Zero-point noise in a nonstationary dielectric cavity

M. Artoni,[1] A. Bulatov,[2] and J. Birman[2]
[1]*Photonics Division 715, NASA Goddard Space Flight Center, Greenbelt, Maryland 20771*
[2]*Department of Physics, The City College of New York, Convent Avenue at 138th Street, New York, New York 10031*
(Received 3 May 1995)

The vacuum field in a dielectric cavity where the medium has a rapidly changing time-dependent refractive index exhibits nonclassical features that are directly related to the distortion of the vacuum fluctuations of the field. For appropriate parameters, these nonclassical effects are appreciable enough that a suitable measure of the change in the zero-point noise of the cavity field could provide a way to probe distortions of the vacuum fluctuations.

PACS number(s): 12.20.−m

I. INTRODUCTION

The quantum fluctuations of the electromagnetic vacuum field can be altered by rapid distortions of the vacuum field. A typical and quite fashionable example consists in changing the position of a perfectly reflecting mirror in an empty electromagnetic cavity. The time-dependent boundary condition of a moving mirror can alter the structure of the cavity vacuum field. Relevant work on cavity quantum electrodynamics in the presence of moving boundaries can be found, e.g., in Refs. [1–3]. Another example consists in changing the refractive index of a material in time. In this case the structure of the vacuum is altered due to the dependence of the zero energy of the field, i.e., $\hbar c(k_x^2+k_y^2+k_z^2)^{1/2}/2n$, on the index of refraction n, if the k's denote the mode wave-vector components and c the speed of light in vacuum. Yablonovitch suggested that a rapidly growing plasma produced by short optical pulses [4,5], or virtual photoconductivity in the transparent region of a semiconductor [6,5], could produce a large rate of change in the index of refraction.

Physically, the moving mirror and the time-dependent medium produce similar effects and the major interest in this type of systems has been perhaps the possibility of photon creation that one can interpret as originating from a rapid distortion of the vacuum. Such a distortion of the vacuum may also be considered as a "dynamical" Casimir effect [7], if one regards as "static" the one [8] in which the change in the zero-point electromagnetic energy is due to the presence of matter whose geometry is held fixed.

In these proposals the effect of photon production is practically zero so that in any realistic experimental situations it would be particularly difficult measuring photon numbers to probe distortion of the zero-point quantum fluctuations of the field. However, since the emission of photons is a purely quantum effect, we expect the photon statistics to exhibit nonclassical properties. In this paper we show that a change in time of the permeability of the medium inside a dielectric cavity modifies the quantum statistics of the cavity field. In particular, distortions of the vacuum field can be easily recovered by measuring modifications in the cavity zero-point noise. Moreover, we show that already in a quasiadiabatic regime these modifications become appreciable enough to be measurable and the relevant conditions are discussed. Our analysis relies on a suitable scheme for the quantization of the electromagnetic field in a time-dependent medium [9]. The changes in the quantum statistics of the cavity field due to the modulation of the medium are determined by using a simple but exact approach based on the formal analogy with a quantum-mechanical scattering problem.

II. MODEL

We begin by considering a cavity consisting of two perfectly reflecting [10] plane mirrors where only axial modes can propagate. The space between the mirrors contains a linear, lossless, and nondispersive [11] medium. For simplicity, we let the magnetic permeability constant throughout the cavity, while the dielectric permeability varies in time according to

$$\varepsilon(t)=\varepsilon_0\left[1-(1-r_{0f})\,\frac{e^{2\pi t/\tau}}{1+e^{2\pi t/\tau}}\right]^{-1}\quad\left(r_{0f}\equiv\frac{\varepsilon_0}{\varepsilon_f}\right). \quad (1)$$

