

Inversionless lasing and photon statistics in a V-type atomic system

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(Received 8 June 1995)

We analyze intensity and statistical properties of lasers without population inversion in a closed three-level V-type system. We derive the threshold condition and examine the intensity dependence on various system parameters. Unlike other inversionless laser systems that generate amplitude-squeezed light, the intensity fluctuation of the inversionless laser from the three-level V-type system is above the shot-noise limit in a wide parameter range studied here.

PACS number(s): 42.55.Ah, 42.50.Hz, 32.80.Bx

I. INTRODUCTION

Recently considerable attention has been directed to the study of lasing without the requirement of population inversion (LWI). Quite a few models have been proposed, and the conditions for the onset of lasing action have been examined [1–10]. Experimentally, laser action related to a noninverted population in a strongly driven two-level system has been demonstrated before [11–13], and light amplification without population inversion in multilevel atomic systems has been reported in a number of recent publications [14–18]. Interestingly, the optical coherence and quantum interference associated with the light amplification may lead to unusual statistical properties in inversionless lasers. Agarwal showed that lasers without inversion may have a narrower linewidth than that of conventional lasers [19]. Gheri and Walls [20] found that amplitude-squeezed light can be generated in an inversionless, three-level Λ system. Amplitude-squeezed lasing may also be found in a four-level cyclic atomic system pumped by a single coherent field [21].

Sub-Poissonian photon statistics have been measured in diode lasers with noise-suppressed pump current [22]. Recently it has been shown that sub-Poissonian light can also be generated by dynamic pump noise suppression [23–26]. The basic principle is that the recycling of many incoherent steps leads to highly regular pumping, and results in sub-Poissonian photon statistics. In an inversionless laser, two factors may contribute to the noise deduction: first, the fast coherent cycling of electrons between states connected by a two-phonon scattering process leads to the highly regulated absorption and emission processes; second the disappearance of the population inversion leads to a depleted atomic population in the upper lasing state, which decreases the spontaneous emission noises. The combination of these two mechanisms can reduce the laser amplitude noise to below the shot noise limit. For laser without inversion in the three-level Λ system, overall the maximum amplitude squeezing of 50% is predicted [20]. For the four-level inversionless system, the amplitude squeezing may reach a level of more than 50% below the shot-noise limit [21]. Naturally, a question arises: does an inversionless laser always generate amplitude-squeezed light? In this paper, we present an analysis of laser

intensity and statistical properties of an inversionless three-level V system, and show that under reasonable operating conditions, no amplitude-squeezed light can be generated in the inversionless V system.

II. THREE-LEVEL V-TYPE SYSTEM

Our model consists of an ensemble of N closed three-level V-type atoms confined in an optical cavity with photon loss rate 2κ . The atoms have ground state $|1\rangle$, and excited states $|2\rangle$ and $|3\rangle$, as illustrated in Fig. 1. The transition $|1\rangle \leftrightarrow |2\rangle$ of frequency ω_{21} is driven by a strong coherent field of frequency ω_1 with Rabi frequency 2Ω . The transition $|1\rangle \leftrightarrow |3\rangle$ of frequency ω_{31} is incoherently pumped with a rate Λ . g is the cavity-atom coupling coefficient. $2\gamma_{ij}$ is the spontaneous decay rate from state $|i\rangle$ to state $|j\rangle$. We treat classically the external coherent field which drives the transition $|1\rangle \leftrightarrow |2\rangle$, but keep the cavity field quantized. In the electric-dipole and rotating-wave approximations, the system Hamiltonian (setting $\hbar = 1$) can be written as

$$\hat{H} = \sum_{j=1}^N \{ \omega_{31} \hat{\sigma}_{33j} + \omega_{21} \hat{\sigma}_{22j} + \Omega (e^{-i\omega_1 t} \hat{\sigma}_{21j} + e^{i\omega_1 t} \hat{\sigma}_{12j}) \} + \sum_{j=1}^N g (\hat{a} \hat{\sigma}_{31j} + \hat{a}^\dagger \hat{\sigma}_{13j}) + \omega_c \hat{a}^\dagger \hat{a}, \quad (1)$$

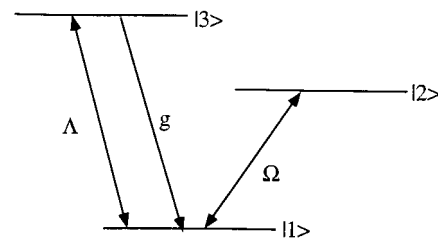


FIG. 1. V-type three-level model for lasing without population inversion. $|3\rangle \rightarrow |1\rangle$ is the lasing transition.

