, m)$  are fixed by the relation  $J = nN + m$  (i.e.,  $n = J \text{div } N$  and  $m = J \bmod N$ ). Further, the unitary operator  $\hat{U}_c^{(J)}$  acts only on the subspace containing particle  $c$  and is given by [1]

$$\hat{U}_c^{(J)} \equiv \sum_{\ell=0}^{N-1} \exp(2\pi i \ell n / N) |\ell\rangle_{cc} \langle(\ell+m) \bmod N|. \quad (3)$$

Equation (1) allows the teleportation of a (possibly unknown) state  $|\phi\rangle_a$  to particle  $c$  through the following protocol: (i) Particle  $b$  from an entangled state  $|\Psi^{(0)}\rangle_{bc}$  is placed near particle  $a$ . (ii) A joint measurement having nondegenerate eigenvalues and eigenvectors given by  $|\Psi^{(J)}\rangle_{bc}$  is made on the two-particle subspace of  $a$  and  $b$ —such a measurement is called a Bell measurement [7]. (iii) The result of this measurement, labeled  $J$ , is transmitted to the location of par-

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particle  $c$ , where (iv) the unitary operator  $\hat{U}_c^{(J)}$  is performed on particle  $c$  yielding in every case the state  $|\phi\rangle_c$ .

At the end of step (ii) there are  $2\log_2 N$  bits of classical information independent of the initial state  $|\phi\rangle$  and the state of particle  $c$ , which by itself is the completely mixed state  $\sum_{c=0}^{N-1} |\phi\rangle_{cc} \langle\phi|/\sqrt{N}$ . Thus, taken individually, none of the subsystems remaining appear to contain any information about the original state  $|\phi\rangle$ . By recasting this procedure entirely at the amplitude level we will be able to see where this information lies “hidden.”

To give a description at the amplitude level we shall need to introduce an auxiliary particle, here labeled  $d$ , with which to correlate the results of the “measurement.” In other words, this particle will represent the internal degrees of freedom of the detector (and environment). The initial state of the system is given by

$$|\psi_0\rangle = |\phi\rangle_a |\Psi^{(0)}\rangle_{bc} |0\rangle_d, \quad (4)$$

where  $|J\rangle_d$  is the  $N^2$ -state system of the detector with  $0 \leq J \leq N^2 - 1$ . This initial state corresponds to step (i) in the standard protocol. Step (ii) is performed with the Hamiltonian

$$\hat{H}_1 = \frac{\pi\hbar}{2} \delta(t) \sum_{J=0}^{N^2-1} |\Psi^{(J)}\rangle_{ab} (\hat{\Sigma}_J^x - \hat{1})_{ab} \langle\Psi^{(J)}|. \quad (5)$$

In this equation

$$\hat{\Sigma}_J^x \equiv \begin{cases} \hat{1}, & J=0 \\ |0\rangle\langle J| + |J\rangle\langle 0| + \sum_{k \neq 0, J}^{N^2-1} |k\rangle\langle k|, & 0 < J < N^2, \end{cases} \quad (6)$$

where this operator acts on the detector degrees of freedom only. This unitary operator maps the fiducial state of the detector  $|0\rangle_d$  to state  $|J\rangle_d$ . The evolution operator associated with Hamiltonian  $\hat{H}_1$  in Eq. (5) is given by

$$\hat{U}_1 = \sum_{J=0}^{N^2-1} |\Psi^{(J)}\rangle_{ab} \hat{\Sigma}_J^x \langle\Psi^{(J)}|; \quad (7)$$

it is easy to check that  $\hat{U}_1^\dagger \hat{U}_1 = \hat{U}_1 \hat{U}_1^\dagger = \hat{1}$ .

The action of this evolution operator yields the intermediate state

$$\hat{U}_1 |\psi_0\rangle = \sum_{J=0}^{N^2-1} |\Psi^{(J)}\rangle_{ab} \hat{U}_c^{(J)\dagger} |\phi\rangle_c |J\rangle_d / N, \quad (8)$$

from which we can study the absence or presence of information about  $|\phi\rangle$ . The key difference between this equation and the decomposition in Eq. (1) is that the measurement results are incorporated here explicitly; otherwise these equations are identical. Unlike an ordinary measurement this corresponds to a reversible procedure.