Here τ denotes the time duration of the change, while ε_0 and ε_f denote the value of the permeability before and after the modulation. We specifically consider the case in which $\varepsilon_f<\varepsilon_0$. For the simplest geometry of a cavity of volume $V=a\cdot b\cdot c$ the frequency of the mode would vary according to

$$\Omega_\lambda(t)=\frac{c\sqrt{(l\pi/a)^2+(m\pi/b)^2+(n\pi/c)^2}}{\sqrt{\varepsilon_0}}\times\left[1-(1-r_{0f})\,\frac{e^{2\pi t/\tau}}{1+e^{2\pi t/\tau}}\right]^{1/2}, \quad (2)$$

where $\{\lambda\}\to l,m,n=0,1,2,...,$ with the restriction that only one integer at a time can be zero. We consider only the case of a linearly polarized field. Since there are no sources, the Lagrangian of the electromagnetic field inside the cavity is given by

$$L=\frac{1}{8\pi}\int_V d\vec{r}[\varepsilon(t)(\vec{E}(\vec{r},t))^2-(\vec{B}(\vec{r},t))^2]. \quad (3)$$

We express the field in terms of a vector potential $\vec{A}$ so that in the Coulomb gauge $\vec{\nabla}\cdot\vec{A}(\vec{r},t)=0$ one has

$$\vec{E}(\vec{r},t) = -\frac{1}{c}\frac{\partial \vec{A}(\vec{r},t)}{\partial t}, \quad \vec{B}(\vec{r},t) = \vec{\nabla}\times\vec{A}(\vec{r},t). \tag{4}$$

Introducing the expansion over the cavity normal modes

$$\vec{A}(\vec{r},t) = \sum_{\lambda} Q_{\lambda}(t)\vec{A}_{\lambda}(\vec{r}) \tag{5}$$

and assuming that the relative rate of change of the permeability is slow compared to the frequency of the excited cavity mode, i.e., $\partial\varepsilon/\varepsilon\partial t < \Omega_{\lambda}$, the Lagrangian becomes [12]

$$L = \frac{1}{2}\sum_{\lambda}[\dot{Q}_{\lambda}^2 - \Omega_{\lambda}^2(t)Q_{\lambda}^2]. \tag{6}$$

The generalized momenta are $P_{\lambda} = \delta L/\delta\dot{Q}_{\lambda} = \dot{Q}_{\lambda}$ and the corresponding Hamiltonian may be written as

$$H = \frac{1}{2}\sum_{\lambda}[P_{\lambda}^2 + \Omega_{\lambda}^2(t)Q_{\lambda}^2]. \tag{7}$$

The radiation field in a nonstationary dielectric can therefore be described by an infinite set of uncoupled harmonic oscillators with time-dependent frequency. The quantization of the field is straightforward. Making use of standard approaches, we associate Hermitian operators $\hat{Q}_{\lambda}$ and $\hat{P}_{\lambda}$ with the classical variables Q_{λ} and P_{λ} and we postulate the familiar commutation relations

$$[\hat{Q}_{\lambda}, \hat{P}_{\lambda'}] = i\hbar\,\delta_{\lambda,\lambda'}. \tag{8}$$

The following non-Hermitian photon destruction and creation operators $\hat{a}_{\lambda}(t)$ and $\hat{a}_{\lambda}^{\dagger}(t)$ can be introduced,

$$\hat{a}_{\lambda}(t) = [\Omega_{\lambda}(t)/2\hbar]^{1/2}\{\hat{Q}_{\lambda} + i\Omega_{\lambda}(t)^{-1}\hat{P}_{\lambda}\},$$

$$\hat{a}_{\lambda}^{\dagger}(t) = [\Omega_{\lambda}(t)/2\hbar]^{1/2}\{\hat{Q}_{\lambda} - i\Omega_{\lambda}(t)^{-1}\hat{P}_{\lambda}\}, \tag{9}$$

whose commutation rules directly follow from Eq. (8),

$$[\hat{a}_{\lambda}(t), \hat{a}_{\lambda'}^{\dagger}(t)] = \delta_{\lambda,\lambda'}, \quad [\hat{a}_{\lambda}(t), \hat{a}_{\lambda'}(t)] = [\hat{a}_{\lambda}^{\dagger}(t), \hat{a}_{\lambda'}^{\dagger}(t)] = 0. \tag{10}$$

