Controlling optical bistability using electromagnetic-field-induced transparency and quantum interferences

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We demonstrate the application of electromagnetic-field-induced transparency and quantum interference effects in the cooperative phenomenon, such as optical bistability. The control field used in tandem with the usual electromagnetic field of the two-level scheme results in a considerable lowering of the threshold intensity. We discuss the transient response of the system in the mean-field limit and describe the regression to the steady state when perturbed away from it; the regression exponent is itself dependent on the control field. We also demonstrate the possibility of control-field-induced multistability in two-level systems.

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I. INTRODUCTION

In recent years many remarkable applications of electromagnetic-field-induced transparency and quantum interference have appeared [1–3]. These include the possibility of enhancing the efficiencies of nonlinear optical processes [1,3–5], and the possibility of producing laser action without population inversion [6]. Three-level schemes are usually the most popular ones in considerations of field-induced transparency. The transparency arises from the Autler-Townes splitting of the absorption line as well as from interferences which make absorption proportional to the decay of the atomic coherence between two states that are not directly connected by a dipole transition.

So far all applications have involved either single-atom situations or those which are equivalent to noninteracting atoms, for example, in the context of phase matching of pulses [7]. In the present work we investigate the role of atomic coherences and interferences in the context of collective phenomenon inside a resonator. We thus use an external electromagnetic field which is tuned (or close to resonance) to an appropriate transition in the atomic system. We examine the modification of the bistability characteristics such as the thresholds, switching times, etc. We find that the control field can even lead to multistability. The outline of the paper is as follows: In Sec. II we discuss the theory governing the proposed schemes, and outline the method involved in calculations. In Sec. III we present the numerical results, and also discuss the effects of field-induced transparency and quantum interferences that lead to the lowering of the threshold for switching from the off state to the on state. In Sec. IV we examine the transient response of the system, and in Sec. V we demonstrate the possibility of multistability and the fine control one has on its various characteristics due to the applied electromagnetic field.

II. MODEL CALCULATIONS

In order to keep the analysis as simple as possible, we consider a unidirectional ring cavity (Fig. 1) with mirrors 3 and 4 with 100% reflectivity and mirrors 1 and 2 with the reflection and transmission coefficient (R and T, respectively), such that $R+T=1$. This is a standard model of optical bistability given by Bonifacio and Lugiato [8,9]. The atomic system is a collection of N homogeneously broadened two-level atoms which have their excited states coupled to yet another level. The electric field at the atom can be written as

$$\mathbf{E}=\mathbf{E}_1\exp(-i\omega_1 t)+\mathbf{E}_2\exp(-i\omega_2 t)+\text{c.c.}, \tag{1}$$

where the subscripts 1 and 2 refer to the transitions $|1\rangle\leftrightarrow|2\rangle$ and $|2\rangle\leftrightarrow|3\rangle$ respectively. The coherent field $\mathbf{E}_2$ applied to the transition $|2\rangle\leftrightarrow|3\rangle$ corresponds to the usual two-level scheme, and $|1\rangle\leftrightarrow|2\rangle$ is the transition on which the control field $\mathbf{E}_1$ is applied. The control field does not circulate in the cavity, and thus its dynamical evolution can be ignored. We consider the system (a) as shown in Fig. 2, where level $|1\rangle$ decays through spontaneous emission to level $|2\rangle$ with the Einstein A coefficient $2\gamma_1$ and level $|2\rangle$ decays to $|3\rangle$ with the Einstein A coefficient $2\gamma_2$. Similarly, in system (b), level $|2\rangle$ decays to levels $|1\rangle$ and $|3\rangle$ with the Einstein A coefficients $2\gamma_1$ and $2\gamma_2$ respectively, and level $|1\rangle$ decays to $|3\rangle$ with the decay rate of 2ν (Fig. 2). We describe the dynamics [10] of

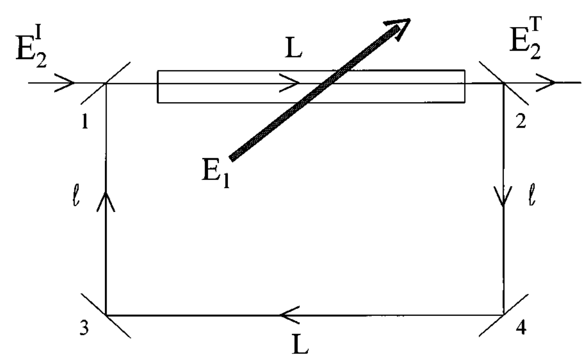

FIG. 1. Unidirectional ring cavity with an atomic sample of length L, with N homogeneously broadened atoms with an atomic configuration as given in Fig. 2. E_2^I and E_2^T are the incident and transmitted fields, respectively, and E_1 is the control field. For mirrors 1 and 2, $R+T=1$, and mirrors 3 and 4 have $R=1$.

