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Preparation energy for electromagnetically induced transparency

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We discuss a requirement for the laser energy that is necessary to initiate electromagnetically induced transparency: The number of photons in the coupling laser pulse must exceed the product of the number of atoms in the laser path times the ratio of the oscillator strengths of the probe and coupling laser transitions.

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It has been shown experimentally that if two lasers are applied to a resonant three-state system (Fig. 1), the atoms will be driven to a population trapped state, and a medium that is opaque to a probe laser can, by applying both lasers simultaneously, be made transparent [1–4]. When both lasers are pulsed, the dynamics of this process is especially interesting and the extent to which transparency is established depends on the relative pulse shapes, their rate of change relative to the instantaneous Rabi frequency (adiabaticity), and the timing of the applied pulses [5–9].

One method for establishing electromagnetically induced transparency (EIT) is to use matched pulses; i.e., pulses that have identical complex envelopes but arbitrary absolute amplitude and phase. In Ref. [6], EIT with matched pulses has been studied for the special case where the linewidth of state $|3\rangle$ is large compared to both the Rabi frequencies and the rate of change of the applied pulses. Here, we extend this work to allow for an arbitrary linewidth of state $|3\rangle$ and focus on the time scale and minimum laser energy that is necessary to establish transparency in an ideal, Doppler-free medium. We give numerical evidence and heuristic arguments that show that the time scale for establishing transparency with adiabatic matched pulses is much shorter than the time scale for establishing a population trapped state of a single atom; and for pulses that vary slowly as compared to the Pythagorean sum of the Rabi frequencies on the two transitions, pulsewidth is irrelevant and the requirement for the initiation of transparency is that the *number of photons in the* $|2\rangle\rightarrow|3\rangle$ *transition laser pulse exceed the number of atoms in the laser path times the ratio of the oscillator strengths of the probe and coupling laser transitions.*

Our notation is established in Fig. 1. At time $t=0$, all of the atoms are in the ground state $|1\rangle$. The envelopes of the probe and coupling laser pulses are denoted by the complex (dimensionless) quantities $f(z,t)$ and $g(z,t)$, respectively [10]. We consider one-dimensional propagation and write the electromagnetic fields in terms of their Rabi frequencies

$$\Omega_p(z,t)=\mathrm{Re}\{\Omega_p f(z,t)\exp[j(\omega_p t-k_p z)]\},$$

$$\Omega_c(z,t)=\mathrm{Re}\{\Omega_c g(z,t)\exp[j(\omega_c t-k_c z)]\},$$

where the quantities Ω_p and Ω_c are real and independent of space and time.

To avoid the need for density-matrix notation we assume that state $|3\rangle$ decays to states that are outside the system. We study the ideal case where the linewidth of the $|1\rangle\rightarrow|2\rangle$ transition is zero and where inhomogeneous broadening can be neglected. (As discussed below, these are reasonable approximations for short laser pulses.) We make the rotating-wave approximation and neglect the contribution of other transitions to the propagation constant. Working in local time $\tau=t-z/c$, the equations for the probability amplitudes a_i and electromagnetic fields $f(z,\tau)$ and $g(z,\tau)$ are

$$\frac{\partial a_1}{\partial\tau}=\frac{j}{2}\Omega_p f a_3,\quad \frac{\partial a_2}{\partial\tau}=\frac{j}{2}\Omega_c g a_3,$$
$$\frac{\partial a_3}{\partial\tau}+\frac{\Gamma_3}{2}a_3=\frac{j}{2}\Omega_p f^* a_1+\frac{j}{2}\Omega_c g^* a_2, \tag{1a}$$

$$\Omega_p\frac{\partial f}{\partial z}=-j\frac{\alpha_p}{2}\Gamma_3 a_3^* a_1,\quad \Omega_c\frac{\partial g}{\partial z}=-j\frac{\alpha_c}{2}\Gamma_3 a_3^* a_2. \tag{1b}$$

The quantities α_p and α_c are defined as the power absorption coefficients of the probe and coupling laser with all of the atoms in state $|1\rangle$ or state $|2\rangle$, respectively; i.e., $\alpha_p=N\sigma_p$ and $\alpha_c=N\sigma_c$, where $\sigma_p=2\omega_p|\mu_{13}|^2/(c\epsilon_0\hbar\Gamma_3)$, $\sigma_c=2\omega_c|\mu_{23}|^2/(c\epsilon_0\hbar\Gamma_3)$, N is the atom density, and the μ_{ij} are the transition matrix elements.

