Green-function approach to the radiation-field quantization for homogeneous and inhomogeneous Kramers-Kronig dielectrics

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A quantization scheme for the radiation field in dispersive and absorptive linear dielectrics is developed, which applies to both bulk material and multilayer dielectric structures. Starting from the phenomenological Maxwell equations, where the properties of the dielectric are described by a permittivity consistent with the Kramers-Kronig relations, an expansion of the field operators is performed that is based on the Green function of the classical Maxwell equations and preserves the equal-time canonical field commutation relations. In particular, in frequency intervals with approximately vanishing absorption the concept of quantization through mode expansion for dispersive dielectrics is recognized. The theory further reveals that weak absorption gives rise to space-dependent mode operators that spatially evolve according to quantum Langevin equations in the space domain. To illustrate the applicability of the theory to inhomogeneous structures, the quantization of the radiation field in a dispersive and absorptive one-interface dielectric is performed.

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I. INTRODUCTION

To describe the quantum features of the propagation of light through dielectric matter, such as optical fibers or dielectric multislab devices, quantization of the phenomenological Maxwell equations that includes both dispersion and absorption and applies to both homogeneous and inhomogeneous matter is desired. The problem of quantum electrodynamics in the presence of linear dielectrics has widely been studied and various concepts have been developed to allow for matter with real permittivity. In this context, both dispersionless matter [1–10] and dispersive dielectrics [11–14] have been considered and extensions to nonlinear dielectrics [9,13,15–23] have been given.

A central problem related to all these approaches is the inclusion of absorption into the propagation of light in dielectric matter. It is well known that the permittivity of a linear dielectric is a complex function of frequency, where the real and imaginary parts describing the effects of dispersion and absorption, respectively, are related by the Kramers-Kronig relations. Since dispersion is always associated with absorption, the losses must necessarily be included into a rigourous quantization scheme. In particular, the quantization should be performed in such a way that it is consistent with the Kramers-Kronig relations and the familiar (equal-time) canonical commutation relations for the overall electric and magnetic fields.

The problem has been considered by a number of authors [24–27]. To our knowledge, a systematic and quantum-theoretically consistent description of quantized radiation in dispersive and absorptive linear bulk dielectrics was first developed by Huttner and Barnett [25]. Using the microscopic Hopfield model of a dielectric [28] and representing the matter by a collection of interacting harmonic-oscillator fields (a polarization field and a continuum of reservoir fields), they diagonalized the coupled radiation-matter Hamiltonian. Since the influence of the medium can be entirely described in terms of the complex frequency-dependent permittivity, the Huttner-Barnett model can be extended to dielectrics other than microscopic harmonic-oscillator media in order to obtain a general quantization scheme for radiation in dispersive and absorptive linear dielectrics of (phenomenologically) given permittivities [29].

To generalize this quantization scheme to radiation in multilayer dielectric structures, in the present paper we analyze it in terms of the Green function of the classical, phenomenological Maxwell equations that describe the propagation of radiation in dispersive and absorptive linear dielectrics in the absence of external sources. Allowing for a frequency-dependent complex permittivity that is consistent with the Kramers-Kronig relations and introducing a (random) operator noise source associated with the absorption of radiation, we regard these equations as quantum-theoretical operator equations. Their solution through a Green-function expansion of the operator of the vector potential may be regarded as the natural generalization of the familiar mode expansion applicable to source-free radiation in (approximately) lossless dielectrics. If in a chosen frequency interval the absorption may be disregarded, in this frequency interval the Green function expansion reduces to a mode expansion with real frequency-dependent permittivity. Clearly, when the absorption vanishes for all frequencies the permittivity must be unity anywhere, which just corresponds to radiation in free space. For weak absorption the theory yields (in the chosen frequency interval) space-dependent mode operators whose spatial evolution is governed by quantum Langevin equations.

The Green-function approach is not restricted to a particular model of the medium, but applies to any linear Kramers-Kronig dielectric. Further, it applies not only to bulk material but also to multilayer dielectrics, such as multislab devices, their spatial structures being included in the Green function of the classical problem. The latter can advantageously be used in the study of the action of optical instruments, such as macroscopic dielectric bodies that respond linearly to the radiation field under study. In this context, the correct quantum optical input-output relations that include the noise contributions associated with the absorption of radiation can be derived in a very systematic way.

The paper is organized as follows. In Sec. II the concept of quantization for homogeneous dielectrics is outlined and the main features of the theory are illustrated by considering linearly polarized radiation propagating in one dimension. In particular the connection between the Green-function approach and the concept of mode expansion is discussed. In Sec. III the theory is extended to multilayer dielectric structures, and the problem of a one-interface dielectric is considered in more detail. A summary and conclusions are given in Sec. IV. Somewhat lengthy calculations are outlined in the Appendixes.

II. HOMOGENEOUS DIELECTRICS

A. Classical description

In the phenomenological classical Maxwell theory ([30,31]) the propagation of radiation in a dispersive and lossy linear (homogeneous and isotropic) dielectric that is free of external sources is frequently described by the equations

$$\mathrm{curl}\mathbf{E} = -\dot{\mathbf{B}}, \quad \mathrm{div}\mathbf{B} = 0, \tag{1}$$

$$\mathrm{curl}\mathbf{H} = \dot{\mathbf{D}}, \quad \mathrm{div}\mathbf{D} = 0, \tag{2}$$

where $\mathbf{B} = \mu_0 \mathbf{H}$ and the displacement field $\mathbf{D}(\mathbf{r},t)$ is related to the electric field $\mathbf{E}(\mathbf{r},t)$ as

$$\mathbf{D}(\mathbf{r},t) = \epsilon_0 \left[\mathbf{E}(\mathbf{r},t) + \int_0^\infty d\tau\, \chi(\tau) \mathbf{E}(\mathbf{r},t-\tau) \right]. \tag{3}$$

Expressing the electric and magnetic fields in terms of the vector potential,

$$\mathbf{E} = -\dot{\mathbf{A}}, \quad \mathbf{B} = \mathrm{curl}\mathbf{A}, \tag{4}$$

the Maxwell equations (1) are automatically satisfied and Eqs. (2) yield

$$\Delta \mathbf{A}(\mathbf{r},t) - \frac{1}{c^2} \left[\ddot{\mathbf{A}}(\mathbf{r},t) + \int_0^\infty d\tau \chi(\tau) \ddot{\mathbf{A}}(\mathbf{r},t-\tau) \right] = \mathbf{0}, \tag{5}$$

where $\mathrm{div}\mathbf{A} = 0$. We now introduce the Fourier transforms,

$$\mathbf{A}(\mathbf{r},t) = \int_0^\infty d\omega\; e^{-i\omega t} \underline{\mathbf{A}}(\mathbf{r},\omega) + \mathrm{c.c.}, \tag{6}$$

$$\mathbf{E}(\mathbf{r},t) = \int_0^\infty d\omega\; e^{-i\omega t} \underline{\mathbf{E}}(\mathbf{r},\omega) + \mathrm{c.c.}, \tag{7}$$

$$\underline{\mathbf{E}}(\mathbf{r},\omega) = i\omega \underline{\mathbf{A}}(\mathbf{r},\omega), \tag{8}$$

$$\mathbf{B}(\mathbf{r},t) = \int_0^\infty d\omega\; e^{-i\omega t} \underline{\mathbf{B}}(\mathbf{r},\omega) + \mathrm{c.c.}, \tag{9}$$

$$\underline{\mathbf{B}}(\mathbf{r},\omega) = \mathrm{curl}\underline{\mathbf{A}}(\mathbf{r},\omega), \tag{10}$$

$$\mathbf{D}(\mathbf{r},t) = \int_0^\infty d\omega\; e^{-i\omega t} \underline{\mathbf{D}}(\mathbf{r},\omega) + \mathrm{c.c.}, \tag{11}$$

$$\underline{\mathbf{D}}(\mathbf{r},\omega) = \epsilon_0 \epsilon(\omega) \underline{\mathbf{E}}(\mathbf{r},\omega), \tag{12}$$

where

$$\epsilon(\omega) = 1 + \int_0^\infty d\tau\; e^{i\omega\tau} \chi(\tau) \tag{13}$$

is the frequency-dependent complex permittivity introduced phenomenologically. The wave equation (5) is satisfied when

$$\Delta \underline{\mathbf{A}}(\mathbf{r},\omega) + \frac{\omega^2}{c^2} \epsilon(\omega) \underline{\mathbf{A}}(\mathbf{r},\omega) = \mathbf{0}. \tag{14}$$