The Hamiltonian operator can then be written as

$$\hat{H}(t) = \sum_{\lambda}\hbar\Omega_{\lambda}(t)[\hat{a}_{\lambda}^{\dagger}(t)\hat{a}_{\lambda}(t) + \tfrac{1}{2}], \tag{11}$$

so that before and after the modulation $\hat{H}$ is the usual Hamiltonian for a simple harmonic oscillator with a given constant frequency. For times $t_1 \ll -\tau/2$ and $t_2 \gg \tau/2$ the Hamiltonians are, respectively,

$$\hat{H}(t_1) = \hbar\Omega_1[\hat{a}_{\lambda}^{\dagger}(t_1)\hat{a}_{\lambda}(t_1) + \tfrac{1}{2}],$$

$$\hat{H}(t_2) = \hbar\Omega_2[\hat{a}_{\lambda}^{\dagger}(t_2)\hat{a}_{\lambda}(t_2) + \tfrac{1}{2}], \tag{12}$$

where we denote by $\Omega_1 \equiv \Omega_{\lambda}(t_1) = \omega_{\lambda}/\sqrt{\varepsilon_o}$ and $\Omega_2 \equiv \Omega_{\lambda}(t_2) = \omega_{\lambda}/\sqrt{\varepsilon_f}$ the initial and final frequencies, which are both assumed to propagate in the cavity. Here ω_{λ} is instead the unperturbed cavity vacuum frequency.

We now derive an expression for the photon operators $\hat{a}_{\lambda}(t_2) \equiv \hat{a}_2$ and $\hat{a}_{\lambda}^{\dagger}(t_2) \equiv \hat{a}_2^{\dagger}$ in terms of those before the modulation of the dielectric, i.e., $\hat{a}_{\lambda}(t_1) \equiv \hat{a}_1$ and $\hat{a}_{\lambda}^{\dagger}(t_1) \equiv \hat{a}_1^{\dagger}$. Using a formalism originally due to Brown and Carson [13], such an expression can be obtained from the asymptotic solutions of the equation of motion for the photon oscillator complex coordinate, or from Eq. (6) with $Q_{\lambda} = q_{\lambda} + q_{\lambda}^{*}$,

$$\ddot{q}_{\lambda} + \Omega_{\lambda}^2(t)q_{\lambda} = 0. \tag{13}$$

Asymptotic solutions can be found noting that Eq. (13) is formally identical to a one-dimensional quantum-mechanical barrier penetration problem. In fact, this equation turns into a one-dimensional Schrödinger equation if we make the following replacements: time $t \rightarrow$ space coordinate x, oscillator coordinate $q(t) \rightarrow$ wave function $\psi(x)$, and frequency $\Omega^2(t) \rightarrow 2m[E - V(x)]$, where E and $V(x)$ are the total and potential energy of a particle of mass m. The two asymptotic wave functions in the barrier penetration problem, say, ψ_f and ψ_b, are described by a 2×2 unitary and symmetric scattering matrix S [13]. Its elements give the transition amplitudes for a particle incident from the right to be reflected to the right (R_f) or transmitted to the left (T_b) and the transition amplitudes for a particle incident from the left to be reflected to the left (R_b) or transmitted to the right (T_f). The behavior of ψ_f and ψ_b is the same as the asymptotic solutions of Eq. (13), say, q_f and q_b, which can then be described by the same matrix S [13]. In particular, for the present photon problem the elements of S give the transition amplitudes for a photon moving backward in time to be reflected forward (R_f) or scattered backward (T_b) in time and the transition amplitudes for a photon moving forward in time to be reflected backward (R_b) or scattered forward (T_f) in time.

By evaluating the Wronskian of $\hat{Q}_{\lambda}$, from Eq. (9), and either of the two scattering solutions q_f and q_b for $t_1 \ll -\tau/2$ and $t_2 \gg \tau/2$ and then equating these two asymptotic values, $\hat{a}_2$ and $\hat{a}_2^{\dagger}$ are found to be linearly related by elements of S to the initial operators $\hat{a}_1$ and $\hat{a}_1^{\dagger}$ [13]. Following this procedure for the case of photons moving forward in time the Wronskian operator $W[\hat{Q}_{\lambda}, q_f]$ yields

$$\hat{a}_2 = \frac{T_f}{|T_f|^2}e^{i(\Omega_1 t_1 - \Omega_2 t_2)}\hat{a}_1 - \frac{R_b^{*}T_f}{|T_f|^2}e^{-i(\Omega_1 t_1 + \Omega_2 t_2)}\hat{a}_1^{\dagger}, \tag{14}$$

while for photons moving backward in time the Wronskian operator $W[\hat{Q}_{\lambda}, q_b]$ yields

$$\hat{a}_1 = \frac{T_b^{*}}{|T_b|^2}e^{-i(\Omega_1 t_1 - \Omega_2 t_2)}\hat{a}_2 - \frac{R_f T_b^{*}}{|T_b|^2}e^{-i(\Omega_1 t_1 + \Omega_2 t_2)}\hat{a}_2^{\dagger}. \tag{15}$$