We define new variables

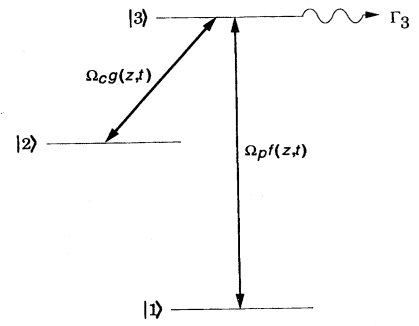

FIG. 1. Energy-level schematic for the analysis. Pulse shapes $f(\tau)=g(\tau)$ are applied at $z=0$.

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \frac{1}{\Omega_s}\begin{bmatrix} \Omega_c & -\Omega_p \\ \Omega_p & \Omega_c \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix},$$
$$\begin{bmatrix} \Omega_s h \\ \Omega_s s \end{bmatrix} = \frac{1}{\Omega_s}\begin{bmatrix} \Omega_c & -\Omega_p \\ \Omega_p & \Omega_c \end{bmatrix}\begin{bmatrix} \Omega_p f \\ \Omega_c g \end{bmatrix}. \quad (2)$$

The probability amplitude of state a_3 is the same in either basis; i.e., $b_3 = a_3$ and $\Omega_s^2 = \Omega_p^2 + \Omega_c^2$.

Equations (1a) and (1b) are then

$$\frac{\partial b_1}{\partial \tau} = \frac{j}{2}\Omega_s h b_3, \quad \frac{\partial b_2}{\partial \tau} = \frac{j}{2}\Omega_s s b_3,$$
$$\frac{\partial b_3}{\partial \tau} + \frac{\Gamma_3}{2} b_3 = \frac{j}{2}\Omega_s h^* b_1 + \frac{j}{2}\Omega_s s^* b_2; \quad (3a)$$

$$\Omega_s \frac{\partial h}{\partial z} = -\frac{j}{2}\Gamma_3 b_3^*(\alpha_1 b_1 + \alpha_3 b_2),$$
$$\Omega_s \frac{\partial s}{\partial z} = -\frac{j}{2}\Gamma_3 b_3^*(\alpha_3 b_1 + \alpha_2 b_2), \quad (3b)$$

where $\alpha_1 \equiv (\Omega_c^2\alpha_p + \Omega_p^2\alpha_c)/\Omega_s^2$, $\alpha_2 \equiv (\Omega_p^2\alpha_p + \Omega_c^2\alpha_c)/\Omega_s^2$, and $\alpha_3 \equiv (\Omega_p\Omega_c/\Omega_s^2)(\alpha_p - \alpha_c)$.

For matched pulses applied to a medium with all atoms initially in the ground state, the initial condition on the atoms at $\tau = 0$, for all z, is $a_1(z,0) = 1$, $a_2(z,0) = a_3(z,0) = 0$, and therefore $b_1(z,0) = \Omega_c/\Omega_s$, $b_2(z,0) = \Omega_p/\Omega_s$, and $b_3(z,0) = 0$. The boundary condition on the fields at $z = 0$, for all τ, is $f(0,\tau) = g(0,\tau)$ and therefore $h(0,\tau) = 0$ and $s(0,\tau) = f(0,\tau)$.

We observe that if either $b_2 = h = b_3 = 0$ or $b_1 = s = b_3 = 0$, then all derivatives are zero; and in this sense, the paired variables b_1, s and b_2, h, both with $b_3 = 0$, are the normal modes of the system [6]. It is apparent from Eq. (3) that if matched pulses are applied at $z = 0$, then *irrespective of adiabaticity*, ultimately the system will evolve into the first of these normal modes, and thereafter the medium will be transparent. But as Γ_3 decreases, the time scale for establishing the normal mode and transparency becomes ever longer.