As before, any of the particles taken separately in Eq. (8) yield completely mixed states with all outcomes being equally likely. That is, we have reproduced the “disembodied” characteristic of this form of transportation of the quantum information. Where has the information gone? It is sit-

ting in the very large part of Hilbert space which is inaccessible to single particle measurements; it is within the correlations between particle  $c$  and the environmental state  $d$ . How large is this part of Hilbert space? If  $c$  is an  $N$ -state system and the detector state corresponds to an  $N^2$ -state system we have  $D_{\text{corr}} = N^3 - N^2 - N$  degrees of freedom in the correlations between these two subsystems. For  $N=2$  it is  $D_{\text{corr}}=2$ , for  $N=3$  it is  $D_{\text{corr}}=15$  — asymptotically it grows as  $N^3$  [8].

Stage (iii) in our new protocol is identical to that of the standard procedure except now the single  $N^2$ -state system of particle  $d$  (instead of  $2\log_2 N$  classical bits) are transported to a location near particle  $c$  where a local interaction will be performed for the final stage. In stage (iv) we must perform a different unitary evolution on particle  $c$  depending on the state of particle  $d$ . Before we give the form of the interaction capable of doing this we note that any unitary operator may be written in the form  $\hat{U} = \exp(-i\hat{h})$ , where  $\hat{h}$  is an Hermitian operator [9]. Given this we may implicitly rewrite the operators  $\hat{U}^{(J)}$  of Eq. (3) as

$$\hat{U}^{(J)} \equiv \exp(-i\hat{h}^{(J)}), \quad (9)$$

for each of the  $J=0, \dots, N^2-1$ .

Using this we write a Hamiltonian which produces stage (iv):

$$\hat{H}_2 = \hbar \delta(t) \sum_J |J\rangle_d \hat{h}_c^{(J)} \langle J|. \quad (10)$$

The unitary operator corresponding to this Hamiltonian is easily calculated to be

$$\begin{aligned} \hat{U}_2 &= \exp\left(-i \sum_J |J\rangle_d \hat{h}_c^{(J)} \langle J|\right) \\ &= \hat{1} + \sum_J |J\rangle_d [\exp(-i\hat{h}_c^{(J)}) - \hat{1}]_c \langle J| \\ &= \sum_J |J\rangle_d \hat{U}_c^{(J)} \langle J|; \end{aligned} \quad (11)$$

again it is easy to check that  $\hat{U}_2^\dagger \hat{U}_2 = \hat{U}_2 \hat{U}_2^\dagger = \hat{1}$ .

After this second evolution operator the system now takes the form

$$\hat{U}_2 \hat{U}_1 |\psi_0\rangle = |\phi\rangle_c \sum_{J=0}^{N^2-1} |\Psi^{(J)}\rangle_{ab} |J\rangle_d / N, \quad (12)$$

where we have moved the state of particle  $c$  out of the sum to show explicitly that it is independent of the outcome of the measurement. Note that the outcome of our reversible measurement is still faithfully recorded in the state of particle  $d$ .

The action of the operator  $\hat{U}_1$  unitarily copies the information from the Bell state of the  $ab$  subsystem into the blank detector state. It is thus curious that this detector state is not restored to its initial blank state at the end of the procedure when it is no longer needed. Because the detector  $d$  is entangled with remote particles  $a$  and  $b$  [see Eq. (12)], there is no local reversible way of restoring it to a pure state. However, if  $d$  were brought back to the vicinity of  $a$  and  $b$ , a

local unitary operation could be used to restore all three particles to their standard states. We might ask what would happen if the detector variables decohered enroute to particle  $c$ ? The teleportation would work, but the restoration process just described would fail.

We have shown that quantum teleportation can be achieved without the irreversible amplification of an intermediate detector. In this way we have implicitly incorporated the correlations between the system and the environment which hide the relevant information in the teleportation scheme. In general, the state of the environment is inaccessible and so its action is an irreversible one on the system.

This occurs when orthogonal states in the environment are correlated to various system states. Yet this is precisely the case we have studied here. The difference is that we have artificially designated these orthogonal outcomes to correspond to the macroscopic states in a detector. The surprising lesson appears to be that there is in principle no distinction between those environments from which we can extract useful information and those from which we cannot (at least as long as their evolution is not in some sense chaotic).

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  - [8] With this excess of space for  $N > 2$  one wonders whether a smaller state may be used to describe the detector yet still yield a fully successful teleportation for the system. In fact, since  $N^2$  distinct classical messages are required, a smaller state would not be possible.
  - [9] This follows from the fact that all the eigenvalues of a unitary operator are unimodular; this means that the logarithm of a unitary operator is well defined. The Hermiticity of  $\hat{h}$  follows automatically.