Note that $[\hat{a}_2, \hat{a}_2^{\dagger}] = [\hat{a}_1, \hat{a}_1^{\dagger}]$, which preserves unitarity.

We now proceed to give an expression for the coefficients of the transformation in Eqs. (14) and (15). In practice, only R_b needs to be determined. This is easily done by observing that the barrier penetration problem associated with Eq. (13) and the potential in Eq. (2) corresponds to a well-known example of a one-dimensional scattering above the barrier for which an exact analytical solution is available [14]. The reflection coefficient associated with the potential in Eq. (2) is [14].

$$|R_{b,\Omega_1,r_{0f}}(\tau)|^2 = \frac{\sinh^2\left\{\frac{\Omega_1\tau}{2}\,[1-\sqrt{r_{0f}}]\right\}}{\sinh^2\left\{\frac{\Omega_1\tau}{2}\,[1+\sqrt{r_{0f}}]\right\}}. \qquad (16)$$

For typical values of the parameters used in our photon problem $|R_{b,\Omega_1,r_{0f}}(\tau)|^2 \ll 1$.

III. FIELD STATISTICS

We proceed to analyze the squeezing generated by the time-varying permeability of the medium inside the cavity. This effect occurs when the quantum fluctuations in one of the quadrature components of the cavity field drops below the vacuum level. Such an effect is characterized by states of the electromagnetic field having negative normally ordered variances, i.e., states with no classical analog. The two Hermitian quadrature operators $\hat{X}$ and $\hat{Y}$ are defined as [15]

$$\hat{a} = \hat{X} + i\hat{Y}, \quad \hat{a}^\dagger = \hat{X} - i\hat{Y}, \qquad (17)$$

so that if $\Delta X = \langle[\hat{X} - \langle\hat{X}\rangle]^2\rangle$ and $\Delta Y = \langle[\hat{Y} - \langle\hat{Y}\rangle]^2\rangle$ denote the corresponding fluctuations, one has, after the modulation,

$$\left.\begin{matrix}\Delta X\\ \Delta Y\end{matrix}\right\} = \tfrac{1}{4}\{1 + 2\langle\hat{a}_2^\dagger\hat{a}_2\rangle \pm 2\,\mathrm{Re}\langle\hat{a}_2^2\rangle\} \mp \tfrac{1}{4}[\langle\hat{a}_2^\dagger\rangle \pm \langle\hat{a}_2\rangle]^2. \qquad (18)$$

Since $[\hat{X},\hat{Y}] = i/2$ the uncertainty relation $\Delta X \Delta Y \geqslant \frac{1}{16}$ must hold and a state of the field is said to be squeezed when ΔX or ΔY reduces below $\frac{1}{4}$. With the help of Eqs. (14) and (10) one can express $\langle\hat{a}_2\rangle$, $\langle\hat{a}_2^2\rangle$, and $\langle\hat{a}_2^\dagger\hat{a}_2\rangle$ in terms of the complex reflection amplitudes $R_b = |R_b|e^{i\phi_b}$ and $R_f = |R_f|e^{i\phi_f}$. The latter are determined by the modulation of the medium and the cavity parameters. From the properties of the matrix S it follows that $|R_b| = |R_f| \equiv |R|$. For a cavity initially in a coherent state, i.e., $\hat{a}_1|\alpha\rangle = |\alpha|e^{-i\Omega_1 t_1 - i\phi}|\alpha\rangle$ and $\langle\alpha|\hat{a}_1^\dagger = |\alpha|e^{i\Omega_1 t_1 + i\phi}\langle\alpha|$, one has

