RAPID COMMUNICATIONS

PHYSICAL REVIEW A VOLUME 52, NUMBER 2 AUGUST 1995

Decoherence from spontaneous emission

Ole Steuernagel and Harry Paul

Arbeitsgruppe "Nichtklassische Strahlung" der Max-Planck-Gesellschaft an der Humboldt-Universität zu Berlin, Rudower Chaussee 5, 12484 Berlin, Germany

(Received 16 February 1995)

Starting from a generalization of the Weisskopf-Wigner model for the case of a two-level atom with a largely spread center-of-mass wave function, we show that spatial correlations are destroyed to some extent by a spontaneously emitted photon. We derive a particularly simple form of the corresponding decoherence function and determine the dependence of the decoherence on orientation and size of a detector registering the outgoing photon.

PACS number(s): 03.75.Be, 03.65.Bz, 32.80.−t

I. INTRODUCTION

When describing an excited atom that is spontaneously emitting a photon one usually considers the atom strongly localized, i.e., pointlike compared to the wavelength of the emitted light. Though this assumption is usually adequate in regard to the extension of the electron cloud around the center of mass of the atom, it need not be so in regard to the atom's center-of-mass wave function $\psi(\boldsymbol{x},t)$. Through modern experimental technology it has become possible to prepare atoms whose uncertainty in position exceeds considerably the wavelength of the emitted light [1–3]. It was shown in a recent pioneering experiment by Pfau *et al.* [4] that in this case the emitted light decoheres the spatial correlations of the atomic density matrix. These authors also gave a short theoretical treatment of the decoherence effect. Focusing on one spatial dimension (z), they started from the observation that the averaged density matrix for the center-of-mass coordinate, $g^{(1)}(z)=\int\rho(z'-z,z')\,dz'$, the transverse *one-point* coherence function, is just the Fourier transform of the distribution for the z component of the momentum. The final momentum distribution is readily calculated [5], and Fourier transforming the result yields the final coherence function $g_f^{(1)}(z)$ in the product form

$$g_f^{(1)}(z)=g_i^{(1)}(z)D(z)\ , \tag{1}$$

where $g_i^{(1)}$ is the initial coherence function, and the decoherence function $D(z)$ proves to be the Fourier transform of the distribution for the momentum (in the z direction) of the emitted photon.

In the present paper we will generalize this result in three ways: (i) we will consider a wavepacket in the full three-dimensional space; (ii) we will study the change of the atomic density matrix $\rho(\boldsymbol{x},\boldsymbol{x}')$ itself, i.e., the true two-point correlation function; and (iii) we will evaluate the decoherence function on condition that the emitted photon has hit a detector with variable size. We will show that a relation similar to Eq. (1) holds, namely [6]

$$\rho_{\text{emitt}}(\boldsymbol{x},\boldsymbol{x}+\boldsymbol{r},t)=\rho(\boldsymbol{x},\boldsymbol{x}+\boldsymbol{r},t)\ D(\boldsymbol{r})\ , \tag{2}$$

where the density matrix ρ_{emitt} describes atoms that have undergone spontaneous emission, whereas ρ refers to atoms that have not (provided they were treated identically except for excitation and hence subsequent spontaneous emission). We will derive an explicit expression for the decoherence function $D(\boldsymbol{r})$ and discuss its properties. Moreover, we will clarify the physical assumptions and approximations on which Eq. (2) rests.

First of all, let us illustrate the physical meaning of the decoherence function in an atomic interference experiment of Young's type. We assume the impinging wavepacket to be "quasimonochromatic"; i.e., to have a well-defined de Broglie wavelength. Then, in perfect analogy to classical optics, the "intensity"; that is, the probability to detect an atom, at a given position in the observation plane, can be written as

$$I\sim\rho(\boldsymbol{x},\boldsymbol{x})+\rho(\boldsymbol{x}+\boldsymbol{r},\boldsymbol{x}+\boldsymbol{r})+2\ \text{Re}\ \{\rho(\boldsymbol{x},\boldsymbol{x}+\boldsymbol{r})e^{i\phi(\tau)}\}\ , \tag{3}$$

where the argument τ in the phase factor denotes the difference of the propagation times from the locations of the holes $\boldsymbol{x}$ and $\boldsymbol{x}+\boldsymbol{r}$ in the interference screen to the observation point. One learns from Eq. (3) that the visibility of the interference pattern is given by

$$\frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}}=\frac{2|\rho(\boldsymbol{x},\boldsymbol{x}+\boldsymbol{r})|}{\rho(\boldsymbol{x},\boldsymbol{x})+\rho(\boldsymbol{x}+\boldsymbol{r},\boldsymbol{x}+\boldsymbol{r})}\ . \tag{4}$$

The influence of decoherence according to Eq. (2) is now readily explained: It follows from Eq. (3) that the phase of the decoherence factor $D(\boldsymbol{r})$ gives rise to a shift of the interference pattern, whereas its modulus, according to Eq. (4), describes a reduction of the visibility.

