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Complementarity and quantum erasure with dispersive atom-field interactions

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The "*welcher Weg*" (which-path) detector as described by Scully *et al.* [Nature **351**, 111 (1991)] employs a pair of initially empty micromaser cavities placed in front of a double-slit apparatus in an atomic interferometer. Laser excited atoms spontaneously emit a photon into either cavity thereby marking the atoms' path and thereby destroying the interference. I propose an alternative method wherein at least one of the cavities is prepared in a coherent state with a strong amplitude. Which-path information is obtained by a nonresonant, dispersive type of atom-field interaction associated with quantum nondemolition measurements. Ground-state atoms passing through the cavity remain in the ground state but impart a phase shift to the cavity fields. Velocity selection is shown to affect the visibility of the fringes. An associated quantum eraser is also discussed.

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Recently there has been much interest in demonstrating Bohr complementarity [1] while avoiding uncontrollable, irreversible interactions associated with the measurement process. The prototype for demonstrating complementarity is, of course, the double-slit experiment wherein the particlelike or wavelike behavior is observed depending on whether or not respectively, "*welcher-Weg*" (which-path) detectors are present [2]. In Einstein's version involving recoiling slits [3], it is Heisenberg's uncertainty principle associated with the complementarity variables x and p_x that is responsible for wiping out which-path information. However, it has been shown that the uncertainty relation is not the only mechanism by which complementary is enforced. Scully *et al.* [4] have studied a micromaser which-path detector for an atomic beam. A plane wave of atoms is incident on wide double slits behind which are a set of collimators which direct the resultant two beams into a pair of high-quality micromasers. Upon emerging from the micromasers the beams illuminate two narrow double slits from which originates an interference pattern on the screen if no which-path information is available (Fig. 1). Without such information, the center-of-mass wave function for the atoms near the screen is

$$\Psi(\vec{r}) = \frac{1}{\sqrt{2}}[\psi_1(\vec{r}) + \psi_2(\vec{r})]|i\rangle, \tag{1}$$

where ψ_1 and ψ_2 are the center-of-mass wave functions associated with paths 1 and 2 and $|i\rangle$ is an internal state of the atom. The probability density for the atoms striking the screen is

$$\begin{aligned} P(\vec{r}) &= |\Psi(\vec{r})|^2 \\ &= \tfrac{1}{2}[|\psi_1(\vec{r})|^2 + |\psi_2(\vec{r})|^2 + \psi_1^*(\vec{r})\psi_2(\vec{r}) \\ &\quad + \psi_2^*(\vec{r})\psi_1(\vec{r})]\langle i|i\rangle, \end{aligned} \tag{2}$$

the cross terms $\psi_1^*\psi_2 + \psi_2^*\psi_1$ giving rise to the interference. On the other, if which-path information is available, the interference will be removed. Let $|D_1\rangle$ and $|D_2\rangle$ represent the states of the cavities (the detectors) placed in front of the double slit. In the scheme of Scully *et al.* [4] the atoms are assumed to have ground and excited states, $|g\rangle$ and $|e\rangle$, respectively, such that the frequency of transitions between these states ω_a is resonance with the frequency of the cavity mode ω_c. Before the atoms enter the cavities, a laser excites them to the state $|e\rangle$. The atoms are assumed to be Rydberg atoms so that state $|e\rangle$ is long lived. It is further assumed that the cavities are initially empty. Upon passing through the cavities the atoms make spontaneous emissions to the ground state, emitting photons into either cavity 1 or 2. The center-

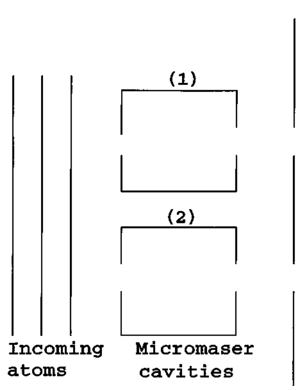

FIG. 1. Double-slit configuration with micromaser cavities as path detectors.