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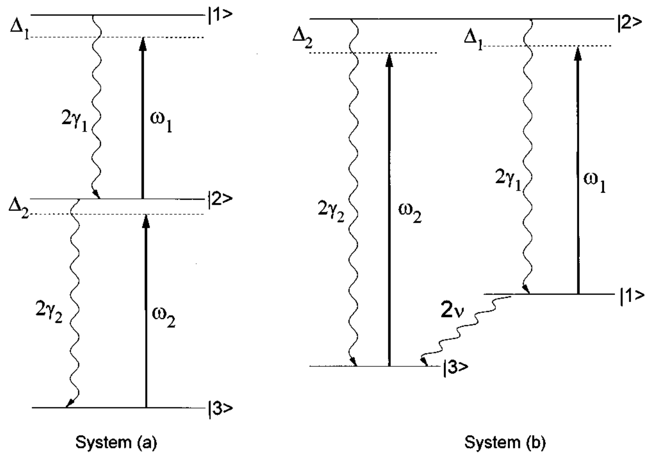

FIG. 2. The transitions labeled $|2\rangle$ and $|3\rangle$ are the bistable transitions. The control field $\mathbf{E}_1$ couples the excited state $|2\rangle$ to another level above [for system (a)] or below [for system (b)] the excited state. Here the Δ_i's are the detunings, and the γ_i's are the decays of the corresponding levels.

the atom plus the radiation fields by the well-known Maxwell-Bloch equations. The unperturbed Hamiltonian of the atom is given as

$$H_0=\hbar\omega_{13}|1\rangle\langle 1|+\hbar\omega_{23}|2\rangle\langle 2|, \tag{2}$$

where energies are measured from level $|3\rangle$. In the dipole approximation the interaction Hamiltonian between the atom and the external fields is given by $H_{\text{int}}=-\mathbf{d}\cdot\mathbf{E}$, where $\mathbf{d}$ is the atomic dipole moment operator having only the off-diagonal elements

$$\mathbf{d}=\mathbf{d}_{12}|1\rangle\langle 2|+\mathbf{d}_{23}|2\rangle\langle 3|+\text{c.c.} \tag{3}$$

The total Hamiltonian for the system is given by $H=H_0+H_{\text{int}}$. Density-matrix formalism is used to study the evolution of the system. Incoherent processes like spontaneous emission from different levels are included in the standard way.

We take the fields in the slowly varying envelope approximation, and neglect the rapidly oscillating terms like $\exp(\pm 2i\omega_1 t)$ and $\exp(\pm 2i\omega_2 t)$, thus transforming the density matrix equations $\rho_{\alpha\beta}$ for system (a) to those for slowly varying quantities $\tilde{\rho}_{\alpha\beta}$, where

$$\begin{aligned}
\tilde{\rho}_{ii}&=\rho_{ii}, \quad i=1,2,3,\\
\tilde{\rho}_{12}&=\rho_{12}\exp(i\omega_1 t),\\
\tilde{\rho}_{23}&=\rho_{23}\exp(i\omega_2 t),\\
\tilde{\rho}_{13}&=\rho_{13}\exp[i(\omega_1+\omega_2)t].
\end{aligned} \tag{4}$$

The equations of evolution of the density matrix are

$$\dot{\tilde{\rho}}_{11}=-2\gamma_1\tilde{\rho}_{11}+iG_1\tilde{\rho}_{21}-iG_1^*\tilde{\rho}_{12}, \tag{5}$$

$$\dot{\tilde{\rho}}_{12}=-(\gamma_1+\gamma_2+i\Delta_1)\tilde{\rho}_{12}+iG_1(\tilde{\rho}_{22}-\tilde{\rho}_{11})-iG_2^*\tilde{\rho}_{13},$$

$$\dot{\tilde{\rho}}_{13}=-[\gamma_1+i(\Delta_1+\Delta_2)]\tilde{\rho}_{13}+iG_1\tilde{\rho}_{23}-iG_2\tilde{\rho}_{12},$$

$$\dot{\tilde{\rho}}_{22}=2\gamma_1\tilde{\rho}_{11}-2\gamma_2\tilde{\rho}_{22}-iG_1\tilde{\rho}_{21}+iG_1^*\tilde{\rho}_{12}+iG_2\tilde{\rho}_{32}-iG_2^*\tilde{\rho}_{23},$$

$$\dot{\tilde{\rho}}_{23}=-(\gamma_2+i\Delta_2)\tilde{\rho}_{23}+iG_1^*\tilde{\rho}_{13}+iG_2(\tilde{\rho}_{33}-\tilde{\rho}_{22}),$$

$$\dot{\tilde{\rho}}_{33}=2\gamma_2\tilde{\rho}_{22}-iG_2\tilde{\rho}_{32}+iG_2^*\tilde{\rho}_{23}.$$