Regarding ϵ as a function of the complex frequency Ω, $\epsilon(\Omega)$ satisfies the relation

$$\epsilon(-\Omega^*) = \epsilon^*(\Omega) \tag{15}$$

and is analytical in the upper complex half plane without zeros. Further, the real and imaginary parts $\epsilon_{\mathrm{R}}(\omega) = \mathrm{Re}\{\epsilon(\omega)\}$ and $\epsilon_{\mathrm{I}}(\omega) = \mathrm{Im}\{\epsilon(\omega)\}$, respectively, satisfy the well-known Kramers-Kronig relations

$$\epsilon_{\mathrm{R}}(\omega) - 1 = \frac{\mathrm{P}}{\pi} \int_{-\infty}^{\infty} d\omega' \, \frac{\epsilon_{\mathrm{I}}(\omega')}{\omega' - \omega}, \tag{16}$$

$$\epsilon_{\mathrm{I}}(\omega) = -\frac{\mathrm{P}}{\pi} \int_{-\infty}^{\infty} d\omega' \, \frac{\epsilon_{\mathrm{R}}(\omega') - 1}{\omega' - \omega}, \tag{17}$$

where P is the principal value.

B. Quantum description

Since $\epsilon(\omega)$ is necessarily complex, Eq. (14) [or Eq. (5)] cannot be valid as an operator equation for $\hat{\underline{\mathbf{A}}}(\mathbf{r},\omega)$[or $\hat{\mathbf{A}}(\mathbf{r},t)$] in the sense of quantum theory. Otherwise, $\hat{\underline{\mathbf{A}}}(\mathbf{r},\omega)$ would be spatially damped and the equal-time field commutators $[\hat{A}_i(\mathbf{r},t), \hat{E}_j(\mathbf{r}',t)]$ would not be preserved for any space points $\mathbf{r}$ and $\mathbf{r}'$. From the quantum theory of damped systems it is well known that reinterpreting classical evolution equations for damped systems as quantum-theoretical operator equations necessarily requires the introduction into the equations of additional operator noise sources [32]. With regard to radiation in dielectric matter, we may therefore include in the theory the effect of quantum noise associated with propagation-assisted damping as follows. We regard the radiation-field quantities as operators and supplement the Maxwell equations as given in Sec. II A with an operator noise current $\hat{\mathbf{j}}_{\mathrm{n}}$. Introducing the Fourier transform of the (Schrödinger) operator of the vector potential,

$$\hat{\mathbf{A}}(\mathbf{r}) = \int_0^\infty d\omega\; \hat{\underline{\mathbf{A}}}(\mathbf{r},\omega) + \mathrm{H.c.} \tag{18}$$

[cf. Eq. (6)], the conditional equation for the operator $\hat{\underline{\mathbf{A}}}(\mathbf{r},\omega)$ now reads as [in place of Eq. (14)]

$$\Delta\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega)+\frac{\omega^2}{c^2}\epsilon(\omega)\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega)=\underline{\hat{\mathbf{j}}}_{\mathrm{n}}(\mathbf{r},\omega). \quad (19)$$

Using the method of Green function, the solution of Eq. (19) can be written as

$$\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega)=\int d^3\mathbf{r}'\,G(\mathbf{r},\mathbf{r}',\omega)\underline{\hat{\mathbf{j}}}_{\mathrm{n}}(\mathbf{r}',\omega), \quad (20)$$

where the Green function $G(\mathbf{r},\mathbf{r}',\omega)$ satisfies the equation

$$\Delta G(\mathbf{r},\mathbf{r}',\omega)+\frac{\omega^2}{c^2}\epsilon(\omega)G(\mathbf{r},\mathbf{r}',\omega)=\delta(\mathbf{r}-\mathbf{r}') \quad (21)$$

and the boundary condition that it vanishes at infinity, which [for $\epsilon_{\mathrm{I}}(\omega)>0$] corresponds to the well-known Sommerfeld condition [33] that is usually applied when the permittivity is real. Clearly, when the presence of (inhomogeneous) dielectric bodies gives rise to surfaces of discontinuity, then $G(\mathbf{r},\mathbf{r}',\omega)$ is also required to be continuously differentiable at these surfaces, so that the tangential components of $\hat{\mathbf{E}}$ and the normal components of $\hat{\mathbf{D}}$ are continuous (cf. Sec. III). Note that when $\epsilon_{\mathrm{I}}(\omega)>0$, adding to Eq. (20) a nontrivial solution of the homogeneous wave equation [Eq. (19), with $\underline{\hat{\mathbf{j}}}_{\mathrm{n}}(\mathbf{r},\omega)=\mathbf{0}$] would violate the boundary condition at infinity. In particular, for homogeneous dielectrics the Green function is given by

$$G(\mathbf{r},\mathbf{r}',\omega)=-\frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|}\exp\left[i\frac{\omega}{c}\sqrt{\epsilon(\omega)}|\mathbf{r}-\mathbf{r}'|\right]. \quad (22)$$

Using Eqs. (18) and (20), the (Schrödinger) operators of the vector potential and the electric-field strength can be represented as

$$\hat{\mathbf{A}}(\mathbf{r})=\int_0^\infty d\omega\int d^3\mathbf{r}'\,G(\mathbf{r},\mathbf{r}',\omega)\underline{\hat{\mathbf{j}}}_{\mathrm{n}}(\mathbf{r}',\omega)+\text{ H.c.}, \quad (23)$$

$$\hat{\mathbf{E}}(\mathbf{r})=-\hat{\dot{\mathbf{A}}}(\mathbf{r})$$

$$=i\int_0^\infty d\omega\,\omega\int d^3\mathbf{r}'\,G(\mathbf{r},\mathbf{r}',\omega)\underline{\hat{\mathbf{j}}}_{\mathrm{n}}(\mathbf{r}',\omega)+\text{ H.c.}. \quad (24)$$

As already mentioned, the source term $\underline{\hat{\mathbf{j}}}_{\mathrm{n}}(\mathbf{r},\omega)$ is required in order to take into account the dissipation-assisted quantum noise. It is therefore clear that when the imaginary part of the permittivity becomes (in certain frequency domains) negligibly small [$\epsilon_{\mathrm{I}}(\omega)\to 0$], then (in these domains) $\underline{\hat{\mathbf{j}}}_{\mathrm{n}}(\mathbf{r},\omega)$ may be disregarded as well [$\underline{\hat{\mathbf{j}}}_{\mathrm{n}}(\mathbf{r},\omega)\to\mathbf{0}$]. Mathematically, the introduction of $\underline{\hat{\mathbf{j}}}_{\mathrm{n}}(\mathbf{r},\omega)$ must ensure the preservation of the well-known canonical field commutation relations

$$[\hat{A}_i(\mathbf{r}),\hat{E}_j(\mathbf{r}')]=-\frac{i\hbar}{\epsilon_0}\delta_{ij}^{\perp}(\mathbf{r}-\mathbf{r}') \quad (25)$$

in the presence of absorption [$\delta_{ij}^{\perp}(\mathbf{r}-\mathbf{r}')$, transverse δ function]. This can be achieved by choosing [29]

$$\underline{\hat{\mathbf{j}}}_{\mathrm{n}}(\mathbf{r},\omega)=\frac{\omega}{c^2}\sqrt{\frac{\hbar}{\pi\epsilon_0}\epsilon_{\mathrm{I}}(\omega)}\hat{\mathbf{f}}(\mathbf{r},\omega), \quad (26)$$

$$[\hat{f}_i(\mathbf{r},\omega),\hat{f}_j^\dagger(\mathbf{r}',\omega')]=\delta_{ij}^{\perp}(\mathbf{r}-\mathbf{r}')\,\delta(\omega-\omega'), \quad (27)$$

$$[\hat{f}_i(\mathbf{r},\omega),\hat{f}_j(\mathbf{r}',\omega')]=[\hat{f}_i^\dagger(\mathbf{r},\omega),\hat{f}_j^\dagger(\mathbf{r}',\omega')]=0, \quad (28)$$

which is in agreement with the microscopic Huttner-Barnett model [25]. In the Heisenberg picture the operator basic field obviously evolves as

$$\hat{\mathbf{f}}(\mathbf{r},\omega,t)=\hat{\mathbf{f}}(\mathbf{r},\omega,t')e^{-i\omega(t-t')}, \quad (29)$$

which corresponds to the equation of motion

$$\dot{\hat{\mathbf{f}}}=-i\omega\hat{\mathbf{f}}=\frac{1}{i\hbar}[\hat{\mathbf{f}},\hat{H}], \quad (30)$$

where the Hamiltonian reads as

$$\hat{H}=\int d^3\mathbf{r}\int_0^\infty d\omega\;\hbar\omega\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)\cdot\hat{\mathbf{f}}(\mathbf{r},\omega). \quad (31)$$