If the matched pulses are adiabatic ($\partial f/\partial\tau \ll \Omega_s$), preparation occurs much more quickly, and depends only on the energy of the pulses. Figure 2 shows this result: At $z = 0$, we apply matched pulses with Rabi frequencies $\Omega_p = 1$ and $\Omega_c = 10$, and numerically solve Eq. (1) to obtain the pulse shapes at the end of a medium that is 50 units long. The decay rate Γ_3 of state $|3\rangle$ is decreased so that in part (a) it is 10 and in part (b) it is 0.01. The absorption coefficients of the probe and coupling lasers are equal, $\alpha_p = \alpha_c$, and vary inversely as Γ_3, so that the product of the absorption coefficient and decay rate is constant. This normalization keeps the total number of atoms in the laser path unchanged as Γ_3 is reduced.

The solid and dashed curves in each part of Fig. 2 show the probe pulse in local time at the start and end of the medium. Irrespective of the ratio of Γ_3 to the Rabi frequencies, the pattern is the same. For early times the medium is opaque and the probe pulse is absorbed. At a critical time,

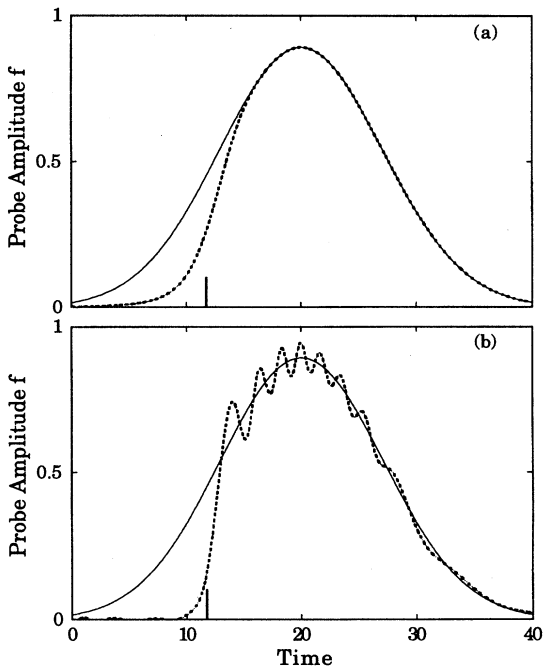

FIG. 2. Probe amplitude vs local time (s) at $z = 0$ (solid lines) and $z = 50$ (dashed lines). In each figure, $\Omega_p = 1$, $\Omega_c = 10$, and the applied pulse is ten units long. In the successive figures, the decay rate is reduced and the absorption coefficient $\alpha_p = \alpha_c \equiv \alpha$ is increased so that the atom density length product is constant. (a) $\Gamma_3 = 10$, $\alpha = 0.1$. (b) $\Gamma_3 = 0.01$, $\alpha = 100$. The tick mark at $\tau = 11.7$ denotes the time at which the integral from zero to τ of the coupling laser photons is equal to the number of atoms in the laser path.

which is roughly the same in both figures, the medium rather abruptly becomes transparent. The upward tick at $\tau = 11.7$ in each figure denotes that time at which the integral of the coupling laser photons from zero to $\tau = 11.7$ is equal to the number of atoms in the path of the laser beam. We thus observe that the medium becomes *nearly transparent* at a time when the number of coupling laser photons that have passed through it is equal to the (oscillator strength weighted) number of atoms in the laser path.

In Fig. 3 we focus on the case where $\Gamma_3 = 0.01$ and examine the behavior of the atoms in both the bare and in the normal mode bases. The solid and dashed curves show the probability amplitudes at the start ($z = 0$) and end ($z = 50$) of the medium. At $z = 0$, the optical pulses are exactly matched and the atoms exhibit strong oscillations in their probability amplitudes. For matched pulses, these oscillations occur when $\Gamma_3 < \Omega_s$, and decay on a time scale of $1/\Gamma_3$. This is the time scale for establishing transparency for a single atom or of an optically thin sample. In this example, this time scale is much longer than the applied pulse. At $z = 50$, the atom behavior is different. Here, the oscillations in the probability amplitudes are much less pronounced and, as the probe pulse passes, the atoms may be viewed as making a transition to a reasonably good normal mode, thereby establishing transparency in a time much less than $1/\Gamma_3$.