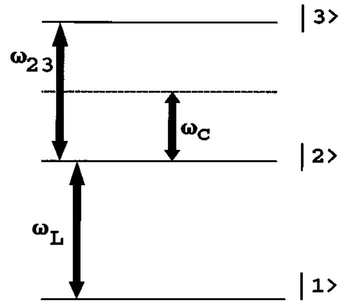

FIG. 2. Atomic-energy-level configuration. Levels 1 and 2 are coupled by laser excitation of resonant frequency ω_L. Levels 2 and 3 are coupled nonresonantly to the cavity field.

of-mass motion is unaffected. In this case, the atoms and micromaser cavities are in the correlated (entangled) state

$$\Psi(\vec{r})=\frac{1}{\sqrt{2}}[\psi_1(\vec{r})|D_1\rangle+\psi_2(\vec{r})|D_2\rangle]|g\rangle, \tag{3}$$

where $|D_1\rangle=|1_1\rangle|0_2\rangle$ denotes the state with one photon in cavity 1, the vacuum in cavity 2, and vice versa for $|D_2\rangle=|0_1\rangle|1_2\rangle$. The probability density on the screen is now

$$P(\vec{r})=\tfrac{1}{2}[|\psi_1(\vec{r})|^2+|\psi_2(\vec{r})|^2+\psi_1^*(\vec{r})\psi_2(\vec{r})\langle D_1|D_2\rangle + \psi_2^*(\vec{r})\psi_1(\vec{r})\langle D_2|D_1\rangle]\langle g|g\rangle. \tag{4}$$

Since $\langle D_1|D_2\rangle=\langle D_2|D_1\rangle=0$, the coherence in the atomic beam is lost and the interference disappears:

$$P(\vec{r})=\tfrac{1}{2}[|\psi_1(\vec{r})|^2+|\psi_2(\vec{r})|^2]. \tag{5}$$

On the other hand, if the cavities are prepared in coherent states, the emission of one photon has little effect on the cavity fields, and interference is again possible depending on the length of the interaction time [5]. At long interaction times the interference disappears due to the dephasing of the Rabi oscillations in the atom-field interaction. It is interesting to note that the decoherence of the atomic beam occurs even when no which-path information is available.

In this paper, I propose an alternative which-path micromaser detector using the ideas related to the quantum nondemolition (QND) measurement of the photon number of cavity fields [6] and the generation of macroscopic superposition states (Schrödinger-cat states) [7]. I assume the cavities are prepared in coherent states $|\alpha_1\rangle$ and $|\alpha_2\rangle$, where

$$|\alpha\rangle=e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \tag{6}$$

and where at least one of the field amplitudes $|\alpha_1|^2$ or $|\alpha_2|^2$ is large. Such fields can be generated by driving the cavities with classical currents. I further assume a beam of atoms, as in Fig. 1, passes through the cavities and that ω_c is the cavity resonant frequency. The level structure of the atom is given in Fig. 2. The $|2\rangle\leftrightarrow|3\rangle$ transition is coupled to the cavity field; however, the detuning is taken to be large. That is, if ω_{23} is the corresponding atomic transition frequency then $|\Delta|=|\omega_{23}-\omega_c|$ is so large that only virtual transitions occur between states $|2\rangle$ and $|3\rangle$. Letting $a_1(a_1^\dagger)$ and a_2 $(a_2^\dagger)$ represent the annihilation (creation) operators of the modes of the two cavities, the effective interaction Hamiltonian for the atom-field interaction is [8]

$$H_I^i=\hbar\eta_i a_i^\dagger a_i\sigma_z^{23}, \qquad i=1,2, \tag{7}$$

where $\sigma_z^{23}=|3\rangle\langle 3|-|2\rangle\langle 2|$, $\eta_i=\hbar^2/2\Delta_i$, and where λ is the atomic dipole moment and Δ_i is the detuning of the ith cavity. The above Hamiltonian is valid under the assumption that $\lambda^2 n\ll\Delta_i^2+\gamma$, where n is a characteristic photon number and γ is the spontaneous-emission rate [8]. This type of interaction has been previously discussed in connection with QND measurements of photon numbers [6,7].

I assume that the atom is laser pumped to state $|2\rangle$, also a long-lived Rydberg state. Using the relation

$$e^{\pm i\phi a^\dagger a}|\alpha\rangle=|\alpha e^{\pm i\phi}\rangle, \tag{8}$$

after the atom passes through the cavity the atom-cavity state has again the form of Eq. (3) but now with the detector states

$$|D_1\rangle=|\alpha_1 e^{i\phi_1}\rangle|\alpha_2\rangle,$$
$$|D_2\rangle=|\alpha_1\rangle|\alpha_2 e^{i\phi_2}\rangle, \tag{9}$$

where $\phi_1=\eta_1 t_1$ and $\phi_2=\eta_2 t_2$, t_1 and t_2 being the atom-cavity interaction times. These interaction times can be adjusted by the velocity selection of the atoms. If L_i is the length of the ith cavity then $t_i=L_i/v$, where v is the velocity of the atom. It is the alternation of the phase of the coherent state that tags the path of the atom.