It is worth noting that when (in a certain frequency interval) $\epsilon_{\mathrm{I}}(\omega)\to 0$ and hence $\underline{\hat{\mathbf{j}}}_{\mathrm{n}}(\mathbf{r},\omega)\to 0$, the vector potential $\hat{\mathbf{A}}(\mathbf{r})$ in the form (23) does not vanish. A careful inspection of the $\mathbf{r}'$ integral in Eq. (23) reveals that, on using Eq. (22) and Eqs. (26) – (28), $\hat{\mathbf{A}}(\mathbf{r})$ tends to the familiar representation of the source-free field through mode decomposition [29], cf. Eq. (50) for radiation propagating in the x direction. In particular, in the absence of dielectric matter [$\epsilon(\omega)\to 1\ \forall\ \omega$] $\hat{\mathbf{A}}(\mathbf{r})$ in Eq. (23) reduces to the source-free field in free space. Hence, the Green-function expansion (23) [together with Eq. (22) and Eqs. (26) – (28)] may be regarded as a natural extension of the familiar mode expansion of radiation in free space to damped radiation in a dispersive and absorptive linear dielectric. The effect of additional sources embedded in the dielectric matter (and resonantly interacting with the radiation) may then be described by an overall Hamiltonian that consists of the radiation-dielectric Hamiltonian (31), a source Hamiltonian, and a standard radiation-matter interaction energy, such as $\propto\hat{\mathbf{p}}\cdot\hat{\mathbf{A}}$ ($\hat{\mathbf{p}}$, momentum of a charged particle). Here, the vector potential $\hat{\mathbf{A}}$ can be expressed, in principle, in terms of the basic-field variables $\hat{\mathbf{f}}$ and $\hat{\mathbf{f}}^\dagger$, on using Eqs. (23) and (26), and (nonlinearly coupled) Heisenberg equations of motion for the source variables and the field variables $\hat{\mathbf{f}}$ and $\hat{\mathbf{f}}^\dagger$ can be derived. Clearly, the current density $\hat{\mathbf{j}}_{\mathrm{s}}$ associated with such sources is, in general, quite different from $\underline{\hat{\mathbf{j}}}_{\mathrm{n}}$. It should be noted that the canonical formalism (of Heisenberg equations of motion) implies preservation of equal-time commutation relations.

For the sake of clarity, let us illustrate the main features of the concept for linearly polarized radiation propagating in the x direction [$\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega)\to\underline{\hat{A}}_y(x,\omega)\equiv\underline{\hat{A}}(x,\omega)$, $\hat{\mathbf{f}}(\mathbf{r},\omega)\to\hat{f}_y(x,\omega)\equiv\hat{f}(x,\omega)$], which effectively reduces the system to one spatial dimension. In this case the inhomogeneous wave equation (19) [together with Eq. (26)] takes the form

$$\frac{\partial^2}{\partial x^2}\underline{\hat{A}}(x,\omega)+\frac{\omega^2}{c^2}\epsilon(\omega)\underline{\hat{A}}(x,\omega)=\frac{\omega}{c^2}\sqrt{\frac{\hbar}{\pi\epsilon_0\mathcal{A}}\epsilon_{\mathrm{I}}(\omega)}\hat{f}(x,\omega), \quad (32)$$

where $\mathscr{A}$ is the normalization area perpendicular to the x direction, and the basic-field commutation relations (27) and (28) read as

$$[\hat{f}(x,\omega),\hat{f}^{\dagger}(x',\omega')]=\delta(x-x')\,\delta(\omega-\omega'), \tag{33}$$

$$[\hat{f}(x,\omega),\hat{f}(x',\omega')]=[\hat{f}^{\dagger}(x,\omega),\hat{f}^{\dagger}(x',\omega')]=0. \tag{34}$$

The Green-function expansion of $\hat{A}(x,\omega)$ is given by

$$\hat{A}(x,\omega)=\frac{\omega}{c^2}\sqrt{\frac{\hbar}{\pi\epsilon_0\mathscr{A}}\epsilon_{\rm I}(\omega)}\int_{-\infty}^{\infty}dx'\,G(x,x',\omega)\hat{f}(x',\omega), \tag{35}$$

where $G(x,x',\omega)$ satisfies the equation

$$\frac{\partial^2}{\partial x^2}G(x,x',\omega)+\frac{\omega^2}{c^2}\epsilon(\omega)G(x,x',\omega)=\delta(x-x'). \tag{36}$$

It can easily be proved that the solution of Eq. (36) that satisfies the correct boundary conditions at $\pm\infty$ is

$$G(x,x',\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dk\; e^{ik(x-x')}\frac{c^2}{\omega^2\epsilon(\omega)-c^2k^2}$$
$$=\left[2i\frac{\omega}{c}n(\omega)\right]^{-1}\exp\left[i\frac{\omega}{c}n(\omega)|x-x'|\right], \tag{37}$$

where

$$n(\omega)=\sqrt{\epsilon(\omega)} \tag{38}$$

is the complex refractive index [note that $\epsilon_{\rm I}(\Omega)>0$ for $\mathrm{Re}\{\Omega\}>0$].

To show that the (Schrödinger) operators of the vector potential

$$\hat{A}(x)=\int_0^{\infty}d\omega\;\frac{\omega}{c^2}\sqrt{\frac{\hbar}{\pi\epsilon_0\mathscr{A}}\epsilon_{\rm I}(\omega)}$$
$$\times\int_{-\infty}^{\infty}dx'\,G(x,x',\omega)\hat{f}(x',\omega)+\mathrm{H.c.} \tag{39}$$

and the electric-field strength

$$\hat{E}(x)=i\int_0^{\infty}d\omega\;\frac{\omega^2}{c^2}\sqrt{\frac{\hbar}{\pi\epsilon_0\mathscr{A}}\epsilon_{\rm I}(\omega)}$$
$$\times\int_{-\infty}^{\infty}dx'\,G(x,x',\omega)\hat{f}(x',\omega)+\mathrm{H.c.} \tag{40}$$

satisfy the canonical commutation relation

$$[\hat{A}(x),\hat{E}(x')]=-\frac{i\hbar}{\mathscr{A}\epsilon_0}\delta(x-x'), \tag{41}$$

we note that the integral relation

$$\frac{\omega^2}{c^2}\epsilon_{\rm I}(\omega)\int_{-\infty}^{\infty}dy\,G(x,y,\omega)G^*(x',y,\omega)$$
$$=\frac{1}{2}i[G(x,x',\omega)-G^*(x',x,\omega)] \tag{42}$$

is valid (see Appendix B). Combining Eqs. (39) and (40), recalling the commutation relations (33) and (34), and using the relation (42), straightforward calculation yields

$$[\hat{A}(x),\hat{E}(x')]=\frac{\hbar}{\pi\epsilon_0c^2\mathscr{A}}\int_{-\infty}^{\infty}d\omega\,\omega G(x,x',\omega) \tag{43}$$

[note that $G^*(x,x',\omega)=G(x,x',-\omega)$ and $G(x,x',\omega)=G(x',x,\omega)$]. Substituting in Eq. (43) for the Green functions the explicit expressions as given in Eq. (37), we find that

$$[\hat{A}(x),\hat{E}(x')]=\frac{\hbar}{2\pi i\epsilon_0c\mathscr{A}}$$
$$\times\int_{-\infty}^{\infty}d\omega\;n^{-1}(\omega)\exp\left[i\frac{\omega}{c}n(\omega)|x-x'|\right]. \tag{44}$$

Performing the ω integral, on using the properties of $\epsilon(\omega)$, indeed yields the commutation relation (41) (see Appendix C).