$$\langle\hat{a}_2\rangle = \frac{|\alpha|}{\sqrt{1-|R|^2}}\{e^{-i\Omega_2 t_2 - i\phi + i\phi_f} - |R|e^{-i\Omega_2 t_2 + i\phi + i\phi_f - i\phi_b}\}, \qquad (19)$$

$$\langle\hat{a}_2^2\rangle = \frac{|\alpha|^2}{1-|R|^2}\{e^{-2i\Omega_2 t_2 - 2i\phi + 2i\phi_f} + |R|^2 e^{-2i\Omega_2 t_2 + 2i\phi + 2i\phi_f - 2i\phi_b}\} - \frac{|R|(1+2|\alpha|^2)}{1-|R|^2}\, e^{-2i\Omega_2 t_2 - i\phi_b + 2i\phi_f}, \qquad (20)$$

and

$$\langle\hat{a}_2^\dagger\hat{a}_2\rangle = \frac{|R|^2 + |\alpha|^2[1 + |R|^2 - 2|R|\cos\Phi]}{1-|R|^2}. \qquad (21)$$

Here ϕ and $|\alpha|^2$ denote the initial phase and average number of photons in the field, both of which can be controlled experimentally, while $\Phi \equiv 2\phi - \phi_b$. When $|\alpha|^2 = 0$ we recover the case of the cavity vacuum state. The expression for the quadrature fluctuations readily follows:

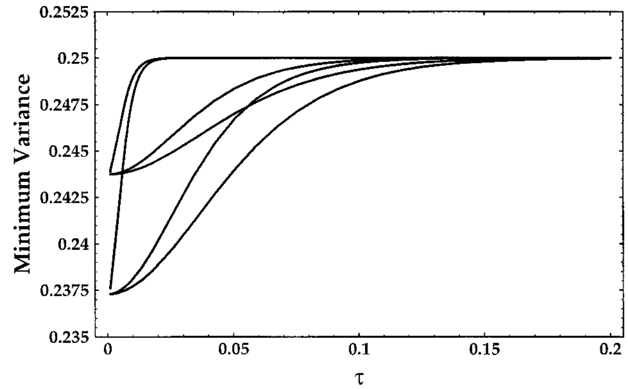

FIG. 1. Minimum zero-point noise in the field of a dielectric cavity as a function of the modulation time τ (in psec) of the medium. The vacuum level limit 0.25 corresponds to the unperturbed cavity, while smaller values are achieved with a modulation of the medium. The upper (lower) set of curves corresponds to 5% (10%) relative change in the permeability with the following parameters (from top to bottom): $\{\lambda = 5\ \mu\mathrm{m},\ \varepsilon_0 = 1\}$, $\{\lambda = 30\ \mu\mathrm{m},\ \varepsilon_0 = 1\}$, and $\{\lambda = 30\ \mu\mathrm{m},\ \varepsilon_0 = 2\}$. For a 5% (10%) change in the dielectric permeability the maximum noise reduction is ~2.5% (5%).

$$\left.\begin{matrix}\Delta X_\chi\\ \Delta Y_\chi\end{matrix}\right\} = \frac{1}{4} + \frac{|R|}{2[1-|R|^2]}\,[|R| \mp \cos(2\chi - 2\phi_f + \phi_b)]. \qquad (22)$$

The t_2 time dependence has been replaced by the phase χ: this angle determines the quadrature phase of the field and is governed by the detection scheme that is used [16]. In view of the physical mechanism that produces squeezing, one can interpret the photon creation originating from the distortion of the zero-point quantum fluctuations of the field as a nonlinear excitation of the vacuum so that photon pairs can be created. This two-photon character of the field is clearly apparent in the transformation in Eq. (14). The quantum effects induced by this nonlinear excitation are strictly determined by the time duration of the excitation, by the material and cavity mode parameters. We plot in Fig. 1 the minimum value of the variance in Eq. (22) as a function of the modulation time for different cavity and material parameters. The system exhibits maximum squeezing (~5%) for infrared cavity modes and times of the order of fractions of a picosecond. In general, an increase of the resonant wavelength, or larger values of the initial dielectric constant ε_0, make squeezing available at slightly longer times. The modulation depth $r_{0,f}$ and the modulation speed τ (fixed λ,ε_0), consistently with $\partial\varepsilon/\varepsilon\,\partial t < \Omega_\lambda$, ultimately determine the maximum achievable squeezing. It is instructive to observe that squeezing is independent of the initial phase and amplitude of the field: equal contractions in the quadrature fluctuations obtain for either an initial *vacuum state* or an initial *coherent state*. The size of the electromagnetic vacuum distortions can then be recovered through a measure of *either* the zero-point noise reduction *or* the noise reduction in a cavity mode initially populated by a large coherent mean field.