where

$$\begin{aligned}\hat{\sigma}_{12j} &= e^{-i\vec{k}_1 \cdot \vec{r}_j} |1_j\rangle\langle 2_j|, \quad \hat{\sigma}_{13j} = e^{-i\vec{k}_c \cdot \vec{r}_j} |1_j\rangle\langle 3_j|, \\ \hat{\sigma}_{23j} &= e^{-i(\vec{k}_c - \vec{k}_1) \cdot \vec{r}_j} |2_j\rangle\langle 3_j| \quad (j=1-N),\end{aligned}$$

and together with their Hermitian conjugates are the atomic raising or lowering operators with their phase factors for the j th atom. \vec{k}_1 (\vec{k}_c) is the k vector of the external driving field (cavity field). $\sigma_{mmj} = |m_j\rangle\langle m_j|$ ($m=1-3$, $j=1-N$) are the atomic population operators for the j th atom. \hat{a} (\hat{a}^\dagger) is the annihilation (creation) operator for the cavity photons. Ω_{lm} is the transition frequency from state $|l\rangle$ to state $|m\rangle$. We define the collective atomic polarization and population operators J_{lm} as

$$\hat{J}_{lm} = \sum_{j=1}^N \hat{\sigma}_{lmj} \quad (l, m=1-3). \quad (2)$$

Considering the closure of the system, we have $J_{11} + J_{22} + J_{33} = N$. We then choose the operators in a vector form with normal ordering such that

$$\begin{aligned}\vec{\hat{\alpha}} &= (\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \dots, \hat{\alpha}_{10}) \\ &= (\hat{a}, \hat{a}^\dagger, \hat{J}_{12}, \hat{J}_{13}, \hat{J}_{23}, \hat{J}_{22}, \hat{J}_{33}, \hat{J}_{23}^\dagger, \hat{J}_{13}^\dagger, \hat{J}_{12}^\dagger).\end{aligned} \quad (3)$$

The P representation in the ten-dimensional complex phase space can be defined as

$$P(\vec{\alpha}, t) = \text{Tr}\{\hat{\rho}(t) \delta^c(\vec{\alpha} - \vec{\alpha})\}, \quad (4)$$

where

$$\delta^c(\vec{\alpha} - \vec{\tilde{\alpha}}) = \prod_{i=1}^{10} \delta^2(\alpha_i - \tilde{\alpha}_i), \quad (5)$$

and α_i is the expectation value of operator $\hat{\alpha}_i$. In the rotating frame and with the use of the standard procedure [27], we derive the Fokker-Planck equation

$$\begin{aligned}\frac{\partial P(\vec{\alpha}, t)}{\partial t} &= \text{Tr}\left\{\frac{\partial \hat{\rho}(t)}{\partial t} \delta^c(\vec{\alpha} - \vec{\tilde{\alpha}})\right\} \\ &= \left\{-\frac{\partial}{\partial \alpha_\mu} A_\mu(\vec{\alpha}) + \frac{1}{2} \frac{\partial^2}{\partial \alpha_\mu \partial \alpha_\nu} D_{\mu\nu}(\vec{\alpha})\right\} P(\vec{\alpha}, t).\end{aligned} \quad (6)$$

We then transform the Fokker-Planck equation (6) with positive-definite diffusion matrix $D_{\mu\nu}(\vec{\alpha})$ into a set of stochastic differential equations. The stochastic differential equations are

$$\frac{d\vec{\alpha}}{dt} = \vec{A}(\vec{\alpha}) + \vec{F}(t), \quad (7)$$

where the noise correlation term F_i satisfies

$$\langle F_i(t) \rangle = 0 \quad (i=1-10) \quad (8a)$$

and

$$\langle F_i(t) F_j(t') \rangle = D_{ij}(\vec{\alpha}) \delta(t-t') \quad (i, j=1-10). \quad (8b)$$

D_{ij} are the diffusion coefficients derived from the Fokker-Planck equation (6). Explicitly, the stochastic differential equations are

$$\frac{d\alpha_1}{dt} = -[\kappa + i(\omega_c - \omega_l)]\alpha_1 - ig\alpha_4 + F_1,$$

$$\frac{d\alpha_2}{dt} = -[\kappa - i(\omega_c - \omega_l)]\alpha_2 + ig\alpha_9 + F_2,$$

$$\begin{aligned}\frac{d\alpha_3}{dt} &= -(\gamma_{21} + \Lambda/2 + i\Delta_1)\alpha_3 + i\Omega(2\alpha_6 + \alpha_7 - N) \\ &\quad + ig\alpha_1\alpha_8 + F_3,\end{aligned}$$

$$\begin{aligned}\frac{d\alpha_4}{dt} &= -(\gamma_{31} + \Lambda + i\Delta_2)\alpha_4 + ig\alpha_1(2\alpha_7 + \alpha_6 - N) \\ &\quad + i\Omega\alpha_5 + F_4,\end{aligned}$$

$$\begin{aligned}\frac{d\alpha_5}{dt} &= -[\gamma_{21} + \gamma_{31} + \Lambda/2 + i(\Delta_2 - \Delta_1)]\alpha_5 - ig\alpha_1\alpha_{10} \\ &\quad + i\Omega\alpha_4 + F_5,\end{aligned}$$