C. Limiting case of mode decomposition

It should be pointed out that the quantization scheme also applies to cases when the losses in the dielectric may be disregarded, that is, $\epsilon_{\rm I}\to0$. These systems are usually treated by applying the method of mode decomposition. Clearly, assuming $\epsilon_{\rm I}=0$ and allowing for $\epsilon_{\rm R}\neq1$ in the whole frequency domain, one strongly violates the Kramers-Kronig relations. Therefore, quantization through mode decomposition is, apart from the vacuum, an approximation that can only be applied to radiation whose relevant frequencies are in intervals where absorption may be disregarded.

To show that quantization through mode decomposition is involved in the quantization scheme developed as a limiting case, we use the Green function (37) and rewrite Eq. (39) as

$$\hat{A}(x)=\int_0^{\infty}d\omega\sqrt{\frac{\hbar}{4\pi c\omega\epsilon_0\beta(\omega)\mathscr{A}}}\frac{\beta(\omega)}{n(\omega)}[e^{i\beta(\omega)\omega x/c}\hat{a}_+(x,\omega)+e^{-i\beta(\omega)\omega x/c}\hat{a}_-(x,\omega)]+\mathrm{H.c.}, \tag{45}$$

where

$$\hat{a}_{\pm}(x,\omega)=\frac{1}{i}\sqrt{2\gamma(\omega)\frac{\omega}{c}}\exp\left[\mp\gamma(\omega)\frac{\omega}{c}x\right]\times\int_{-\infty}^{\pm x}dx'\exp\left[-i\frac{\omega}{c}n(\omega)x'\right]\hat{f}(\pm x',\omega). \tag{46}$$

In Eqs. (45) and (46) the notation

$$n(\omega)=\sqrt{\epsilon(\omega)}=\beta(\omega)+i\gamma(\omega) \tag{47}$$

has been used. The subscripts + and − denote propagation to the right and left, respectively. From Eq. (46) we derive, on recalling Eq. (33),

$$[\hat{a}_{\pm}(x,\omega),\hat{a}^{\dagger}_{\pm}(x',\omega')]=e^{-\gamma(\omega)\omega|x-x'|/c}\delta(\omega-\omega'). \tag{48}$$

So far, Eqs. (45) – (48) are exact. Let us now assume that in a chosen frequency interval $\Delta\omega$ the absorption is sufficiently small, so that the condition

$$\gamma(\omega)\ll\beta(\omega) \tag{49}$$

is satisfied. Equations (46) and (48) imply that when $|-\gamma(\omega)\omega|x-x'|/c|\to 0$ the operators $\hat{a}_{\pm}(x,\omega)$ become independent of x. In this case Eq. (45) reduces, in the chosen frequency interval $\Delta\omega$, to the familiar mode-expansion result

$$\hat{A}_{\Delta\omega}(x)=\int_{(\Delta\omega)}d\omega\sqrt{\frac{\hbar}{4\pi c\omega\epsilon_0\beta(\omega)\mathscr{A}}}[e^{i\beta(\omega)\omega x/c}\hat{a}_{+}(\omega)+e^{-i\beta(\omega)\omega x/c}\hat{a}_{-}(\omega)]+\text{H.c.}, \tag{50}$$

where the associated photon operators $\hat{a}_{\pm}(\omega)$ and $\hat{a}^{\dagger}_{\pm}(\omega)$ satisfy the commutation relations

$$[\hat{a}_{\pm}(\omega),\hat{a}^{\dagger}_{\pm}(\omega')]=\delta(\omega-\omega'). \tag{51}$$

Moreover, let us assume that the width $\Delta\omega$ of the chosen frequency interval is small compared with its midfrequency ω_{m}, so that $\beta(\omega)\approx\beta(\omega_{\mathrm{m}})$, and confine ourselves to resolving distances Δx that are large compared with the characteristic length $c/(\beta(\omega_{\mathrm{m}})\Delta\omega)$,

$$\omega_{\mathrm{m}}\gg\Delta\omega\gg\frac{c}{\beta(\omega_{\mathrm{m}})\Delta x}. \tag{52}$$

Using Eqs. (50) and (51) and calculating the commutator $[\hat{A}_{\Delta\omega}(x),\hat{E}_{\Delta\omega}(x')]$ in the approximation given by the inequalities (52), we easily recognize the well-known result that

$$[\hat{A}_{\Delta\omega}(x),\hat{E}_{\Delta\omega}(x')]=-\frac{i\hbar}{\mathscr{A}\epsilon_{\mathrm{R}}(\omega_{\mathrm{m}})\epsilon_0}\delta(x-x') \tag{53}$$

[$\epsilon_{\mathrm{R}}(\omega_{\mathrm{m}})=\beta^2(\omega_{\mathrm{m}})$, note that the δ function is defined on a length scale of the order of magnitude of $c/(\beta(\omega_{\mathrm{m}})\Delta\omega)$, because of the coarse-graining condition $\Delta x\gg c/(\beta(\omega_{\mathrm{m}})\Delta\omega)$]. Equation (53) reveals that with respect to the chosen frequency interval, $\hat{A}_{\Delta\omega}(x)$ and $\hat{D}_{\Delta\omega}(x)=\epsilon_{\mathrm{R}}(\omega_{\mathrm{m}})\hat{E}_{\Delta\omega}(x)$ may be regarded as canonically conjugate field variables. This commutation rule is commonly used in the standard theory of quantization of the radiation field in dielectric matter characterized by real and constant ϵ (see, e.g., [13]).

D. Quantum Langevin equations

To make contact with the standard quantum theory of damping (see, e.g., [32]), we note that the operators $\hat{a}_{\pm}(x,\omega)$ defined in Eq. (46) may be regarded as system operators that describe the space-dependent amplitudes of monochromatic radiation-field excitations in dispersive and absorptive dielectric matter. Classically, these amplitudes are exponentially damped out in space, the characteristic length being given by $[\gamma(\omega)\omega/c]^{-1}$. In quantum theory, the spatial evolution of the operators $\hat{a}_{\pm}(x,\omega)$ is therefore expected to be governed by quantum Langevin equations, where the quantum noise associated with the damping is taken into account by operator Langevin noise sources. Indeed, from Eq. (46) we easily derive

$$\frac{\partial}{\partial x}\hat{a}_{\pm}(x,\omega)=\mp\gamma(\omega)\frac{\omega}{c}\hat{a}_{\pm}(x,\omega)+\hat{F}_{\pm}(x,\omega), \tag{54}$$

where the

$$\hat{F}_{\pm}(x,\omega)=\pm\frac{1}{i}\sqrt{2\gamma(\omega)\frac{\omega}{c}}\exp\left[\mp i\beta(\omega)\frac{\omega}{c}x\right]\hat{f}(x,\omega) \tag{55}$$

have all the properties of operator Langevin noise sources. We first calculate the commutators $[\hat{F}_{\pm}(x,\omega),\hat{F}^{\dagger}_{\pm}(x',\omega')]$. Recalling Eq. (33), straightforward calculation yields

$$[\hat{F}_{\pm}(x,\omega),\hat{F}^{\dagger}_{\pm}(x',\omega')]=2\gamma(\omega)\frac{\omega}{c}\delta(x-x')\delta(\omega-\omega'). \tag{56}$$

Next, from Eqs. (55) and (46) together with Eq. (33) we find that

$$[\hat{a}_{\pm}(x,\omega),\hat{F}^{\dagger}_{\pm}(x',\omega')]=0\quad\text{if}\quad \pm x\mp x'<0. \tag{57}$$

Equations (56) and (57) reveal that (54) indeed represents quantum Langevin equations in the space domain, which is in agreement with [25]. Their solutions may be written as

$$\hat{a}_{\pm}(x,\omega)=\hat{a}_{\pm}(x',\omega)\exp\left[\mp\gamma(\omega)\frac{\omega}{c}(x-x')\right]+\int_{x'}^{x}dy\hat{F}_{\pm}(y,\omega)\exp\left[\mp\gamma(\omega)\frac{\omega}{c}(x-y)\right], \tag{58}$$

where $\pm x\mp x'\geqslant 0$, which fully agrees with Eq. (46). Substituting in the commutators $[\hat{a}_{\pm}(x,\omega),\hat{a}^{\dagger}_{\pm}(x',\omega')]$ for the operators $\hat{a}_{\pm}(x,\omega)$ the solutions (58) of the Langevin equations and using the relations (56) and (57), we easily see that the correct commutation relations (48) are satisfied.