The *photon-counting statistics*, on the other hand, is analyzed in terms of deviations from classical Poisson statistics: This is conveniently assessed in terms of the Mandel Q

factor [15] $Q \equiv \langle [\Delta n]^2 \rangle - \langle n \rangle / \langle n \rangle$ with $\langle [\Delta n]^2 \rangle = \langle n^2 \rangle - \langle n \rangle^2$. The sign of Q determines whether $\langle [\Delta n]^2 \rangle$ exceeds $\langle n \rangle$ or $\langle n \rangle$ exceeds $\langle [\Delta n]^2 \rangle$, i.e., whether the photon-counting statistics is super- or sub-Poissonian. Since for a classical stochastic field Q must be non-negative, sub-Poissonian photon statistics is regarded as an essential quantum feature of the field [15]. For a cavity field initially in a coherent state the photon-number fluctuations after the modulation are

$$\langle [\Delta \hat{n}_2]^2 \rangle = \frac{2|R|^2 + |\alpha|^2[1 + 8|R|^2 - 4|R|\cos \Phi]}{(1-|R|^2)^2}, \quad (23)$$

so that

$$Q_2 = \frac{|R|^4 + |R|^2 + |R||\alpha|^2[|R|^3 + 8|R| - 2(1+|R|^2)\cos \Phi]}{(1-|R|^2)\{|R|^2 + |\alpha|^2[1+|R|^2 - 2|R|\cos \Phi]\}}. \quad (24)$$

Unlike for squeezing, the counting statistics exhibits nearly opposite behaviors, depending on the initial number of photons in the cavity. For an initial *vacuum state* the photon-number fluctuations after the modulation always exceed the photon-number mean value: in this case the dielectric originates radiation with a super-Poissonian statistics and Q_2 is close to unity. This is a typical feature of the effect of squeezing on the vacuum [15]. Therefore a rapid change in time of the permeability somehow reorganizes the phase of the vacuum, which is inherently random. For a *coherent state* with an initial nonvanishing mean number of photons the number distribution may instead become slightly narrower than before the modulation for appropriate values of the phase Φ (fixed τ) or of τ (fixed Φ). In this case the phase uncertainty for the state resulting from a change of the refractive index becomes larger than that of the original coherent state to satisfy the usual number-phase uncertainty [15]. Figure 2 (upper frame) represents the variation of Q_2 with $N \equiv |\alpha|^2$ and Φ, for a modulation of fixed duration. The addition of an appropriately large coherent amplitude can convert the super-Poissonian statistics of the perturbed vacuum into a sub-Poissonian photon-number distribution. This nonclassical effect, however, survives only for very short modulations. Figure 2 (lower frame) represents the variation of Q_2 with N and the length τ of the modulation. In this case, for each τ, we take ϕ_b and ϕ so that the phase difference Φ is fixed and equal to 0. For the parameters considered here only a few percent narrowing of the photon-number distribution takes place for modulations shorter than some fractions of picosecond and completely disappears thereafter. The sub-Poissonian regime takes place provided $|R|^3 + |R| + |\alpha|^2[|R|^3 - 2|R|^2 + 8|R| - 2] < 0$, when $\Phi = 0$. Since $|R|$ is much smaller than unity this condition sets, for each $|R|$, a lower bound on the initial number of cavity photons leading to a narrowing of the photon-counting distribution. For the cavity and pumping parameters we examine here, $|\alpha_{\min}|^2 \approx 10^{-2}$, which produces in practice a very sharp super- to sub-Poissonian transition.