$$\frac{d\alpha_6}{dt} = -2\gamma_{21}\alpha_6 + i\Omega(\alpha_3 - \alpha_{10}) + F_6,$$

$$\frac{d\alpha_7}{dt} = \Lambda N - \Lambda\alpha_6 - 2(\Lambda + \gamma_{31})\alpha_7 + ig(\alpha_2\alpha_4 - \alpha_1\alpha_9) + F_7,$$

$$\begin{aligned}\frac{d\alpha_8}{dt} &= -[\gamma_{21} + \gamma_{31} + \Lambda/2 - i(\Delta_2 - \Delta_1)]\alpha_8 \\ &\quad + ig\alpha_2\alpha_3 - i\Omega\alpha_9 + F_8,\end{aligned}$$

$$\begin{aligned}\frac{d\alpha_9}{dt} &= -(\gamma_{31} + \Lambda - i\Delta_2)\alpha_9 - ig\alpha_2(2\alpha_7 + \alpha_6 - N) \\ &\quad - i\Omega\alpha_8 + F_9,\end{aligned}$$

$$\begin{aligned}\frac{d\alpha_{10}}{dt} &= -(\gamma_{21} + \Lambda/2 - i\Delta_1)\alpha_{10} - i\Omega(2\alpha_6 + \alpha_7 - N) \\ &\quad - ig\alpha_2\alpha_5 + F_{10}.\end{aligned} \quad (9)$$

In Eqs. (9), $\Delta_1 = \omega_{21} - \omega_l$, $\Delta_2 = \omega_{31} - \omega_l$, ω_c is the resonant frequency of the empty cavity, and ω_l is the lasing frequency. For the convenience of the calculation, Ω and g are chosen to have real values.

A. Steady-state inversionless laser intensity

Neglecting the noise correlation operators F_i and setting the derivatives to zero in Eqs. (9), the steady-state laser intensity defined as $I \equiv g^2 \alpha_2 \alpha_1$ and atomic population distribution are obtained by solving the resulting semiclassical, deterministic equations. We consider the resonant coherent pumping ($\Delta_1 = 0$) and assume that the cavity is tuned to the

resonant frequency of the lasing transition $|1\rangle \leftrightarrow |3\rangle$. Then, the dispersive response $\text{Re}(J_{31})$ vanishes and the lasing frequency $\omega_l = \omega_c = \omega_{31}$. From the semiclassical equations of the laser system, we obtain the solution for the steady-state laser intensity above the lasing threshold as

$$I \equiv n g^2 = \frac{-B + \sqrt{B^2 - 4g^2 AC}}{2A}, \quad (10)$$

where $n \equiv \alpha_2 \alpha_1$ is the number of photons in the cavity. The parameters A , B , and C are

$$\begin{aligned} A &= \frac{2\gamma_{21}\kappa}{\Omega[\gamma_{31}(\Lambda/2 + \gamma_{21} + \gamma_{31}) + \gamma_{21}(\Lambda + \gamma_{31})]}, \\ B &= \frac{Ng^2\gamma_{21}\gamma_{31}}{\Omega[\gamma_{31}(\Lambda/2 + \gamma_{31} + \gamma_{21}) + \gamma_{21}(\Lambda + \gamma_{31})]} + \frac{\Omega\kappa(3\Lambda^2 + 7\Lambda\gamma_{31} + 2\Lambda\gamma_{21} + 2\gamma_{31}^2 + 2\gamma_{21}\gamma_{31})}{2(\Lambda + \gamma_{31})[\gamma_{31}(\Lambda/2 + \gamma_{31} + \gamma_{21}) + \gamma_{21}(\Lambda + \gamma_{31})]} \\ &\quad + \frac{\kappa\gamma_{21}[(\Lambda + \gamma_{31})^2 + (\Lambda + 2\gamma_{21})(\Lambda/2 + \gamma_{31} + \gamma_{21})]}{\Omega[\gamma_{31}(\Lambda/2 + \gamma_{31} + \gamma_{21}) + \gamma_{21}(\Lambda + \gamma_{31})]}, \\ C &= \frac{N\Omega[2\gamma_{31}^2 - \Lambda(\gamma_{21} - \gamma_{31})]}{2[\gamma_{31}(\Lambda/2 + \gamma_{31} + \gamma_{21}) + \gamma_{21}(\Lambda + \gamma_{31})]} + \frac{N[\gamma_{31}\gamma_{21}(\Lambda/2 + \gamma_{31} + \gamma_{21})(\Lambda + 2\gamma_{21})]}{2\Omega[\gamma_{31}(\Lambda/2 + \gamma_{31} + \gamma_{21}) + \gamma_{21}(\Lambda + \gamma_{31})]} \\ &\quad + \frac{\Omega(3\Lambda + 4\gamma_{31})(\Lambda/2 + \gamma_{31} + \gamma_{21}) + \frac{\lambda_{21}}{\Omega}(\Lambda + 2\gamma_{21})(\Lambda + \gamma_{31})(\Lambda/2 + \gamma_{31} + \gamma_{21})}{2g^2[\gamma_{31}(\Lambda/2 + \gamma_{31} + \gamma_{21}) + \gamma_{21}(\Lambda + \gamma_{31})]} \kappa \left(\Lambda + \gamma_{31} + \frac{\Omega^2}{\Lambda/2 + \gamma_{31} + \gamma_{21}} \right). \end{aligned}$$