It should be pointed out that both the operators $\hat{a}_{\pm}(x,\omega)$, $\hat{a}^{\dagger}_{\pm}(x,\omega)$ and the operators $\hat{F}_{\pm}(x,\omega)$, $\hat{F}^{\dagger}_{\pm}(x,\omega)$ are related to the basic-field operators $\hat{f}(x,\omega)$, $\hat{f}^{\dagger}(x,\omega)$ originally introduced [cf. Eqs. (46) and (55)]. Clearly, using the Langevin equation scheme, knowledge of the explicit relations is no longer necessary, because all relevant information is now contained in Eqs. (54), (56), and (57). The Langevin-equation formalism enables one to easily derive quantum optical input-output relations of the type (58). In particular, we may let [cf. Eq. (58)]

$$\langle\hat{f}(x,\omega)\rangle=0, \tag{59}$$

$$\langle\hat{a}_{\pm}(x,\omega)\rangle=\langle\hat{a}_{\pm}(x',\omega)\rangle\exp\left[-\gamma(x)\frac{\omega}{c}|x-x'|\right], \tag{60}$$

where $\pm x\mp x'\geqslant 0$, with arbitrary $\langle\hat{a}_{\pm}(x',\omega)\rangle$. Taking the average of $\hat{A}(x,t)$, from Eq. (45) $[\hat{a}_{\pm}(x,\omega)\rightarrow\hat{a}_{\pm}(x,\omega)e^{-i\omega t}]$ together with Eqs. (59) and (60) we just obtain the general solutions of the classical homogeneous wave equations for forward- and backward-propagating radiation. As we will show in a forthcoming presentation, the method also applies to the determination of the (nontrivial) input-output relations observed when quantized radiation falls on spatially structured dielectric bodies of finite extension. In particular, the introduction of system and noise operators by means of the Green function then ensures that the boundary conditions at the surfaces of discontinuity are automatically satisfied.

As long as the condition (49) is satisfied, the operator of the vector potential (in the chosen frequency interval $\Delta\omega$) reads as [see Eq. (45)]

$$\hat{A}_{\Delta\omega}(x)=\int_{\Delta\omega}d\omega\sqrt{\frac{\hbar}{4\pi c\omega\epsilon_0\beta(\omega)\mathscr{A}}}\,[e^{i\beta(\omega)\omega x/c}\hat{a}_{+}(x,\omega)+e^{-i\beta(\omega)\omega x/c}\hat{a}_{-}(x,\omega)]+\text{H.c.} \tag{61}$$

Equations (61) and (54) [together with (56) and (57)] may be regarded as the basic equations describing the propagation of quantum light pulses in dispersive and absorptive dielectrics, provided that the absorption is not too strong. When the condition (49) is not satisfied the exact expansion (45) must be used in place of the quasimode expansion (61). The condition (49) may therefore be regarded as being the analogy to the Markov condition in the standard theory of temporally damped (quasi)modes. We finally mention that in many cases when light pulses propagate through dielectric matter, such as optical fibers, the absorption may only be expected to be significant in the direction of pulse propagation. Disregarding the absorption perpendicular to the direction of propagation, the field structure in this direction may be obtained from standard mode expansion.

III. MULTILAYER DIELECTRICS

A. Basic equations

In the (approximative) quantization scheme through mode expansion the classical problem consists of solving the homogeneous wave equation (Helmholtz equation) to determine the (orthonormal) set of radiation-field modes and express the field operators in terms of the bosonic creation and destruction operators associated with the modes. Allowing for complex permittivities, the analogous problem now consists of the determination of the classical Green function. The resulting expansion (39) may be regarded as the natural generalization of the mode expansion (50) to the case of radiation in dispersive and lossy dielectrics. It is well known that the method of quantization through mode expansion can be extended to inhomogeneous dielectrics in a straightforward way [8,11]. Similarly, the method of Green function may be expected to also apply to inhomogeneous dielectrics. Let us restrict our attention to multilayer dielectric structures, which are typically used in practice. Since the space can be subdivided into elements where the permittivity is spatially constant, Eqs. (26) – (28) can directly be used to introduce operator noise sources in each element, which implies that the commutation relations (25) are preserved (see Appendix A).

For example, when linearly polarized radiation propagates along the x axis through a multislab dielectric device, Eq. (32) becomes

$$\frac{\partial^2}{\partial x^2}\hat{A}(x,\omega)+\frac{\omega^2}{c^2}\epsilon(x,\omega)\hat{A}(x,\omega)=\frac{\omega}{c^2}\sqrt{\frac{\hbar}{\pi\epsilon_0\mathscr{A}}}\epsilon_{\rm I}(x,\omega)\hat{f}(x), \tag{62}$$

where $\hat{f}(x)$ and $\hat{f}^{\dagger}(x)$ satisfy the commutation relations (33) and (34). Introducing the Green function satisfying the equation

$$\frac{\partial^2}{\partial x^2}G(x,x',\omega)+\frac{\omega^2}{c^2}\epsilon(x,\omega)G(x,x',\omega)=\delta(x-x'), \tag{63}$$

in place of Eq. (35) we now have

$$\underline{\hat{A}}(x,\omega)=\frac{\omega}{c^2}\sqrt{\frac{\hbar}{\pi\epsilon_0\mathscr{A}}}\times\int_{-\infty}^{\infty}dx'\,\sqrt{\epsilon_{\rm I}(x',\omega)}G(x,x',\omega)\hat{f}(x',\omega). \tag{64}$$

The (Schrödinger) operators of the vector potential $\hat{A}(x)$ and the electric-field strength $\hat{E}(x)$ then read as

$$\hat{A}(x)=\int_0^{\infty}d\omega\,\frac{\omega}{c^2}\sqrt{\frac{\hbar}{\pi\epsilon_0\mathscr{A}}}\times\int_{-\infty}^{\infty}dx'\,\sqrt{\epsilon_{\rm I}(x',\omega)}G(x,x',\omega)\hat{f}(x',\omega)+\text{H.c.}, \tag{65}$$

$$\hat{E}(x)=i\int_0^{\infty}d\omega\,\frac{\omega^2}{c^2}\sqrt{\frac{\hbar}{\pi\epsilon_0\mathscr{A}}}\times\int_{-\infty}^{\infty}dx'\,\sqrt{\epsilon_{\rm I}(x',\omega)}G(x,x',\omega)\hat{f}(x',\omega)+\text{H.c.} \tag{66}$$

Since the generalized version of Eq. (42) reads as

$$\frac{\omega^2}{c^2}\int_{-\infty}^{\infty}dy\,\epsilon_{\rm I}(y,\omega)G(x,y,\omega)G^*(x',y,\omega)=\frac{1}{2}i[G(x,x',\omega)-G^*(x',x,\omega)] \tag{67}$$

(see Appendix B), Eq. (43) may easily be proved correct also in the case when the permittivity varies with space,

$$[\hat{A}(x),\hat{E}(x')]=\frac{\hbar}{\pi\epsilon_0c^2\mathscr{A}}\int_{-\infty}^{\infty}d\omega\,\omega G(x,x',\omega). \tag{68}$$

The evaluation of the integral in Eq. (68) requires knowledge of the Green function. Considering an arbitrary multislab dielectric device, the calculation of the Green function is expected to be very lengthy. Nevertheless, in any case the commutation relation

$$[\hat{A}(x),\hat{E}(x')]=-\frac{i\hbar}{\mathscr{A}\epsilon_0}\delta(x-x') \tag{69}$$

is suggested to come out, because of causality reasons [cf. Appendix A].