For system (b), the following transformations are undertaken to neglect the rapidly rotating terms:

$$\begin{aligned}
\tilde{\rho}_{ii}&=\rho_{ii}, \quad i=1,2,3,\\
\tilde{\rho}_{12}&=\rho_{12}\exp(i\omega_1 t),\\
\tilde{\rho}_{23}&=\rho_{23}\exp(i\omega_2 t),\\
\tilde{\rho}_{13}&=\rho_{13}\exp[i(\omega_1-\omega_2)t].
\end{aligned} \tag{6}$$

The equations of motion for system (b) are given by

$$\begin{aligned}
\dot{\tilde{\rho}}_{11}&=2\gamma_1\tilde{\rho}_{22}-2\nu\tilde{\rho}_{11}-iG_1\tilde{\rho}_{12}+iG_1^*\tilde{\rho}_{21},\\
\dot{\tilde{\rho}}_{13}&=-[\nu+i(\Delta_1-\Delta_2)]\tilde{\rho}_{13}+iG_1^*\tilde{\rho}_{23}-iG_2\tilde{\rho}_{12},\\
\dot{\tilde{\rho}}_{21}&=-(\gamma_1+\gamma_2+\nu-i\Delta_1)\tilde{\rho}_{21}+iG_1(\tilde{\rho}_{11}-\tilde{\rho}_{22})+iG_2\tilde{\rho}_{31},\\
\dot{\tilde{\rho}}_{22}&=-2(\gamma_1+\gamma_2)\tilde{\rho}_{22}+iG_1\tilde{\rho}_{12}-iG_1^*\tilde{\rho}_{21}+iG_2\tilde{\rho}_{32}-iG_2^*\tilde{\rho}_{23},\\
\dot{\tilde{\rho}}_{23}&=-(\gamma_1+\gamma_2-i\Delta_2)\tilde{\rho}_{23}+iG_1\tilde{\rho}_{23}-iG_2(\tilde{\rho}_{22}-\tilde{\rho}_{33}),\\
\dot{\tilde{\rho}}_{33}&=2\nu\tilde{\rho}_{11}+2\gamma_2\tilde{\rho}_{22}-iG_2\tilde{\rho}_{32}+iG_2^*\tilde{\rho}_{23}.
\end{aligned} \tag{7}$$

The parameters $2G_1=2\mathbf{d}_{12}\cdot\mathbf{E}_1/\hbar$ and $2G_2=2\mathbf{d}_{23}\cdot\mathbf{E}_2/\hbar$ are the Rabi frequencies associated with the laser fields $\mathbf{E}_1$ and $\mathbf{E}_2$ respectively. The detunings of the field from the atomic transitions are given by $\Delta_1=\omega_{12}-\omega_1$ and $\Delta_2=\omega_{23}-\omega_2$. It is the field at frequency ω_2 that circulates through the cavity and shows bistable behavior, hence we look at the induced polarization on the transition $|2\rangle\leftrightarrow|3\rangle$ which is given by

$$P(\omega_2)=nd_{32}\tilde{\rho}_{23}, \tag{8}$$

where n is the density of atoms.

In the ring cavity (Fig. 1) the coherent field E_2^I enters into the cavity from the semisilvered mirror 1, and drives the atomic sample. The control field further regulates the induced polarization $P(\omega_2)$ through interference effects, which alter the absorption and dispersion profiles of the active medium. The boundary conditions for the ring cavity impose the following conditions between the incident field E_2^I, the transmitted field E_2^T, the fields at different positions in the cavity $E_2(0,t)$ and $E_2(L,t)$:

$$\begin{aligned}
E_2^T(t)&=\sqrt{T}E_2(L,t),\\
E_2(0,t)&=\sqrt{T}E_2^I(t)+R\exp(-i\delta_0)E_2(L,t-\Delta t),
\end{aligned} \tag{9}$$

where L is the length of the sample and $\Delta t=(2l+L)/c$ is the time light takes to travel from mirror 2 to mirror 1. The cavity detuning $\delta_0=(\omega_c-\omega_0)L_T/c$, where ω_c is the fre-