Here we define the laser intensity I as the steady-state photon number n inside the cavity multiplied by the squared cavity-atom coupling coefficient g . Since $A > 0$ and $B > 0$, a non-trivial solution for the laser intensity requires $C < 0$. The first two terms in C containing the atomic number N are dominant when the laser operates above the threshold. This immediately gives the required minimum value of the Rabi frequency of the pump laser, i.e.,

$$\Omega > \left(\frac{\gamma_{31}\gamma_{21}(\Lambda + 2\gamma_{21})(\Lambda/2 + \gamma_{31} + \gamma_{21})}{\Lambda(\gamma_{21} - \gamma_{31}) - 2\gamma_{31}^2} \right)^{1/2}, \quad (11)$$

assuming that the necessary condition for the existence of inversionless gain, $\Lambda(\gamma_{21} - \gamma_{31}) > 2\lambda_{31}^2$, is satisfied by the system [9]. Then, in general, the onset of the inversionless lasing requires fulfillment of the inequality (11) and also the threshold condition for the total number of atoms inside the cavity. In the limit of a strong driving field, i.e., $\Omega \gg \Lambda$, γ_{ij} ($i, j = 1-3$), and the g [the inequality (11) is satisfied], the threshold number of atoms N_{th} is

$$N_{\text{th}} = \frac{\kappa\Omega^2(3\Lambda + 4\gamma_{31})}{g^2[\Lambda(\gamma_{21} - \gamma_{31}) - 2\gamma_{31}^2]}. \quad (12)$$

Below the threshold and for small laser intensities above the threshold, we obtain the atomic population distribution as follows:

$$J_{11} = J_{22} = \frac{\Lambda + 2\gamma_{31}}{3\Lambda + 4\gamma_{31}} N, \quad (13)$$

$$J_{33} = \frac{\Lambda}{3\Lambda + 4\gamma_{31}} N. \quad (14)$$

J_{ii} ($i = 1-3$) is the atomic population in state $|i\rangle$. Since $J_{11} > J_{33}$ and $J_{22} > J_{33}$, there is no population inversion in the basis of the bare atomic states. It is straightforward to derive the population distribution in the dressed states [28]. For the on-resonance coherent pump field ω_1 , the dressed states are simply

$$|+\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) \quad \text{with the energy } E_+ = \Omega, \quad (15)$$

and

$$|-\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle) \quad \text{with the energy } E_- = -\Omega. \quad (16)$$

The dressed-state population distributions are $J_{++} = J_{--} = J_{11} = J_{22}$. Then we have $J_{33} < J_{++}$ and $J_{33} < J_{--}$ in the dressed-state basis. Thus population inversion cannot happen in either the bare atomic states ($|1\rangle$, $|2\rangle$, and $|3\rangle$) or the dressed atomic states ($|+\rangle$, $|-\rangle$, and $|3\rangle$). The laser can only start from light amplification by coherence [9]. This is expected for the present pumping scheme. Since the three-level system is closed and the incoherent pump $|1\rangle \leftrightarrow |3\rangle$ and the coherent pump $|1\rangle \leftrightarrow |2\rangle$ are both reversible, population inversion cannot be created between states $|1\rangle$ and $|3\rangle$ or states $|1\rangle$ and $|2\rangle$. In other words, one always obtains $J_{11} \geq J_{33}$ and $J_{11} \geq J_{22}$ [29]. Far above the threshold, the number of atoms in the cavity, N , is much larger than N_{th} , and the laser intensity approaches the saturation limit I_s :

$$I_s \equiv n g^2 = \frac{\Omega^2[\Lambda(\gamma_{21} - \gamma_{31}) - 2\gamma_{31}^2]}{2\gamma_{31}\gamma_{21}}. \quad (17)$$