Now

$$\langle D_1|D_2\rangle=\langle D_2|D_1\rangle^*=\langle\alpha_1 e^{i\phi_1}|\alpha_1\rangle\langle\alpha_2|\alpha_2 e^{i\phi_2}\rangle =\exp[-|\alpha_1|^2(1-e^{-i\phi_1})-|\alpha_2|^2(1-e^{i\phi_2})]. \tag{10}$$

With $\phi_1=\phi_2=$(odd integer)$\times\pi$ we have

$$\langle D_1|D_2\rangle=\exp[-2(|\alpha_1|^2+|\alpha_2|^2)]\simeq 0 \tag{11}$$

for $|\alpha_1|$ and/or $|\alpha_2|$ large so that interference disappears. On the other hand, for $\phi_1=\phi_2=$(even integer)$\times\pi$, $\langle D_1|D_2\rangle=1$ and interference reappears. Thus the visibility of the interference fringes can be modulated by the velocity selection of the atoms which in turn determines the phase shifts of the cavity fields through the dispersive interaction.

Ideally one should have identical cavities so that $\eta_1=\eta_2$ and arranged so that $t_1=t_2$. However, it is simpler to have just one cavity, say, cavity 1, in a coherent state and the other cavity in the vacuum ($\alpha_2=0$) or equivalently no second cavity at all. In this case the atom-cavity state is

$$\Psi(r)=\frac{1}{\sqrt{2}}[\psi_1(\vec{r})|\alpha_1 e^{i\phi_1}\rangle+\psi_2(\vec{r})|\alpha_1\rangle]|0_2\rangle|g\rangle, \tag{12}$$

where the detector states are now $|D_1\rangle=|\alpha_1 e^{i\phi_1}\rangle$ and $|D_2\rangle=|\alpha_1\rangle$ such that

$$\langle D_1|D_2\rangle=\exp[-|\alpha_1|^2(1-e^{-i\phi_1})]. \tag{13}$$

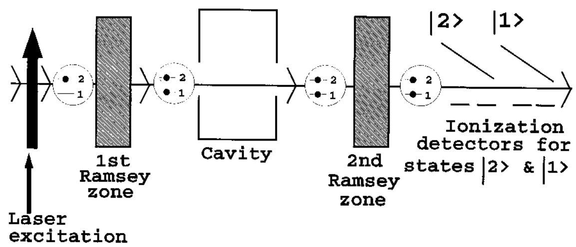

FIG. 3. Setup for the measurement of the parity of the cavity field. The atom is laser excited to state $|2\rangle$ and the first Ramsey zone creates a superposition of states $|1\rangle$ and $|2\rangle$. Passage through the cavity alters the superposition. The second Ramsey zone analyzes the superposition, after which the zone state is detected by selective ionization.

Since only one phase now appears, velocity selection for controlling the visibility of the fringes depends only on the parameters of one cavity.

Finally, I indicate how the quantum eraser [9] idea can be implemented in the present scheme (see Ref. [4] for the scheme of Scully *et al.*). I consider the case with only cavity 1 containing a coherent state with cavity 2 containing the vacuum. Further, I assume that $\phi_1=\pi$ so that the atom-cavity state is

$$\Psi(\vec{r})=\frac{1}{\sqrt{2}}[\psi_1(\vec{r})|-\alpha_1\rangle+\psi_2(\vec{r})|\alpha_1\rangle]|0_2\rangle|g\rangle \quad (14)$$

with $|\alpha_1|$ large enough so that $\langle D_1|D_2\rangle=\langle-\alpha_1|\alpha_1\rangle\simeq 0$, i.e., no interference fringes. It is convenient to define the symmetric and antisymmetric superpositions of coherent field states

$$|S_\pm\rangle=\frac{1}{N_\pm}(|\alpha_1\rangle\pm|-\alpha_1\rangle), \quad (15)$$

where $N_\pm=\sqrt{2(1\pm e^{-2|\alpha_1|^2})}\approx\sqrt{2}$. Then Eq. (14) may be rewritten as