It is important to note that the coupling laser energy that is required to initiate transparency is not absorbed by the me-

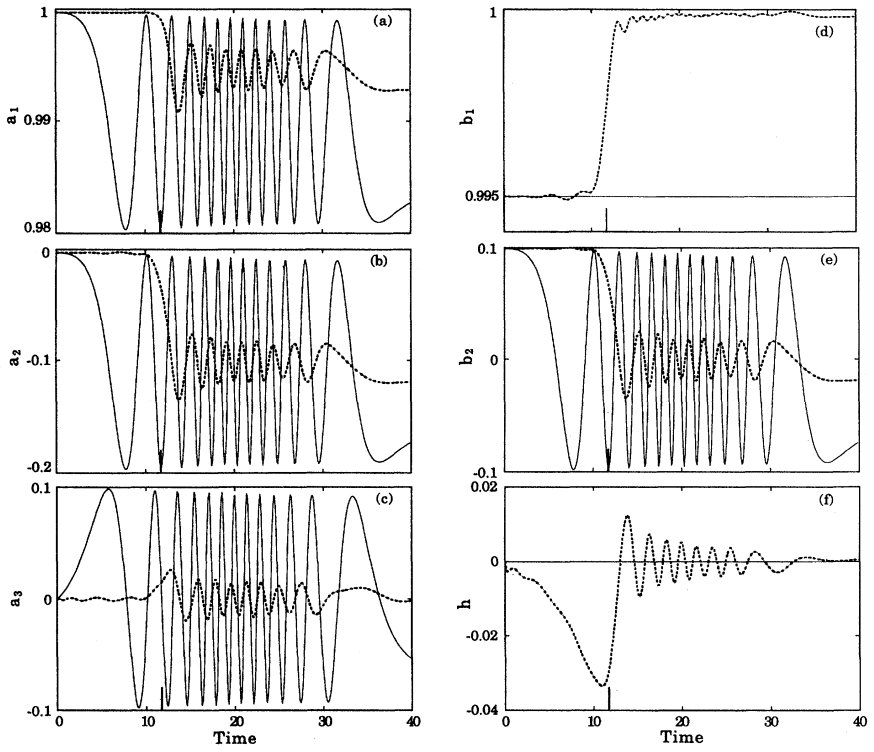

FIG. 3. Atomic probability amplitude vs local time (s) for the conditions of part (b) of Fig. 2. The behavior of the atoms is shown at $z=0$ (solid lines) and $z=50$ (dashed lines) in the bare and normal mode bases. At the time when the medium becomes transparent, the atoms make a transition to a reasonably good normal mode.

dium, and in fact, the front edge of the coupling laser gains energy as the pulse propagates. The need for a preparation energy is therefore, perhaps, better understood as a requirement on preparation time. We first note that for a weak probe, and a time invariant coupling laser, the product of the group velocity delay and the photon density of the coupling laser is equal to the oscillator strength weighted number of atoms in the laser path; i.e., if T_D is the group delay and P_c/A is the power density of the coupling laser at $z=0$, then

$$T_D=\left(\frac{1}{V_G}-\frac{1}{c}\right)L, \quad \frac{1}{\hbar\omega_c}\left(\frac{P_c}{A}\right)T_D=\frac{f_{13}}{f_{23}}NL. \tag{4}$$

This expression is easily derived using the expressions in Ref. [5] and has been noted for a related problem [11]. It is also true if the coupling laser varies adiabatically.

The behavior when adiabatic weak probe matched pulses are applied to a medium is as follows: At first, the atoms are in the ground state, the front edge of the probe pulse is absorbed, and the coupling laser amplified; as the front edge of the probe is reduced in intensity, the coupling laser pulse moves past it, and the probe propagates at a group velocity that corresponds to the instantaneous coupling laser intensity. When the probe reaches the end of the medium, preparation is complete, and thereafter both pulses travel with velocity c. If the coupling laser pulse is shorter than T_D, then it will not overlap the slow probe pulse at the end of a medium of length L.

For matched pulses, the probe pulse loses energy linearly with distance, and energy remains in the medium after the pulse passes. This lost front-edge energy may be estimated as follows: For a Gaussian pulse $f(\tau)$ with pulsewidth T_P and normalized so that $\int_{-\infty}^{+\infty}|f|^2 d\tau=T_P$, the pulsewidth when the number of photons in the coupling laser pulse is equal to the number of atoms in a laser path of length L is $T_L=(\Gamma_3/\Omega_c^2)\alpha_p L$, and the ratio of lost probe energy to total probe energy is T_L/T_P. For the parameters of Fig. 2, $T_L=0.5$, $T_P=10$, and the estimated probe pulse loss is 5%.