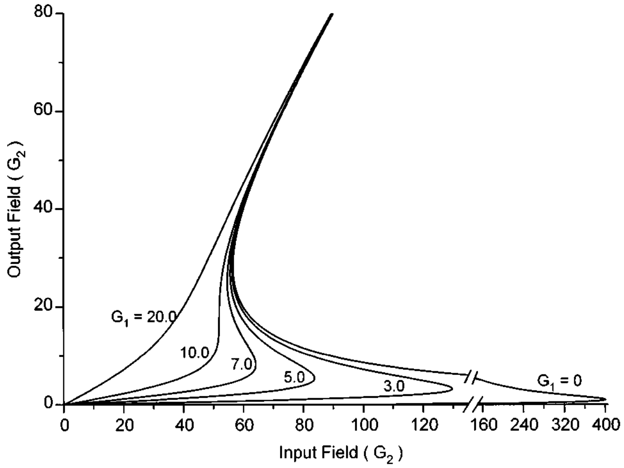

FIG. 3. The decrease in the threshold due to the control field for system (a) compared to the usual two-level bistable system. The threshold can be controlled by changing the control field $G_1/\gamma_2=3$, 5, 7, 10, and 20; $C=400$; and $\Delta_1=\Delta_2=0$. Note the possibility of transistor action for $G_1/\gamma_2=10$.

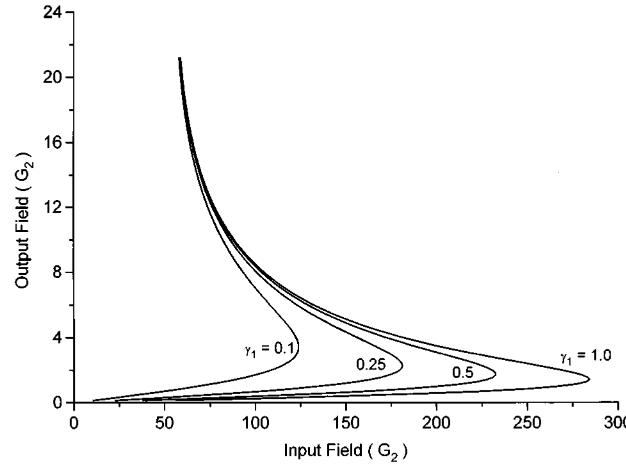

FIG. 4. The quantum-interference-effect-induced decrease in the threshold by having a long-lived state $|1\rangle$, i.e., $\gamma_1/\gamma_2=0.1$, 0.25, 0.5, and 1.0; $G_1/\gamma_2=1.0$; $C=400$; and $\Delta_1=\Delta_2=0$.

quency of the cavity nearest to resonance with the incident field frequency ω_0, and $L_T=2(l+L)$ is the total length of the cavity.

The field equation

$$\frac{\partial E_2}{\partial t}+c\,\frac{\partial E_2}{\partial z}=2\pi i\,\omega_2 d_{32}P(\omega_2), \tag{10}$$

with the boundary conditions (9) is solved in the steady-state limit, i.e., $(\partial E_2/\partial t)=(\partial\tilde{\rho}_{ij}/\partial t)=0$. Unlike the two-level system, where the induced polarization $P(\omega_2)$ reduces to a relatively simple analytical form, we resorted to solving the set of simultaneous coupled equations numerically. To obtain the polarization we solve the set of equations (5) or (7) for system (a) or (b), respectively, then using (8) we integrate equation (10) in the steady-state limit over the length of the sample. The boundary conditions used in the steady-state limit reduce (9) to

$$\begin{aligned} E_2^T&=\sqrt{T}E_2(L),\\ E(0)&=\sqrt{T}E_2^I+R\,\exp(-i\delta_0)E_2(L). \end{aligned} \tag{11}$$

One should note that in the limit that the control field $G_1\rightarrow 0$ for system (a), and the multiple limit $G_1\rightarrow 0$ and $\gamma_1\rightarrow 0$ for system (b), both systems reduce to the conventional two-level scheme, where the absorption coefficient α on the transition $|2\rangle\leftrightarrow|3\rangle$ of the system is given by

$$\alpha=\frac{4\pi\omega_{32}d_{32}^2 n}{\hbar c\gamma_2}. \tag{12}$$

In order to compare the results with the two-level system, we define the usual cooperation parameter C as $C=(\alpha L/2T)$, the same as in the two-level system [9,11].

III. ELECTROMAGNETIC CONTROL OF OPTICAL BISTABILITY

In this section we present details of our numerical results. We do not make use of the mean-field approximation. Figure 3 gives the bistable behavior of the two-level system subjected to a control field on the upper transition. Clearly the control field leads to a lowering of the bistability threshold, owing to the Autler-Townes splitting. The absorption at the line center decreases with an increase in the control field G_1. When the control field becomes too large then bistability disappears, as for large G_1 there is hardly any linear absorption, and even the linear dispersion at the line center is zero.