B. One-interface dielectric

To explicitly show this let us consider an inhomogeneous dielectric consisting of two bulk dielectrics with a common interface in more detail. Since such a one-interface dielectric may be regarded as the basic element of an arbitrary multislab configuration, all the calculations and results given below may be extended to multislab dielectrics in a very straightforward way.

Assuming the interface at $x=0$, the complex permittivity as a function of x may be written as

$$\epsilon(x,\omega)=\Theta(-x)\epsilon_1(\omega)+\Theta(x)\epsilon_2(\omega), \tag{70}$$

where $\Theta(x)$ is the unit step function and $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$, respectively, are the complex permittivities of the left-hand and right-hand bulk dielectrics, so that Eq. (63) reads as

$$\frac{\partial^2}{\partial x^2}G(x,x',\omega)+\frac{\omega^2}{c^2}[\Theta(-x)\epsilon_1(\omega)+\Theta(x)\epsilon_2(\omega)]G(x,x',\omega)=\delta(x-x'). \tag{71}$$

Using standard methods, the solution of Eq. (71) may be written as follows:

$$G(x,x',\omega)=\Theta(-x')[\Theta(-x)G_1^{(0)}(x,x',\omega)+\Theta(-x)r_{12}(\omega)R_1(|x|,|x'|,\omega)+\Theta(x)t_{12}(\omega)T_{12}(|x|,|x'|,\omega)]+\Theta(x')[\Theta(x)G_2^{(0)}(x,x',\omega)+\Theta(x)r_{21}(\omega)R_2(|x|,|x'|,\omega)+\Theta(-x)t_{21}(\omega)T_{21}(|x|,|x'|,\omega)]. \tag{72}$$

Here, the $G_j^{(0)}(x,x',\omega)$, $j=1,2$, are the Green functions of the bulk dielectrics; that is,

$$G_j^{(0)}(x,x',\omega)=\left[2i\frac{\omega}{c}n_j(\omega)\right]^{-1}\exp\left[i\frac{\omega}{c}n_j(\omega)|x-x'|\right] \tag{73}$$

[see Eq. (37)]. The effects of reflection at and transmission through the interface is described by the functions

$$R_j(x,x',\omega)=\left[2i\frac{\omega}{c}n_j(\omega)\right]^{-1}\exp\left[i\frac{\omega}{c}n_j(\omega)(x+x')\right] \tag{74}$$

and

$$T_{jj'}(x,x',\omega)=\left[2i\frac{\omega}{c}n_j(\omega)\right]^{-1}\times\exp\left\{i\frac{\omega}{c}[n_j(\omega)x'+n_{j'}(\omega)x]\right\}, \tag{75}$$

respectively, together with the well-known complex reflection and transmission coefficients

$$r_{jj'}(\omega)=\frac{n_j(\omega)-n_{j'}(\omega)}{n_j(\omega)+n_{j'}(\omega)}, \tag{76}$$

$$t_{jj'}(\omega)=\frac{2n_j(\omega)}{n_j(\omega)+n_{j'}(\omega)}. \tag{77}$$

Note that the functions $R_j(x,x',\omega)$ and $T_{jj'}(x,x',\omega)$ satisfy the homogeneous wave equation.

Since the Green function $G(x,x',\omega)$ given in Eq. (72) is continuous and (except for $x=x'$) continuously differentiable, the vector potential $\hat{A}(x)$, Eq. (65), is continuously differentiable at the interface $x=0$. Hence, the radiation field satisfies the proper boundary conditions at the interface. Substituting in Eqs. (65) and (66) for the Green function the expression (72), explicit expressions for the operators of the vector potential and the electric-field strength may easily be obtained. In particular, quantum Langevin equations for forward- and backward-propagating fields on the left-hand and right-hand sides of the interface can be obtained in a similar way as in Sec. II D. We renounce this procedure here because of the somewhat voluminous formulas.

Using the Green function as given in Eq. (72) enables one to explicitly show that $\hat{A}(x)$ and $\hat{E}(x)$ satisfy the correct commutation relation. Substituting in Eq. (68) for the Green function the expression (72), by straightforward calculation we find that the reflection and transmission terms do not contribute to the ω integral:

$$\int_{-\infty}^{\infty} d\omega\ \omega r_{jj'}(\omega) R_j(|x|,|x'|,\omega)=0 \tag{78}$$

(see Appendix D),

$$\int_{-\infty}^{\infty} d\omega\ \omega t_{jj'}(\omega) T_{jj'}(|x|,|x'|,\omega)=0 \tag{79}$$

(see Appendix E). Hence we obtain

$$[\hat{A}(x),\hat{E}(x')]=\frac{\hbar}{2\pi i\epsilon_0 c\mathscr{A}}\Bigg\{\Theta(-x)\Theta(-x')\int_{-\infty}^{\infty}d\omega\ n_1^{-1}(\omega)\exp\left[i\frac{\omega}{c}n_1(\omega)|x-x'|\right] + \Theta(x)\Theta(x')\int_{-\infty}^{\infty}d\omega\ n_2^{-1}(\omega)\exp\left[i\frac{\omega}{c}n_2(\omega)|x-x'|\right]\Bigg\}, \tag{80}$$

which reveals that (see Appendix C)

$$[\hat{A}(x),\hat{E}(x')]=-\frac{i\hbar}{\mathscr{A}\epsilon_0}\,\delta(x-x'). \tag{81}$$

We would like to mention that when $\epsilon_1(\omega)=1$ the field representations given above correspond to the results recently obtained for a dielectric-vacuum interface [34], by combining the microscopic Huttner-Barnett model [25] for a bulk dielectric and the mode-expansion scheme for the free space with the boundary conditions on the electric and magnetic fields at the surface of discontinuity.

IV. SUMMARY AND CONCLUSIONS

On the basis of a Green-function expansion we have presented a concept of quantization of the phenomenological Maxwell theory for radiation in linear dielectrics, which is consistent with the Kramers-Kronig relations and the proper canonical (equal-time) commutation relation for the vector potential and the electric-field strength. It may be regarded as an extension of the microscopic Huttner-Barnett model [25] to more general linear dielectrics described in terms of a phenomenological complex permittivity that satisfies the Kramers-Kronig relations. These relations ensure that dispersion and absorption are correctly associated with each other. It is worth noting that the quantization scheme also applies to radiation in multilayer dielectric structures. The advantage of the Green-function approach is that all the information on the dielectric, such as its dispersive and absorptive properties and its spatial structure, is included in the classical Green function of light propagation through the dielectric. Clearly, for a given dielectric matter the determination of the Green function can be extensive. Nevertheless, it is a purely classical problem.

It is well known that when the concept of mode expansion applies the classical problem consists of the determination of the mode structure of the radiation field. Apart from the vacuum, a proper mode decomposition is only feasible when the absorption can be disregarded, that is, for real permittivity. From the Kramers-Kronig relations it is clear that such an assumption can only be justified for certain intervals of the frequency spectrum. Exactly for these frequency domains the Green-function expansion takes the familiar form of a mode expansion (with frequency-dependent real refractive index), which shows that the standard formalism that rests on a real permittivity is included in the quantization scheme developed as a limiting case.

When in a given frequency interval the absorption is small compared with the dispersion the theory shows that the effect of absorption can be taken into account by a (quasi)-mode expansion in the sense that ordinary modes are associated with space-dependent "photon" destruction and creation operators that spatially evolve according to quantum Langevin equations. It is worth noting that the Langevin-equation formalism also applies to radiation in strongly absorbing linear dielectrics, when an interpretation as photon destruction and creation operators of the space-dependent amplitude operators that can be associated with monochromatic radiation-field excitations fails.