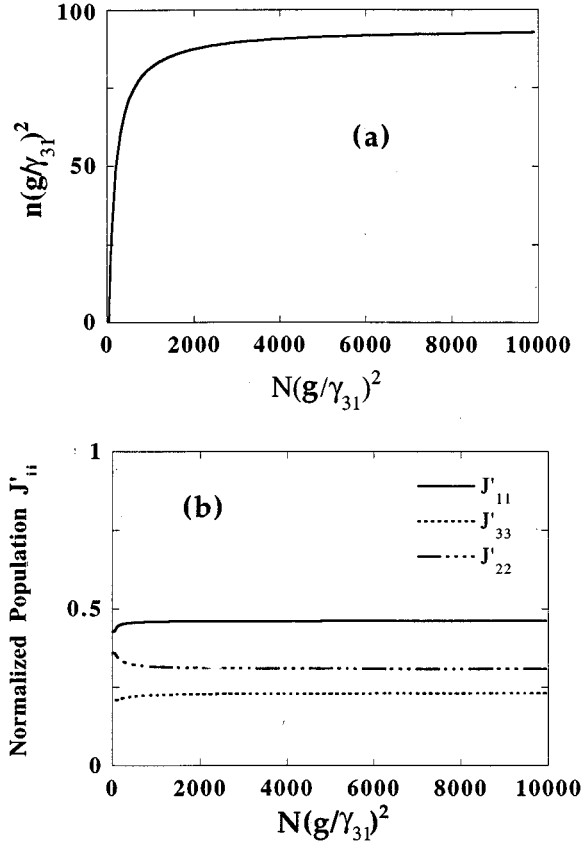


FIG. 2. (a) Calculated steady-state laser intensity and (b) normalized population distribution versus the number of atoms N in a V-type, three-level inversionless laser. The chosen parameters are $\gamma_{21} = 6\gamma_{31}$, $\Lambda = 2\lambda_{31}$, $\Omega = 15\gamma_{31}$, and $\kappa = 0.1\gamma_{31}$.

I_s is independent of N and proportional to the intensity of the strong external driving field. When $\Omega \gg \Lambda$, γ_{ij} ($i, j = 1-3$), and g , the coherent cycling of electrons between $|2\rangle \leftrightarrow |1\rangle \leftrightarrow |3\rangle$, may dominate the incoherent spontaneous emission processes $|2\rangle \rightarrow |1\rangle$ and $|3\rangle \rightarrow |1\rangle$. The population distribution for the laser far above the threshold is given by

$$J_{11} = \frac{N(2\gamma_{21}\gamma_{31} + \Lambda\gamma_{21})}{2[\gamma_{31}(\Lambda/2 + \gamma_{21} + \gamma_{31}) + \gamma_{21}(\Lambda + \gamma_{31})]}, \quad (18)$$

$$J_{22} = \frac{N\gamma_{31}(\Lambda/2 + \gamma_{21} + \gamma_{31})}{\gamma_{31}(\Lambda/2 + \gamma_{21} + \gamma_{31}) + \gamma_{21}(\Lambda + \gamma_{31})}, \quad (19)$$

$$J_{33} = \frac{N\Lambda\gamma_{21}}{2[\gamma_{31}(\Lambda/2 + \gamma_{21} + \gamma_{31}) + \gamma_{21}(\Lambda + \gamma_{31})]}. \quad (20)$$

Again, the population distribution satisfies $J_{33} - J_{11} < 0$ and $J_{22} - J_{11} < 0$, and no population inversion can occur throughout the laser operating range. It is interesting to note that as the inversionless laser reaches the saturation regime, the atomic population at the upper lasing state, J_{33} , is greater than that at or below the threshold. Note that this differs from conventional lasers in which the upper lasing state population below the threshold is greater than the above the threshold. Figure 2 shows the typical behavior of the three-level V-type inversionless laser (the laser intensity and the population distribution) as the number of atoms in the cavity in-

creases from the threshold to the saturation regime. As the laser intensity builds up quickly above the threshold, the atomic population J_{11} and J_{33} slowly increase while the atomic population J_{22} decreases. Well above the threshold, the population distribution does not change appreciably. Under the normal operating conditions, one always has $J_{11} > J_{22} > J_{33}$, i.e., LWI is valid from below the threshold to well above the threshold.

B. Statistical properties

To calculate the statistical properties of the laser field, we follow the procedure described by Reid [30]. Above the threshold (large N values), the quantum fluctuations ($\propto 1/\sqrt{N}$) are small, so we can treat them as linear perturbations. Assuming $\alpha_i = \alpha_{i;0} + \delta\alpha_i$ ($i = 1-10$) where $\alpha_{i;0}$ is the steady-state solution to Eqs. (9), we calculate the small fluctuations around the steady-state expectation values of the atomic and field operators. The linearized stochastic differential equations describe the fluctuations in the field and atomic variables to first order and can be written as

$$\frac{d\delta\vec{\alpha}}{dt} = \underline{M}(\vec{\alpha}_0)\delta\vec{\alpha} + \vec{F}(t), \quad (21)$$

where

$$M_{ij}(\vec{\alpha}_0) = \left. \frac{\partial A_i}{\partial \alpha_j} \right|_{\vec{\alpha} = \vec{\alpha}_0}.$$