For the ladder system of Fig. 2(a), the linear susceptibility on the transition $|2\rangle\leftrightarrow|3\rangle$ is to all orders in the strength of the control field G_1, and the first order in the cavity field G_2 is given by [12]

$$\chi_{(a)}(\omega_2)\propto\left[(\Delta_2-i\gamma_2)-\frac{G_1^2}{(\Delta_1+\Delta_2-i\gamma_1)}\right]^{-1}. \tag{13}$$

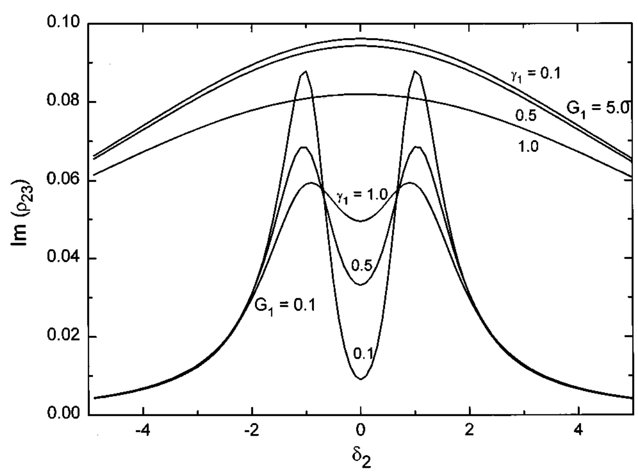

FIG. 5. The all-order nonlinear absorption for different values of the cavity field $G_2/\gamma_2=0.1,5$. The atomic coherence decay from the upper level plays a small role for large G_2.

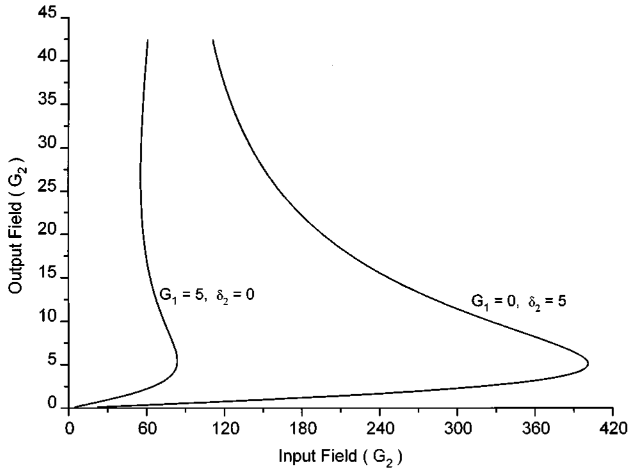

FIG. 6. For the same level of absorption, i.e., $\mathrm{Im}(\rho_{23})=3.85\times 10^{-3}$, the appropriately detuned two-level system ($G_1/\gamma_2=0$, $\Delta_1=0$, and $\Delta_2=5$) requires a much larger threshold than the three-level scheme (a) with the control field for $C=400$, $G_1/\gamma_2=5.0$, and $\Delta_1=\Delta_2=0$.

In the absence of the control field the $\chi_{(a)}(\omega_2)$ in (13) has only the first term; the second term is due to the interference between the two possible absorption channels of the field at the frequency ω_2 to the dressed state doublet created by G_1 on the transition $|1\rangle\leftrightarrow|2\rangle$. Due to this interference effect, the absorption of population at $|3\rangle$ is now dependent on the decay of the coherence ρ_{13}, though the transition $|1\rangle\leftrightarrow|3\rangle$ is not dipole allowed. When $\Delta_1=\Delta_2=0$, then

$$\chi_{(a)}(\omega_2)=\frac{i\gamma_1}{G_1^2+\gamma_1\gamma_2}. \tag{14}$$

The absorption at the line center again decreases with a decrease in γ_1. Thus if level $|1\rangle$ is long lived, then we can achieve a further lowering of the bistability threshold. This is demonstrated in Fig. 4. We do not expect any change in the second bistability threshold (i.e., the threshold for switching from the on state to the off state), as here the field G_2 becomes large, thereby offsetting the advantage of a long-lived state $|1\rangle$. This is also demonstrated by calculating the all order response function on the $|2\rangle\leftrightarrow|3\rangle$ transition. The result of this calculation is shown in Fig. 5. We also compare the bistability in a two-level system where the input field is detuned by an amount so that the linear absorption is the same as that at the line center in the presence of the control field. As Fig. 6 shows, there is a substantial gain in using a control field.