Let us briefly comment on the operator source field $\hat{\mathbf{f}}(\mathbf{r},\omega)$ that was introduced. Formally, this field is required to ensure that the proper (equal-time) commutation relations $[\hat{A}_i(\mathbf{r}),\hat{E}_j(\mathbf{r}')]$ are preserved for any $\mathbf{r}$ and $\mathbf{r}'$. Physically, the sources that build up the dielectric matter are also responsible for the quantum noise associated with the absorption of light. Hence, in the frame of quantum theory the effect of the dielectric matter on the radiation cannot be described only in terms of a (complex) permittivity but is necessarily associated with the inclusion into the Maxwell equations of a matter-assisted quantum source. Clearly, $\hat{\mathbf{f}}(\mathbf{r},\omega)$ cannot be a proper noise operator whose average vanishes at all space points $\mathbf{r}$, otherwise the average of the vector potential would identically vanish. Dependent upon the direction of propagation, only for certain space domains the average of $\hat{\mathbf{f}}(\mathbf{r},\omega)$ can reasonably vanish. In this connection, the basic field $\hat{\mathbf{f}}(\mathbf{r},\omega)$ originally introduced into the theory can be used to define both system and proper noise fields that are associated with each other by quantum Langevin equations. In this way, the averages of the system fields can be related to the solutions of the classical homogeneous wave equation, provided that the averages of the noise fields vanish.

In the paper we have considered quantized radiation in inhomogeneous dielectric matter whose complex permittivity $\epsilon(\mathbf{r},\omega)$ can be regarded as being a multistep function of $\mathbf{r}$, which is typically the case in a number of applications, such as propagation of light through multislab dielectric devices. The space can be subdivided into elements, so that in each element the permittivity is spatially constant and the vector potential can be assumed to be transverse. It should be pointed out that the use of an $\epsilon(\mathbf{r},\omega)$ that continuously depends on $\mathbf{r}$ needs additional considerations, because the ordinary condition of transversality does not apply.

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APPENDIX A: THREE-DIMENSIONAL MULTILAYER SYSTEMS

When the permittivity is spatially varying, from Maxwell's equations we obtain, in place of Eq. (19),

$$-\frac{\partial^2}{\partial x_i \partial x_k}\underline{\hat{A}}_k(\mathbf{r},\omega)+\frac{\partial^2}{\partial x_k \partial x_k}\underline{\hat{A}}_i(\mathbf{r},\omega)+\frac{\omega^2}{c^2}\epsilon(\mathbf{r},\omega)\underline{\hat{A}}_i(\mathbf{r},\omega)=\underline{\hat{j}}_{\mathrm{n}i}(\mathbf{r},\omega), \tag{A1}$$

where the generalized Coulomb gauge

$$\frac{\partial}{\partial x_i}[\epsilon(\mathbf{r},\omega)\underline{\hat{A}}_i(\mathbf{r},\omega)]=0 \tag{A2}$$

has been used [note that $\hat{\mathbf{j}}_{\mathrm{n}}(\mathbf{r},\omega)$ is transverse]. Here and in the following we adopt the convention of summation over repeated vector-component indices. We now introduce the (tensorial) Green function that satisfies the generalized condition of transversality of the type (A2), and solves the equation

$$-\frac{\partial^2}{\partial x_i \partial x_k}G_{ki'}(\mathbf{r},\mathbf{r}',\omega)+\frac{\partial^2}{\partial x_k \partial x_k}G_{ii'}(\mathbf{r},\mathbf{r}',\omega)+\frac{\omega^2}{c^2}\epsilon(\mathbf{r},\omega)G_{ii'}(\mathbf{r},\mathbf{r}',\omega)=\tilde{\delta}^{\perp}_{ii'}(\mathbf{r},\mathbf{r}',\omega), \tag{A3}$$

where the δ function $\tilde{\delta}^{\perp}_{ii'}(\mathbf{r},\mathbf{r}',\omega)$ also satisfies the generalized condition of transversality. When the Green function is known, then the solution of Eq. (A1) can be given by

$$\underline{\hat{A}}_i(\mathbf{r},\omega)=\int d^3\mathbf{r}'\,G_{ii'}(\mathbf{r},\mathbf{r}',\omega)\underline{\hat{j}}_{\mathrm{n}i'}(\mathbf{r}',\omega), \tag{A4}$$

so that

$$\underline{\hat{A}}_i(\mathbf{r})=\int d\omega\int d^3\mathbf{r}'\,G_{ii'}(\mathbf{r},\mathbf{r}',\omega)\underline{\hat{j}}_{\mathrm{n}i'}(\mathbf{r}',\omega)+\mathrm{H.c.}. \tag{A5}$$

In particular, for bulk dielectrics when the permittivity does not vary with space, $\tilde{\delta}^{\perp}_{ii'}(\mathbf{r},\mathbf{r}',\omega)$ reduces to the ordinary transverse δ function $\delta^{\perp}_{ii'}(\mathbf{r}-\mathbf{r}')$ and the now transverse Green function $G_{ii'}(\mathbf{r},\mathbf{r}',\omega)$ satisfies the equation

$$\frac{\partial^2}{\partial x_k \partial x_k}G_{ii'}(\mathbf{r},\mathbf{r}',\omega)+\frac{\omega^2}{c^2}\epsilon(\omega)G_{ii'}(\mathbf{r},\mathbf{r}',\omega)=\delta^{\perp}_{ii'}(\mathbf{r}-\mathbf{r}'), \tag{A6}$$

whose solution reads as

$$G_{ii'}(\mathbf{r},\mathbf{r}',\omega)=\int d\mathbf{r}''G(\mathbf{r},\mathbf{r}'',\omega)\,\delta^{\perp}_{ii'}(\mathbf{r}''-\mathbf{r}'), \tag{A7}$$

where $G(\mathbf{r},\mathbf{r}'',\omega)$ is given by Eq. (22).

When the permittivity varies with space the determination of the Green function is, in general, very difficult. Considering multilayer dielectric structures, we may assume that the space can be subdivided into elements where the permittivity is (spatially) constant. Since in each space element Eq. (A6) applies, the Green function can be obtained by adding to a bulk-material solution of the type (A7) a solution of the homogeneous equation associated with Eq. (A6) in such a way that the boundary conditions at the surfaces of discontinuity are satisfied. Physically, the two contributions to the Green function describe the emission in homogeneous space of radiation from pointlike sources and its reflection at and transmission through the surfaces of discontinuity. The equations given above are of course valid in both the classical and quantum descriptions. Note that in each space element the vector potential (A5) is transverse.

Now, the relations (26) – (28) are used to specify the current in each space element in the integral in Eq. (A5) as an operator noise source associated with dissipation. Calculating the equal-time commutator $[\hat{A}_i(\mathbf{r}),\hat{E}_j(\mathbf{r}')]$, we see that the fields (at the positions $\mathbf{r}$ and $\mathbf{r}'$) that can contribute to the commutator at chosen time arise (directly and/or through reflections and transmissions) from pointlike sources excited at the same time at distances $\mathbf{r}''-\mathbf{r}$ and $\mathbf{r}''-\mathbf{r}'$. From causality arguments, we therefore may conclude that a nonvanishing commutator can only be expected when $\mathbf{r}$ and $\mathbf{r}'$ belong to equal space elements, with $\mathbf{r}''\to\mathbf{r}'\to\mathbf{r}$. The relevant sources must be in the same element as the points of observation and (time-delayed) contributions associated with reflections and/or transmissions can be excluded from consideration, because of the equal-time condition. That is, the only relevant part of the Green function is the bulk-material solution (A7), which just yields (within each space element) the commutation relation (25), as has been shown in [29].