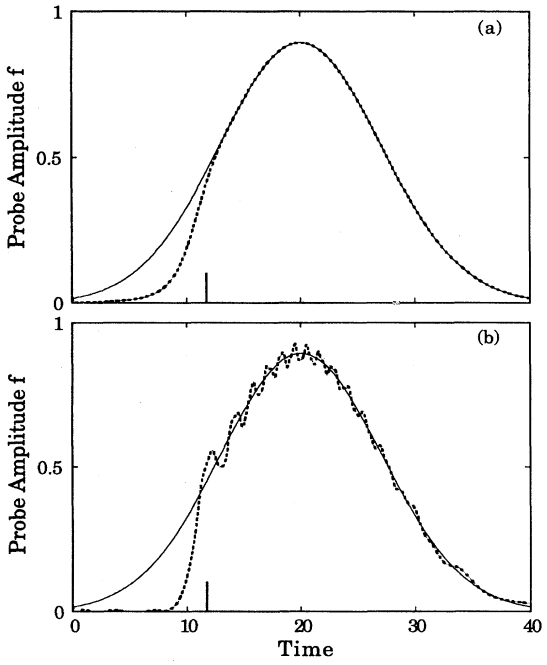

FIG. 4. Probe amplitude vs local time (s) at $z=0$ (solid lines) and $z=50$ (dashed lines). $\Omega_p=\Omega_c=10$. All other conditions are the same as in Fig. 2.

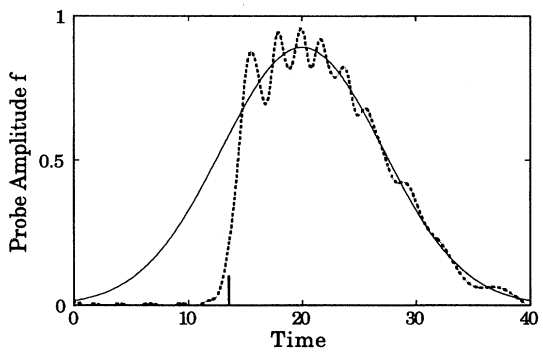

FIG. 5. Probe amplitude vs local time (s) for unequal absorption coefficients and parameters $\Omega_p=1$, $\Omega_c=10$, $\Gamma_3=0.01$, $\alpha_p=200$, and $\alpha_c=100$. $z=0$ (solid line) and $z=50$ (dashed line). The tick mark at $\tau=13.54$ denotes the time at which Eq. (4) is satisfied.

The actual probe pulse losses in Figs. 2(a) and 2(b) are 7.4% and 5.3%, respectively.

In Fig. 4 we show the input and output laser pulses for equal Rabi frequencies. Preparation occurs somewhat sooner than in the case for a small probe; but overall, to within a factor of 2 in preparation energy, the behavior is comparable to the small probe case. Figure 5 shows that we obtain similar behavior when the absorption coefficients α_p and α_c are not equal.

These results suggest the application of picosecond and subpicosecond lasers to EIT experiments in their own right, and to nonlinear processes [12] and laser without inversion experiments that use EIT [13,14]. The requirements on the coupling laser pulse are that (a) it must be short as compared to the inverse linewidth of the $|1\rangle\rightarrow|2\rangle$ transition, (b) it must vary slowly as compared to its instantaneous Rabi frequency, and (c) it must have an energy such that the number of photons exceeds the oscillator strength weighted number of atoms in the laser path.

For metal vapors or gases, these conditions are easily satisfied. Two-photon Doppler widths are usually less than 0.1 cm^{-1} and therefore require pulses shorter than 50 ps. Efficient nonlinear frequency conversion will require an atom-length product of about 10^{16} atoms/cm^2, and therefore several mJ per cm^2 of laser energy density. For matrix elements of 1 a.u., the Rabi frequency at 10^8 W/cm^2 is about 10 cm^{-1} and therefore satisfies the rise-time requirement. For a beam diameter of 1 mm, experiments in metal vapors or gases will require picosecond time-scale pulses, with pulse energies of about 1 mJ.

In essence, group delay allows counterintuitive preparation of EIT on a time scale that is short as compared to the decay or dephasing rate of the upper state. This type of preparation requires pulses that vary slowly as compared to their instantaneous Rabi frequency.

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