II. GENERAL THEORY

We utilize a recently found solution [7] to the problem of spontaneous emission, in the dipole approximation, from an extended wavepacket. This is an extension of the treatment by Weisskopf and Wigner [8], taking the center-of-mass motion into account. We are interested in the atomic state only after the emission has happened; thus we consider the initially excited atoms only at times t larger than the mean lifetime of the excited level γ_0^{-1}. In this case the solution [7] for the wave function $|\Psi(t)\rangle$ of the total system reduces to

$$|\Psi(t)\rangle=\int d^3p\sum_j\ \beta_j(\boldsymbol{p},\boldsymbol{k}_j;t)\ |\boldsymbol{p},\boldsymbol{k}_j\rangle\ , \tag{5}$$

where

$$\beta_j(\boldsymbol{p},\boldsymbol{k}_j;t)=\lambda_j\alpha_0(\boldsymbol{p}+\hbar\boldsymbol{k}_j)\frac{\exp\left\{-i\left[\frac{p^2}{2\hbar M}+\omega_j-\frac{\omega_0}{2}\right]t\right\}}{\frac{\boldsymbol{p}\cdot\boldsymbol{k}_j}{M}+\frac{\hbar k_j^2}{2M}+\omega_0-i\gamma_0-\omega_j}\,. \tag{6}$$

Here $\boldsymbol{p}$ denotes the atomic momentum, and $\boldsymbol{k}_j$ and ω_j are the wave vector and the frequency of the radiation modes. The label j stands for both the wave vector and the polarization, ω_0 denotes the atomic eigenfrequency, M is the atomic mass, and $\alpha_0(\boldsymbol{p})$ denotes the initial center-of-mass wave function, for the atom, in momentum representation. The λ's are the coupling constants (in units of $\hbar$) describing dipole coupling. Due to our assumption $t\gg\gamma_0^{-1}$, the atom is in the ground state with certainty. Hence the internal atomic state does not affect the density operator for the center-of-mass dynamics and has therefore been discarded in Eq. (5).

In what follows, we will assume that no polarization-sensitive measurement is performed on the emitted photon. Choosing the two independent polarization directions associated with the wave vector $\boldsymbol{k}=k(\cos\varphi\sin\vartheta,\sin\varphi\sin\vartheta,\cos\vartheta)$ as $\boldsymbol{u}_1=(\cos\varphi\cos\vartheta,\sin\varphi\cos\vartheta,-\sin\vartheta)$ and $\boldsymbol{u}_2=(-\sin\varphi,\cos\varphi,0)$, we can perform the summation over the two polarizations, for given $\boldsymbol{k}$, with the result

$$\begin{aligned}|\lambda_{\boldsymbol{k}}|^2&\equiv|\lambda_{\boldsymbol{k},1}|^2+|\lambda_{\boldsymbol{k},2}|^2\\&=g_k^2[(d_x\sin\varphi-d_y\cos\varphi)^2\\&\quad+(d_x\cos\varphi\cos\vartheta+d_y\sin\varphi\cos\vartheta-d_z\sin\vartheta)^2]\,,\end{aligned} \tag{7}$$

where $\boldsymbol{d}=(d_x,d_y,d_z)$ is the atomic dipole moment and g_k a universal coupling constant.

We imagine that a detector equipped with an optical imaging system collects radiation corresponding to a certain solid angle $\Delta\Omega$ in $\boldsymbol{k}$ space. Then, according to the axioms of quantum mechanical measurement theory, a response of the detector will give rise to a reduction of the wave function (5) to the density matrix (in $\boldsymbol{p}$ representation)

$$\rho_{\text{emitt}}(\boldsymbol{p},\boldsymbol{p}';t)=\text{const}\sum_{j[\Delta\Omega]}\beta_j(\boldsymbol{p},\boldsymbol{k}_j;t)\beta_j^*(\boldsymbol{p}',\boldsymbol{k}_j;t)\,, \tag{8}$$

where the trace over j is restricted to those modes whose propagation direction falls into the solid angle $\Delta\Omega$.