Such an approximation neglects the phase variation of the complex solution to Eqs. (9), and limits the time of its validation to the inverse of the laser linewidth. Since the laser linewidth is very small, the laser phase diffuses very slowly in comparison with processes relevant to the laser intensity. The amplitude squeezing spectrum is defined as [30]

$$V(X, \omega) = \int_{-\infty}^{\infty} \langle \hat{X}(t+\tau)\hat{X}(t) \rangle e^{i\omega\tau} d\tau, \quad (22)$$

where $X(t) = a_{\text{out}} + a_{\text{out}}^\dagger$ is the quadrature phase amplitude of the cavity output field and $\langle \hat{X}\hat{Y} \rangle = \langle \hat{X}\hat{Y} \rangle - \langle \hat{X} \rangle \langle \hat{Y} \rangle$. After a Fourier transformation, the amplitude squeezing spectrum is given by

$$V(X, \omega) = 1 + 2\kappa(S_{12}(\omega) + S_{21}(\omega) + S_{11}(\omega) + S_{22}(\omega)), \quad (23)$$

where the spectral matrix $\mathbf{S}(\omega)$

$$\underline{S}(\omega) = (\underline{M}(\vec{\alpha}_0) - i\omega I)^{-1} \underline{D}(\vec{\alpha}_0) (\underline{M}^T(\vec{\alpha}_0) - i\omega I)^{-1}. \quad (24)$$

If $V(X, \omega) < 1$, the laser output field is amplitude squeezed. For small fluctuations, the classical amplitude of the laser field can be treated as an in-phase local oscillator for the quantum fluctuations, and the amplitude-squeezing spectrum is equivalent to the intensity fluctuation spectrum. The output intensity fluctuation spectrum of the laser is related to the Mandel Q parameter by [31,32]

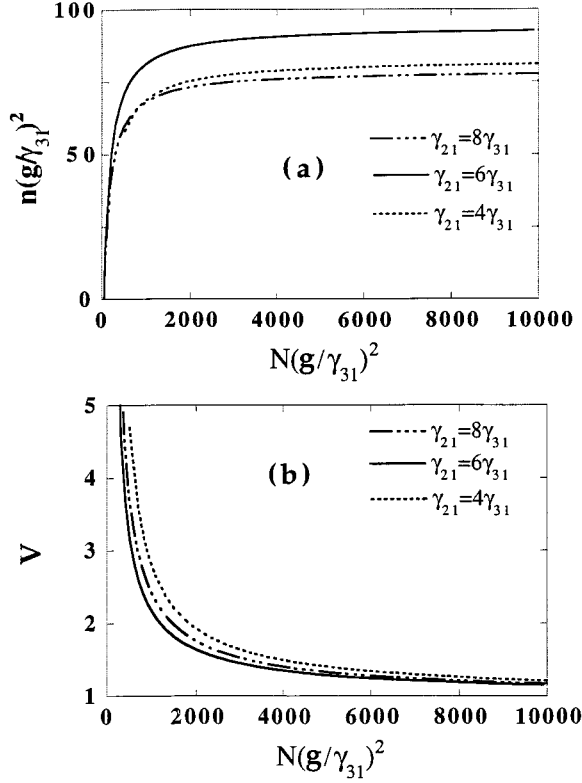


FIG. 3. (a) Calculated inversionless laser intensity I and (b) the amplitude variance $V(\omega=0)$ versus the number of atoms N inside the cavity. The unspecified parameter are the same as that in Fig. 2.

$$V(X, \omega) = 1 + Q \frac{4\kappa\lambda}{\lambda^2 + \omega^2}, \quad (25)$$

where λ^{-1} is the intensity correlation time for the laser and is proportional to κ . Negative Q values ($0 < Q < -0.5$) correspond to sub-Poissonian photon statistics, which is consistent with the laser intensity squeezing [$V(X, \omega) < 1$]. We have calculated numerically the laser intensity and the amplitude spectral variance as functions of various system parameters, and found that the intensity fluctuation is always above the shot noise limit. Figure 3 shows the calculated inversionless laser intensity I and the amplitude variance V versus the number of atoms confined in the cavity for several γ_{21} values. For a given incoherent pump rate Λ , the optimized operation condition (both the laser intensity and the corresponding fluctuation) of the V-type inversionless laser occurs at intermediate γ_{21} values. Although there is no population inversion and light amplification is provided by the atomic coherence J_{23} the amplitude variance V of the inversionless laser is above the shot-noise limit. This may be due to the fact that the coherence term J_{23} is associated with the two excited states $|2\rangle$ and $|3\rangle$, and therefore is fully subject to the quantum noises of the spontaneous emission. Also notice that when the laser is above the threshold, the atomic population in the upper lasing state $|3\rangle$ increases somewhat. The combined effects make the V-type inversionless laser behave statistically just like a conventional inversion laser that approaches Poissonian photon statistics ($Q \rightarrow 0$ or $V \rightarrow 1$) when it is well above the threshold. Plotted in Fig. 4