We next present modifications in bistability characteristics if the control field is applied on the transition as in Fig. 2(b). In the absence of the parameter ν, coherent population trapping [13] occurs if the two detunings are equal; otherwise, on resonance, the absorption is again proportional to the decay ν

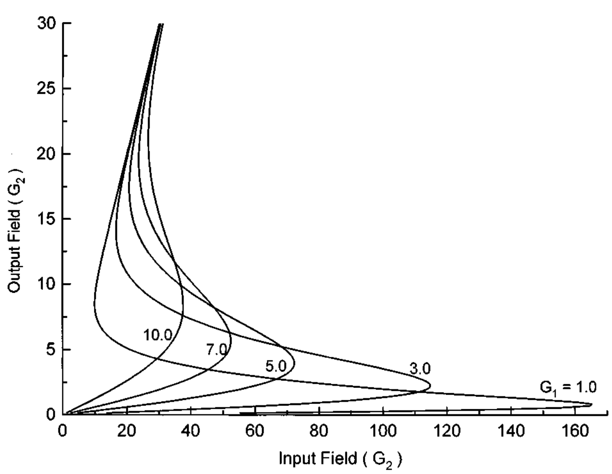

FIG. 7. Effect of quantum interferences for system (b) for $\nu=0$, $C=400$, $\Delta_1=0$, $\Delta_2=1$, and $G_1/\gamma_2=1$, 3, 5, 7, and 10. The behavior is similar to that for the ladder system (Fig. 4).

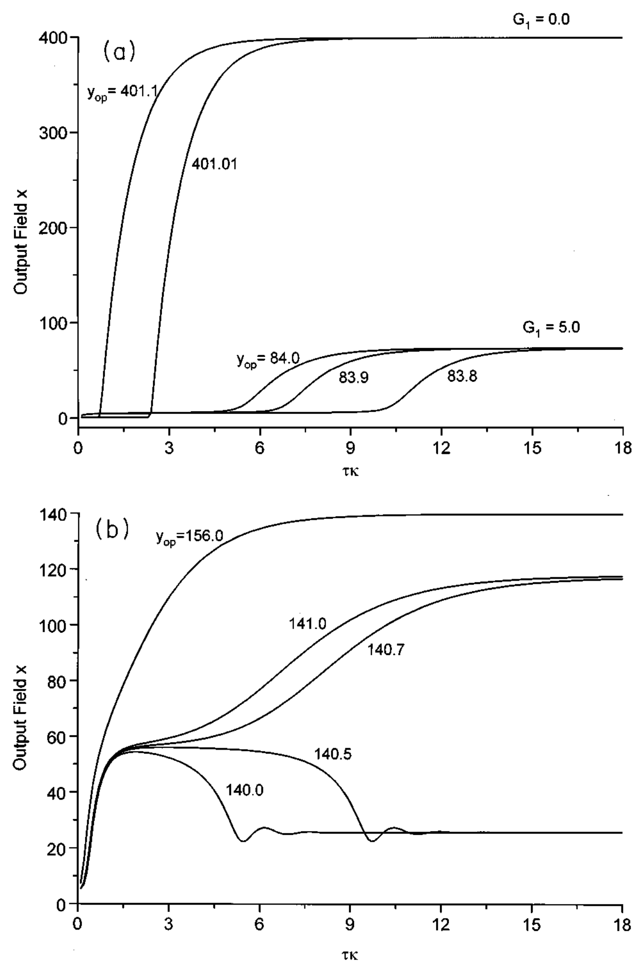

FIG. 8. (a) Comparison between the two-level scheme and the proposed schemes; the rise time is equal to ($\tau\kappa\sim 5.0$). The temporal evolution is shown for different operating points in the two-level system $y_{\mathrm{op}}=401.01$ and 401.1; and for system (a) $y_{\mathrm{op}}=83.8$, 83.9, and 84.0, one observes a critical slowing down phenomenon as one approaches the switch up intensity. We choose $C=400$ and $\Delta_1=\Delta_2=0$. (b) Switching to the second stable state at $y_{\mathrm{op}}=140$ and 140.5 is oscillatory with a small rise time ($\lesssim\tau\kappa=5.0$). Switching to the third stable state at $y_{\mathrm{op}}=140.7$ and 141 takes a longer time ($\gtrsim\tau\kappa=15.0$). Also, the operating point beyond the multistable region $y_{\mathrm{op}}=156$ takes a similar time ($\sim\tau\kappa=15.0$).