In general, the appearance of effects associated with a distortion of the zero-point vacuum fluctuations is restricted to a nonadiabatic regime [1–5]. However, for a dielectric cavity sizeable effects occur also in a quasiadiabatic regime

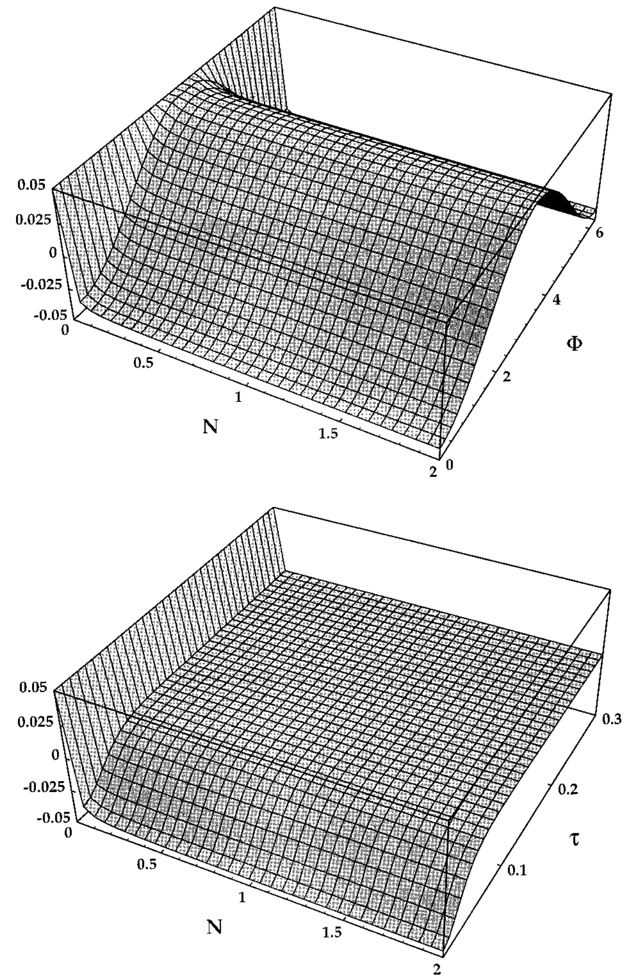

FIG. 2. Upper frame: Mandel Q factor as a function of the photon number N and the field phase Φ for $\tau = 0.01$ psec. Lower frame: Mandel Q factor as a function of the photon number N and the time duration τ (in psec) for $\Phi = 0$. The cavity field is initially in a coherent state with average photon number N, phase Φ, and $Q = 0$. The cavity and material parameters are, respectively, $\lambda = 25$ μm and $r_{0,f} = \varepsilon_0/\varepsilon_f = 1.11$. A super- to sub-Poissonian transition occurs for $N \geqslant 10^{-2}$ and modulations shorter than 0.3 psec (upper frame) or phases $|\Phi| \leqslant \pi/2$ (lower frame), producing up to a 5% maximum narrowing of the distribution. For $N > 2$ the features of this figure are unchanged.

or when the time scales of the dielectric modulation become comparable to those associated with the photon cavity frequency. Notice that in this case one still has $\partial\varepsilon/\varepsilon\,\partial t < \Omega_\lambda$ since the relative change in the dielectric permeability is usually rather small (<10%) [17]. On the other hand, when the modulation time scales are larger than those for the cavity mode frequency nonclassical effects completely disappear. Thus, for slow changes of the permeability the response of the vacuum fluctuations is always adiabatic and no effect is observed, while rapid enough changes can cause real transitions, boost quantum fluctuations into real photons, and modify the quantum statistics of the cavity vacuum.

IV. CONCLUSION

We use a rather straightforward approach to study the quantization of the electromagnetic field in a time-dependent medium and to demonstrate nonclassical effects in the field of a one-dimensional dielectric cavity whose medium has a rapidly changing time-dependent refractive index. Squeezing, in particular, occurs and a measure of the change of the zero-point noise in a dielectric cavity would then provide a way to probe distortions of the zero-point fluctuations of the radiation field. For certain time scales, cavity and material parameters noise reductions of the order of a few tenths of a decibel could be achieved, which are just large enough to be measurable [18], unlike the immeasurable number of photons produced by such a distortion.

ACKNOWLEDGMENT

We should like to acknowledge valuable discussions with Professor E. Yablonovitch.