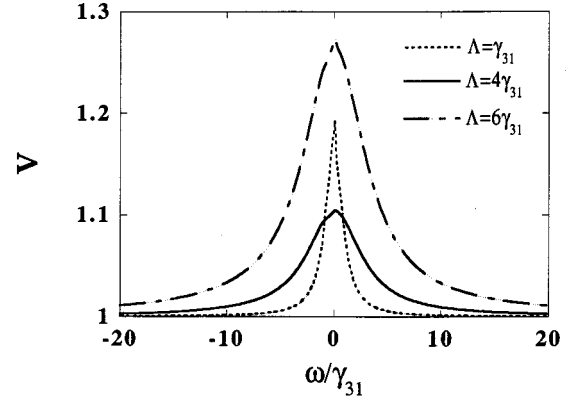


FIG. 4. Calculated intensity fluctuation spectrum V versus the frequency ω for several values of the incoherent pump rate Λ . The normalized number of atoms inside the cavity is $N(g/\gamma_{31})^2 = 20\,000$ and the Rabi frequency $\Omega = 20\gamma_{31}$. The other parameters are the same as that in Fig. 2.

are the intensity fluctuation spectrum V versus the frequency ω for several values of the incoherent pump rate Λ . All curves in Fig. 4 give a positive Mandel Q parameter [see Eq. (25)], indicating super-Poissonian photon statistics. The laser intensity fluctuations are minimized at intermediate Λ values, but a smaller Λ value leads to a smaller linewidth in the fluctuation spectrum V . At small Λ values, the laser intensity is small, which results in a larger V value; at larger Λ values,

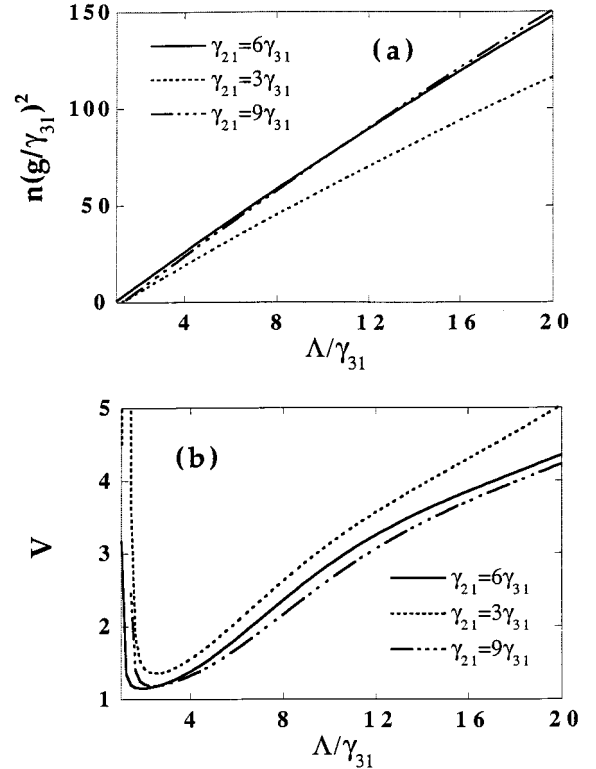


FIG. 5. (a) Calculated inversionless laser intensity I and (b) the amplitude variance $V(\omega=0)$ versus Λ . The normalized number of atoms inside the cavity is $N(g/\gamma_{31})^2 = 20\,000$. The unspecified parameters are the same as that in Fig. 2.

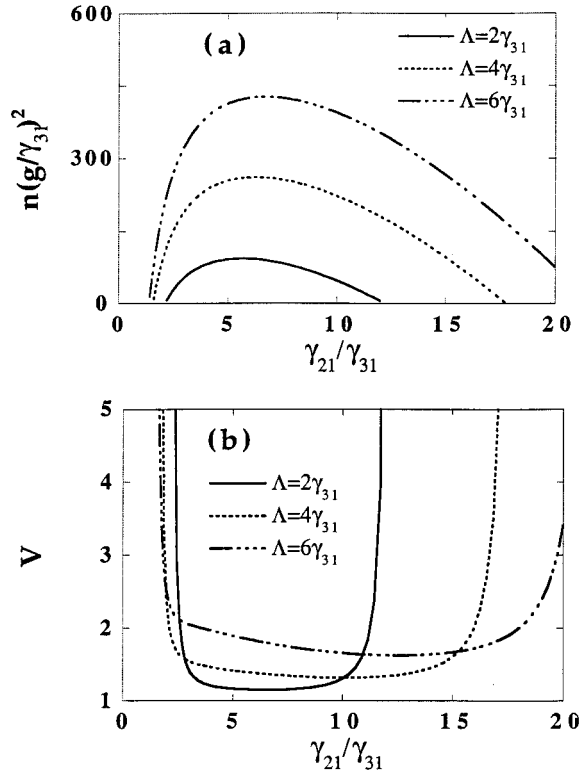


FIG. 6. (a) Calculated inversionless laser intensity I and (b) the amplitude variance $V(\omega=0)$ versus γ_{21} . The normalized number of atoms inside the cavity is $N(g/\gamma_{31})^2=20\,000$. The unspecified parameters are the same as that in Fig. 2.