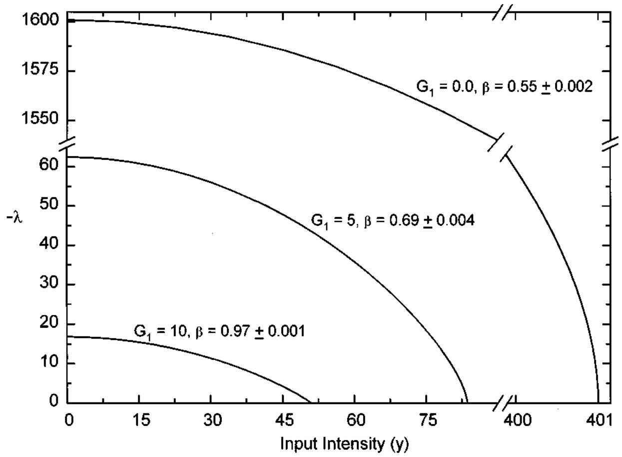

FIG. 9. The regression to the steady state is governed by the eigenvalue $\lambda \propto (y_{\rm th} - y)^{\beta}$, where β depends on the control field. With an increase in control field the system takes a longer time to return to its steady state.

of level $|1\rangle$ which is not dipole connected to level $|3\rangle$. The analog of (13) for the Λ system is

$$\chi_{(b)}(\omega_2) \propto \left[(-\Delta_2 - i[\gamma_2 + \gamma_1]) - \frac{G_1^2}{(\Delta_1 - \Delta_2 - i\nu)} \right]^{-1}. \tag{15}$$

Figure 7 gives the changes in the bistability characteristics as the strength of the control field is changed. We have chosen $\Delta_1 = 0$, $\nu = 0$, and $\Delta_2 \neq 0$. The results are somewhat similar to those for the situation of Fig. 2(a).

IV. CONTROL-FIELD-INDUCED CHANGES IN TRANSIENT RESPONSE

For simplicity we consider the transient response of the system in the mean-field approximation, i.e., in the multiple limit $\alpha L \to 0$, $T \to 0$, and $\delta_0 \to 0$. It is called the mean-field limit as the field inside the cavity does not change very much in each pass due to the weak coupling ($\alpha L \to 0$), but the mean lifetime ($L_T / cT, T \to 0$) of the photons in the cavity is large. Thus the photons even in this weak-coupling limit experience substantial interaction with the atoms due to the many passes they make through the sample owing to their long lifetimes. The limit $\delta_0 \to 0$ implies that the cavity detuning is smaller than the free spectral range, thus ensuring that we operate in a cavity mode resonant with the incident field. It should be noted that our previous discussion on control of the switching threshold is extremely general, and no such approximation is made.

The time evolution of the transmitted field in a good cavity with $\delta_0 = 0$ is given by

$$\kappa^{-1} \frac{\partial x}{\partial t} = -(x - y) - 2CP_0, \tag{16}$$

where $\kappa^{-1} = cT / L_T$ is the cavity lifetime, x and y are the normalized amplitudes of the transmitted and incident fields, respectively, $\gamma_1 = \gamma_2 = \gamma$, $x = d_{32} E^T / (2\hbar^2 \gamma^2 T)^{1/2}$, $y = d_{32} E^I / (2\hbar^2 \gamma^2 T)^{1/2}$, and P_0 is the normalized nonlinear response of the medium defined as $P_0 = [P(\omega_2)/i](N/\sqrt{2})^{-1}$

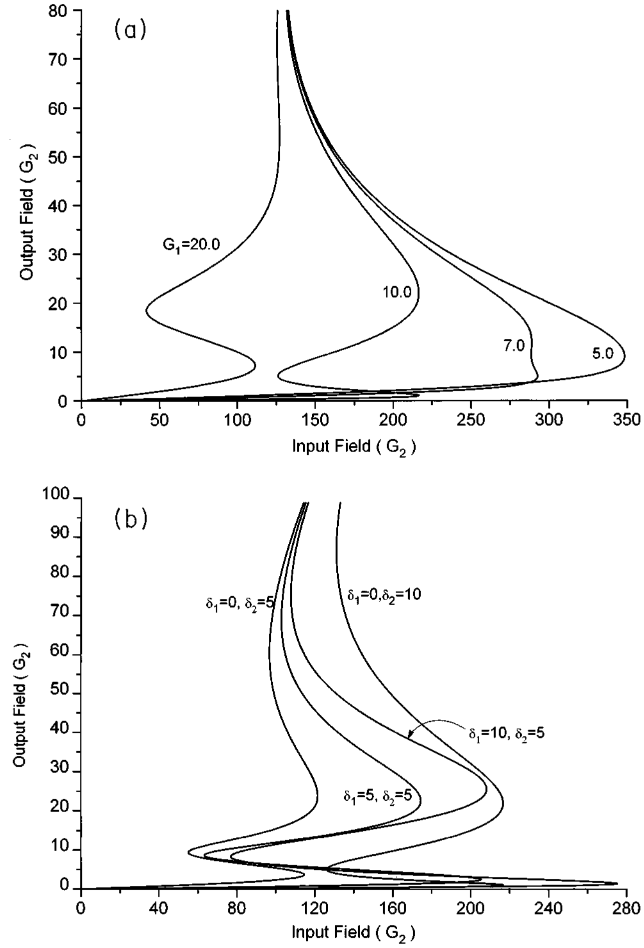

FIG. 10. (a) Control-field-induced multistability for $C = 400$, $\Delta_1 = 0$, $\Delta_2 = 10$, and $G_1/\gamma_2 = 5$, 7, 10, and 20. (b) The region of multistability could be relocated by varying $\Delta_2 = 5$ and 10. The region of input intensities over which there is multistability is broadened by increasing $\Delta_1 = 0$, 5, and 10. Here $C = 400$ and $G_1/\gamma_2 = 10$.