APPENDIX B: PROOF OF EQUATIONS (42) AND (67)

The symmetry of the Green function

$$G(x,y,\omega)=G(y,x,\omega) \tag{B1}$$

implies that

$$\frac{\partial^2}{\partial y^2}G(x,y,\omega)+\frac{\omega^2}{c^2}\epsilon(y,\omega)G(x,y,\omega)=\delta(x-y). \tag{B2}$$

Multiplication of Eq. (B2) by $G^*(x',y,\omega)$ and y integration yields

$$\frac{\omega^2}{c^2}\int_{-\infty}^{\infty}dy\,\epsilon(y,\omega)G(x,y,\omega)G^*(x',y,\omega)=-\int_{-\infty}^{\infty}dy\left[\frac{\partial^2}{\partial y^2}G(x,y,\omega)\right]G^*(x',y,\omega)+G^*(x',x,\omega), \tag{B3}$$

which after integration by parts reads as

$$\frac{\omega^2}{c^2}\int_{-\infty}^{\infty}dy\,\epsilon(y,\omega)G(x,y,\omega)G^*(x',y,\omega)=\int_{-\infty}^{\infty}dy\left[\frac{\partial}{\partial y}G(x,y,\omega)\right]\left[\frac{\partial}{\partial y}G^*(x',y,\omega)\right]+G^*(x',x,\omega) \tag{B4}$$

[note that $\lim_{|y|\to\infty}G(x,y,\omega)=0$]. Taking the complex conjugate of Eq. (B4), interchanging x and x', and subtracting the resulting equation from Eq. (B4), we obtain the integral relation

$$\frac{\omega^2}{c^2}\int_{-\infty}^{\infty}dy\,\epsilon_{\mathrm{I}}(y,\omega)G(x,y,\omega)G^*(x',y,\omega)$$

$$=\frac{1}{2}i[G(x,x',\omega)-G^*(x',x,\omega)], \tag{B5}$$

which in the case when the permittivity does not vary with space reduces to

$$\frac{\omega^2}{c^2}\epsilon_{\mathrm{I}}(\omega)\int_{-\infty}^{\infty}dy\; G(x,y,\omega)G^*(x',y,\omega)$$

$$=\frac{1}{2}i[G(x,x',\omega)-G^*(x',x,\omega)]. \tag{B6}$$

APPENDIX C: PROOF OF EQ. (41)

To calculate

$$C(x,x')=\frac{\hbar}{2\pi i\epsilon_0 c\mathcal{A}}\times\int_{-\infty}^{\infty}d\omega\; n^{-1}(\omega)\exp\left[i\frac{\omega}{c}n(\omega)|x-x'|\right], \tag{C1}$$

we substitute in Eq. (C1) for the exponential the Fourier decomposition [cf. Eq. (37)]

$$\exp\left[i\frac{\omega}{c}n(\omega)|x-x'|\right]=\frac{1}{2\pi}\int_{-\infty}^{\infty}dk\; e^{ik(x-x')}\;\frac{2ic\omega n(\omega)}{\omega^2\epsilon(\omega)-c^2k^2} \tag{C2}$$

and change the order of integrations. We then find that

$$C(x,x')=\frac{\hbar}{2\pi^2\epsilon_0\mathcal{A}}\int_{-\infty}^{\infty}dk\; e^{ik(x-x')}\times\int_{-\infty}^{\infty}d\omega\;\frac{\omega}{\omega^2\epsilon(\omega)-c^2k^2}. \tag{C3}$$

Since $\epsilon(\Omega)$ is an analytical function in the upper complex half plane without zeros, and

$$\lim_{\Omega\to\infty}\epsilon(\Omega)=1, \tag{C4}$$

the ω integral can be evaluated as follows ($\Omega=Re^{i\theta}$):

$$\int_{-\infty}^{\infty}d\omega\frac{\omega}{\omega^2\epsilon(\omega)-c^2k^2}=-\lim_{R\to\infty}\int_0^{\pi}id\theta\frac{R^2e^{2i\theta}}{R^2e^{2i\theta}\epsilon(Re^{i\theta})-k^2c^2}=-i\pi. \tag{C5}$$

Hence, from Eqs. (C3) and (C5) we finally obtain

$$C(x,x')=-\frac{i\hbar}{2\pi\epsilon_0\mathcal{A}}\int_{-\infty}^{\infty}dk\; e^{ik(x-x')}=-\frac{i\hbar}{\epsilon_0\mathcal{A}}\delta(x-x'). \tag{C6}$$

APPENDIX D: PROOF OF EQUATION (78)

Using Eq. (74), Eq. (78) reads as

$$\int_{-\infty}^{\infty} d\omega \;\omega r_{jj'}(\omega) R_j(|x|,|x'|,\omega)$$

$$= \int_{-\infty}^{\infty} d\omega \frac{c r_{jj'}(\omega)}{2 i n_j(\omega)} \exp\left[i \frac{\omega}{c} n_j(\omega)(|x|+|x'|)\right], \quad \text{(D1)}$$

where $r_{jj'}(\omega)$ is given in Eq. (76). Introducing in Eq. (D1) the Fourier decomposition

$$\exp\left[i \frac{\omega}{c} n_j(\omega)(|x|+|x'|)\right] = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk\; e^{ik(x+x')} \frac{2 i c \omega n_j(\omega)}{\omega^2 n_j(\omega) - c^2 k^2}, \quad \text{(D2)}$$

using Eq. (76), and changing the order of integrations leads to the ω integral

$$I_{jj'}(k) = \int_{-\infty}^{\infty} d\omega \frac{n_j(\omega) - n_{j'}(\omega)}{n_j(\omega) + n_{j'}(\omega)} \frac{\omega}{\omega^2 n_j(\omega) - c^2 k^2}. \quad \text{(D3)}$$

Recalling the properties of $\epsilon_j(\Omega)$, from similar arguments as in Appendix C we find that ($\Omega = Re^{i\theta}$)

$$I_{jj'}(k) = -\lim_{R\to\infty} \int_0^{\pi} i d\theta \frac{n_j(Re^{i\theta}) - n_{j'}(Re^{i\theta})}{n_j(Re^{i\theta}) + n_{j'}(Re^{i\theta})} \frac{R^2 e^{2i\theta}}{R^2 e^{2i\theta} \epsilon_j(Re^{i\theta}) - k^2 c^2} = 0. \quad \text{(D4)}$$

Note that

$$\lim_{\Omega\to\infty} [n_j(\Omega) - n_{j'}(\Omega)] = 0. \quad \text{(D5)}$$

APPENDIX E: PROOF OF EQUATION (79)

Combining Eqs. (79) and (75), we may write

$$\int_{-\infty}^{\infty} d\omega \;\omega t_{jj'}(\omega) T_{jj'}(|x|,|x'|,\omega)$$

$$= \int_{-\infty}^{\infty} d\omega \frac{c t_{jj'}(\omega)}{2 i n_j(\omega)} \exp\left\{ i \frac{\omega}{c} [n_j(\omega)|x'| + n_{j'}(\omega)|x|]\right\}, \quad \text{(E1)}$$

where $t_{jj'}(\omega)$ is given in Eq. (77). Using in Eq. (E1) the Fourier decompositions

$$\exp\left\{ i \frac{\omega}{c} [n_j(\omega)|x'| + n_{j'}(\omega)|x|]\right\} = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dk e^{ikx} \int_{-\infty}^{\infty} dk' e^{ik'x'} \times \frac{2 i c \omega n_j(\omega)}{\omega^2 n_j(\omega) - c^2 k'^2} \frac{2 i c \omega n_{j'}(\omega)}{\omega^2 n_{j'}(\omega) - c^2 k^2}, \quad \text{(E2)}$$

using Eq. (77), and changing the order of integrations, we leave with the ω integral

$$I_{jj'}(k,k') = \int_{-\infty}^{\infty} d\omega \frac{2 n_j(\omega)}{n_j(\omega) + n_{j'}(\omega)} \times \frac{\omega^2}{[\omega^2 n_j(\omega) - c^2 k'^2][\omega^2 n_{j'}(\omega) - c^2 k^2]}. \quad \text{(E3)}$$

Following the lines outlined in Appendixes C and D we obtain

$$I_{jj'}(k,k') = -\lim_{R\to\infty} \int_0^{\pi} i d\theta \frac{2 n_j(Re^{i\theta})}{n_j(Re^{i\theta}) + n_{j'}(Re^{i\theta})} \frac{R^3 e^{3i\theta}}{[R^2 e^{2i\theta} n_j(Re^{i\theta}) - c^2 k'^2][R^2 e^{2i\theta} n_{j'}(Re^{i\theta}) - c^2 k^2]}$$

$$= -\lim_{R\to\infty} \int_0^{\pi} i d\theta \frac{1}{Re^{i\theta}} = 0. \quad \text{(E4)}$$

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