the laser intensity is higher, but the random noises introduced by the incoherent pump Λ are much greater, which also leads to a larger V value. This behavior is shown more systematically in Fig. 5, which plots the calculated laser intensity I and the amplitude variance $V(\omega=0)$ versus the incoherent pump rate Λ for several γ_{21} values. The laser intensity I is a monotonically increasing function of Λ , but the amplitude variance has a minimum value occurring at an intermediate Λ value. Since the incoherent pump Λ excites the atoms into the upper lasing state, the larger the Λ value, the larger the laser intensity. However, besides the spontaneous emission from the upper lasing state $|3\rangle$, the incoherent pump causes dephasing of the atomic coherence J_{23} that is responsible for gain in the V system. Once the laser is above the threshold, a larger Λ always adds more noises to the laser amplitude. Figure 6 shows the calculated intensity I and the amplitude variance V versus γ_{21} with several incoherent pump rates Λ for a fixed number of atoms N inside the cavity. It is seen that the inversionless laser only works at intermediate γ_{21} values for a given incoherent pump rate Λ . As shown by Eq. (12), a sufficiently large γ_{21} is required in order for the V system to exhibit the inversionless gain. At a larger γ_{21} value, the inversionless gain is reduced and the lasing threshold is higher, so the laser intensity becomes smaller. Furthermore, a larger γ_{21} adds more noises to the atomic coherence J_{23} , which increases the laser amplitude fluctuations. Therefore the laser amplitude variance increases with γ_{21} after reaching above the threshold. Figure 7 shows the calculated intensity I and the amplitude variance V versus the cavity

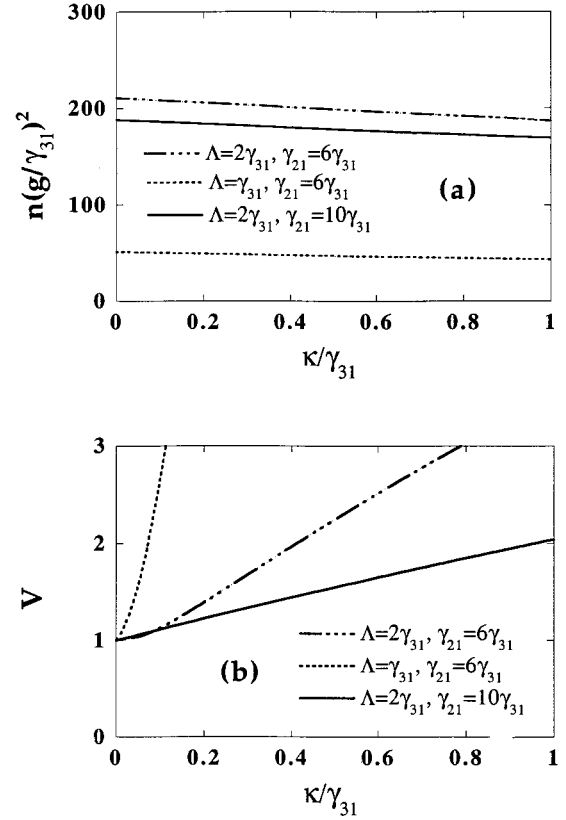


FIG. 7. (a) Calculated inversionless laser intensity I and (b) the amplitude variance $V(\omega=0)$ versus κ . The normalized number of atoms inside the cavity is $N(g/\gamma_{31})^2=20\,000$. The unspecified parameters are the same as that in Fig. 2.

decay rate κ with a fixed number of atoms N inside the cavity. As expected, the intensity fluctuations V increase and the intensity I decreases with increasing κ values. When κ reaches zero, the intensity fluctuations V approach the limiting value 1. This is similar to Poissonian photon statistics for conventional inversion lasers at the high-intensity limit. We also calculated the amplitude fluctuations V as a function of the Rabi frequency Ω under various conditions, and found that V is always greater than 1. The minimum V values occur at an intermediate Ω value and after that the amplitude fluctuations increase with increasing Ω values.

III. CONCLUSION

We have calculated the intensity and statistical properties of a three-level V-type inversionless laser. Well above the threshold, the inversionless laser intensity saturates, becomes independent of the number of the atoms inside the cavity, and is proportional to the pump laser intensity. In contrast to the amplitude squeezing of the inversionless laser from a three-level Λ -type system, no amplitude squeezing is found in the V-type inversionless laser. In both V and Λ systems, light amplification is induced by the atomic coherence gen-

erated in a two-photon scattering process. The atomic coherence associated with the two ground states in a Λ system is largely free from the influence of the quantum fluctuations due to spontaneous emission. In a V system, the atomic coherence is generated between the two excited states and is therefore fully subject to the quantum fluctuations from spontaneous emission. This fact may explain why the amplitude squeezing has not been found in the numerical calcula-

tion for the inversionless laser from the three-level V-type system.

ACKNOWLEDGMENTS

The work of Y.Z. is supported in part by Research Corporation and the U.S. Army Research Office, and the work of M.X. is supported by the Office of Naval Research. A.I.R. is a participant of the NSF REU program at FIU.

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