[1] G. T. Moore, J. Math. Phys. **11**, 2679 (1970).

[2] S. A. Fulling and P. C. W. Davies, Proc. R. Soc. London Ser. A **348**, 393 (1976); P. C. Davies and S. A. Fulling, *ibid*. **356**, 237 (1977).

[3] M. Castagnino and K. Ferraro, Ann. Phys. (N.Y.) **154**, 1 (1984), and references therein.

[4] E. Yablonovitch, Phys. Rev. Lett. **62**, 1742 (1989).

[5] E. Yablonovitch, in *Resonances*, edited by M. D. Levenson, E. Mazur, P. Pershan, and Y. R. Shen (World Scientific, Singapore, 1990); in *Laser Optics of Condensed Matter*, edited by E. Garmire, A. A. Maradudin, and K. K. Rebane (Plenum, New York, 1991), Vol. 2, p. 325.

[6] Y. Yafet and E. Yablonovitch, Phys. Rev. B **43**, 12 480 (1991).

[7] See, e.g., M. T. Jaekel and S. Reynaud, J. Phys. (France) I **2**, 149 (1992); J. Schwinger, Proc. Natl. Acad. Sci. U.S.A. **89**, 4091 (1992); and also Ref. [4].

[8] H. B. G. Casimir, Proc. Kon. Ned. Akad. Wet. **51**, 793 (1948).

[9] See also A. A. Lobashov and V. M. Mostepanenko, Theor. Math. Phys. **88**, 340 (1991), Z. Bialynicka-Birula and I. Bialynicki-Birula, J. Opt. Soc. Am. B **4**, 621 (1987). For a discussion on the quantization in a time-independent but inhomogeneous medium see, e.g., L. Knoll, W. Vogel, and D.-G. Welsh, Phys. Rev. A **36**, 3803 (1987); R. J. Glauber and M. Lewenstein, *ibid*. **43**, 467 (1991).

[10] This is only relevant to the model. One of the mirrors has otherwise a nonvanishing trasmittivity in order to couple the cavity field outside.

[11] We restrict ourselves to a frequency and wave-vector nondispersive medium.

[12] An analogous situation occurs in the case of cavity QED with moving walls: when one of the walls is moving on a time scale that is long compared to the time scale of the cavity mode frequency, the Lagrangian acquires a similar expression. For discussion we refer, e.g., to the recent work of G. Calucci, J. Phys. A **25**, 3873 (1992).

[13] L. S. Brown and L. J. Carson, Phys. Rev. A **20**, 2486 (1979).

[14] P. Epstein, Proc. Natl. Acad. Sci. U.S.A. **16**, 627 (1930); see also V. L. Ginzburg, in *Propagation of Electromagnetic Waves in Plasma* (Nauka, Moscow, 1967), Chap. IV.

[15] See, e.g., R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987); J. Opt. Soc. Am. B **4**, (10) (1987), special issue on squeezed states of electromagnetic fields, edited by H. J. Kimble and D. F. Walls.

[16] Squeezing in the cavity field is typically analyzed by beating the cavity signal through a beam splitter to an external local oscillator whose phase is directly related to the phase angle χ. Phase-sensitive photon noise reduction in the case of a pulsed dielectric cavity could be measured, e.g., with a gated antenna and the quadrature phase χ of the field could be singled out by a suitable synchronous gating of the detector antenna to the pulse generating the cavity modulation.

[17] See, for instance, Fig. 1 ($\lambda=30$ μm, $\varepsilon_0=1$). The time scale for this far-infrared mode is $\Omega_\lambda^{-1}\approx 0.016$ psec. Noticeable effects occur already for modulation times $\tau\cong 1.5\Omega_\lambda^{-1}$ (quasiadiabatic limit) and completely disappear for times $\tau\approx 1$ psec (adiabatic limit). We have $\Delta\varepsilon/\varepsilon\leqslant 0.1$ so that in both limits $\Omega_\lambda^{-1}\partial\varepsilon/\varepsilon\partial t<1$, consistent with the approximation adopted in our treatment (cf. Sec. II).

[18] Current technologies permit us to measure noise reduction up to few hundredths of a decibel.