$$\Psi(\vec{r})=\frac{1}{\sqrt{2}}[\psi_+(\vec{r})|S_+\rangle+\psi_-(\vec{r})|S_-\rangle]|0_2\rangle|g\rangle, \quad (16)$$

where $\psi_\pm=\psi_1\pm\psi_2$. Now the states $|S_\pm\rangle$ are also known as the even and odd coherent states [10], special cases of Schrödinger cat states [11]. If we correlate the atom with the cavity field $|S_+\rangle$, the symmetric interference fringes of Eq. (2) will reappear. If we correlate the atom with the state $|S_-\rangle$ we obtain the antisymmetric fringes

$$P(\vec{r})=\tfrac{1}{2}|\psi_-|^2=\tfrac{1}{2}[|\psi_1|^2+|\psi_2|^2-\psi_1^*\psi_2-\psi_2^*\psi_1]. \quad (17)$$

Since the states $|S_\pm\rangle$ contain only even (+) or odd (−) photon number states it should be sufficient to discover the parity of the cavity field. A procedure for determining this parity has been given by Englert *et al.* [12], which is very closely related to the methods of Brune *et al.* [7] for generating even and odd cat states in a cavity. I adapt these methods here.

I imagine now a second atom passing through the cavity, say, at right angles to the first, in a setup pictured in Fig. 3. A laser excites the atom to level 2 and the microwave Ramsey zones [13] M_1 and M_2 cause the transitions

$$|2\rangle_2\rightarrow\frac{1}{\sqrt{2}}[|2\rangle_2+ie^{i\theta_j}|1\rangle_2] \qquad j=1,2 \text{ (Ramsey zones)},$$

$$|1\rangle_2\rightarrow\frac{1}{\sqrt{2}}[|1\rangle_2+ie^{-i\theta_j}|2\rangle_2] \quad (18)$$

where the subscript 2 on the atom state refers to the second atom. After the first Ramsey zone the atom is in state

$$|\psi_{\text{atom2}}\rangle=\frac{1}{\sqrt{2}}[|2\rangle_2+ie^{i\theta_1}|1\rangle_2]. \quad (19)$$

If the cavity field is in the number state $|n\rangle$ then after the atom passes through the cavity, using the interaction Hamiltonian $H_I=\hbar\eta a_1^\dagger a_1\sigma_z^{23}$ we have

$$|\psi_{\text{atom2-field}}\rangle=\frac{1}{\sqrt{2}}[e^{i\eta nt}|2\rangle_2+ie^{i\theta_1}|1\rangle_2]|n\rangle. \quad (20)$$

(Recall that only level $|2\rangle$ couples to the cavity field.) After the second Ramsey zone M_2 using Eq. (18) we have

$$|\psi_{\text{atom2-field}}\rangle=\tfrac{1}{2}\{-i[e^{i(\theta_1-\theta_2)}+e^{i\eta nt}]|1\rangle_2 -[e^{i(\theta_1-\theta_2)}-e^{i\eta nt}]|2\rangle_2\}|n\rangle. \quad (21)$$

With the choices $\exp[i(\theta_1-\theta_2)]=1$, $\eta t=\pi$, we have

$$|\psi_{\text{atom2,field}}\rangle=\tfrac{1}{2}\{i[1+(-1)^n]|1\rangle_2+[1-(-1)^n]|2\rangle_2\}|n\rangle =\begin{cases} i|n\rangle|1\rangle_2, & n \text{ even}\\ -|n\rangle|2\rangle_2, & n \text{ odd.}\end{cases} \quad (22)$$

Now applied to the combined state of Eq. (16) we obtain after the passage of the second atom

$$\psi(\vec{r},t_p)=\frac{1}{\sqrt{2}}[i\psi_+(\vec{r})|S_+\rangle|1\rangle_2-\psi_-(\vec{r})|S_-\rangle|2\rangle_2], \quad (23)$$

where we have ignored the first atom and the second cavity vacuum field and t_p is the time of passage. Now if the second atom is detected in the ground state $|1\rangle$ this is clearly correlated with field being in the state $|S_+\rangle$ and the interference fringes are revived. On the other hand if the atom is detected in the state $|2\rangle$ the antisymmetric fringes will appear. Thus the detection of the parity of the cavity field by a dispersive atomic probe provides a manifestation of a quantum eraser.

Note added: After this paper was submitted I learned that Storey *et al.* [14] also considered a which-path detector based on dispersive interaction but in a different configuration.

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