[9]. The transmitted amplitude x is complex, whereas the input field y is assumed to be real. We initially begin with no input field, i.e., $y = 0$ and $x = 0$, and then set the input field y to an operating point $y_{\rm op}$ and observe the dynamical evolution of the transmitted field by integrating (16) until the output field reaches its steady-state value corresponding to the input field $y_{\rm op}$. P_0 in (16) is derived by inverting the equations (5) or (7) for scheme (a) or (b) respectively. The transient response with and without the control field has a very similar behavior, as shown in Fig. 8(a). As is shown in Sec. V, multistability [14] is observed for an appropriate choice of the control field strength and detunings. The transient study shows that if we choose the operating point in the multistable region, the output switches to the second stable state in an oscillatory fashion but with a fast response time, Fig. 8(b). The switching times are relatively longer for switching from the second to the third stable state, similar to that of an operating point chosen beyond the multistable region.

When the system is slightly displaced from its stationary state, the regression to the steady state is governed by the eigenvalues of the relaxation matrix obtained from linearizing (16) around the steady state. In (16) we consider $x \to x_0 + \delta x$, where x_0 is the steady-state value and δx is small perturbation such that $\delta x \ll x_0$. We expand $P_0(x, x^*)$ in the Taylor series, and take only the linear terms in δx, i.e.,

$$P_0(x_0+\delta x, x_0^*+\delta x^*) = P_0(x_0, x_0^*) + \delta x\left(\frac{\partial P_0(x_0)}{\partial x_0}\right) + \delta x^*\left(\frac{\partial P_0(x_0)}{\partial x_0}\right)^*. \quad (17)$$

On linearizing (16), we obtain the following eigenvalue equation:

$$\kappa^{-1}\frac{\partial}{\partial t}\begin{bmatrix}\delta x\\ \delta x^*\end{bmatrix} = -\begin{bmatrix} 1+2C\left(\frac{\partial P_0}{\partial x_0}\right) & 2C\left(\frac{\partial P_0}{\partial x_0}\right)^* \\ 2C\left(\frac{\partial P_0}{\partial x_0}\right) & 1+2C\left(\frac{\partial P_0}{\partial x_0}\right)^* \end{bmatrix} \times \begin{bmatrix}\delta x\\ \delta x^*\end{bmatrix}. \quad (18)$$

The lowest eigenvalue of (18) governs the rate of decay to the steady state. As $y \rightarrow y_{\mathrm{th}}$, $\lambda \rightarrow 0$, since the regression to the steady state goes as $\exp(\lambda t)$ we observe the phenomenon of critical slowing down as we approach the threshold point, Fig. 9. The control field changes the slope of the curves in Fig. 9, and the general behavior of the eigenvalue λ near the threshold for the two-level system has the usual $\lambda \propto (y_{\mathrm{th}}-y)^\beta$ dependence; however, *β now depends on the control field*, and with the increase in G, the system takes longer time to revert back to its steady-state value.

V. CONTROL-FIELD-INDUCED MULTISTABILITY

In the presence of the control field, polarization of the medium is found to be the ratio of two polynomials of order 5 and 6 in G_2,

$$P_0(G_2) = \frac{G_2[c_1+c_2|G_2|^2+c_3|G_2|^4]}{[c_4+c_5|G_2|^2+c_6|G_2|^4+c_7|G_2|^6]}, \quad (19)$$

where $c_1 - c_7$ are complex parameters that depend on the detunings, lifetimes, control field strength, etc. For the usual two-level system we have only the first-order term in the numerator, and the term up to the second order in G_2 in the denominator, giving rise only to bistability. Hence in general in the appropriate domain of the control field, detunings, etc. (i.e., c_i's), one has the possibility of multistability. In Fig. 10 we show some representative situations for different strengths and detunings. By changing the external field parameters we can change the region in which this multistate switch can operate.

In conclusion, we have demonstrated schemes with which one can decrease substantially the threshold required for a bistable device using field-induced transparency and quantum interference effects. Such schemes would be very much realizable, and would lead to more efficient devices. Multiple hysteresis for an appropriate choice of control field parameters provides for a possibility of versatile use of the same device configuration in more than one application.

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