We now assume that the atomic velocity is small compared to the velocity of light (nonrelativistic approximation). Then the second term in the denominator of Eq. (6) is negligibly small compared to ω_0, and the same holds true for the first term [9]. Hence in what follows we will drop those terms and pass from Eq. (8) to the $\boldsymbol{x}$ representation

$$\rho_{\text{emitt}}(\boldsymbol{x},\boldsymbol{x}';t)=\text{const}\int d^3p\int d^3p'\int_{[\Delta\Omega]}d^3k\;|\lambda_{\boldsymbol{k}}|^2e^{\,i(\boldsymbol{p}\cdot\boldsymbol{x}-\boldsymbol{p}'\cdot\boldsymbol{x}')/\hbar}\frac{\alpha_0(\boldsymbol{p}+\hbar\boldsymbol{k})\alpha_0^*(\boldsymbol{p}'+\hbar\boldsymbol{k})e^{-i(p^2-p'^2)t/(2\hbar M)}}{(\omega-\omega_0)^2+\gamma_0^2}\,, \tag{9}$$

where Eq. (7) has been used. Upon substituting $\boldsymbol{p}+\hbar\boldsymbol{k}=\boldsymbol{P}$, $\boldsymbol{p}'+\hbar\boldsymbol{k}=\boldsymbol{P}'$, we can rewrite Eq. (9) as

$$\begin{aligned}\rho_{\text{emitt}}(\boldsymbol{x},\boldsymbol{x}';t)&=\text{const}\int d^3P\int d^3P'\int_{[\Delta\Omega]}d^3k\;|\lambda_{\boldsymbol{k}}|^2e^{i(\boldsymbol{P}\cdot\boldsymbol{x}-\boldsymbol{P}'\cdot\boldsymbol{x}')/\hbar}e^{i\boldsymbol{k}\cdot(\boldsymbol{x}'-\boldsymbol{x})}\frac{\alpha_0(\boldsymbol{P})\alpha_0^*(\boldsymbol{P}')e^{-i[(P^2-P'^2)/2\hbar+(\boldsymbol{P}'-\boldsymbol{P})\cdot\boldsymbol{k}]t/M}}{(\omega-\omega_0)^2+\gamma_0^2}\\&=\text{const}\int_{[\Delta\Omega]}d^3k\;|\lambda_{\boldsymbol{k}}|^2\psi_t\left(\boldsymbol{x}+\frac{\hbar\boldsymbol{k}}{M}t\right)\psi_t^*\left(\boldsymbol{x}'+\frac{\hbar\boldsymbol{k}}{M}t\right)\frac{e^{i\boldsymbol{k}\cdot(\boldsymbol{x}'-\boldsymbol{x})}}{(\omega-\omega_0)^2+\gamma_0^2}\,,\end{aligned} \tag{10}$$

where $\omega=ck$.

It is obvious from Eq. (10) that we arrive at an expression of the factorized form (2) if we neglect the kick the atom experiences due to emission, expressed by the correction $\hbar\boldsymbol{k}t/M$ in the argument of the atomic center-of-mass wave function. When will this be justified? Inserting for t the value $10\gamma_0^{-1}$ consistent with the derivation of Eq. (6), and assuming the atomic velocity in the laboratory frame to be of the order of 1000 m/s, we determine the ratio of the shift induced by the recoil of an optical transition ($\omega_0=10^{15}$/s, $\gamma_0=10^8$/s) to the particle's de Broglie wavelength λ_{dB} and find: $\hbar k_0t/(M\lambda_{dB})=2\pi c/(v\omega_0t)\approx2\times10^{-2}$. Clearly such a small relative shift can be neglected.

Dropping this correction in Eq. (10) we arrive at the desired result (2), where the decoherence factor is given by

$$D(\boldsymbol{r})=\text{const}\int_{[\Delta\Omega]}d^3k\;|\lambda_{\boldsymbol{k}}|^2\frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}}}{(\omega-\omega_0)^2+\gamma_0^2}\,, \tag{11}$$

which is, in fact, the Fourier transform of the momentum distribution for the *registered* photons. Thus Eq. (11) is the extension of the previous result [4] in three respects: (i) it is the generalization to three dimensions; (ii) it allows us to study the influence of actual measurements on the emitted photons; and, what is most important, (iii) it describes the decoherence of the full density matrix. Of course, since the decoherence factor depends only on $\boldsymbol{r}$, it also describes the deterioration of the averaged density matrix $\int d^3x\;\rho(\boldsymbol{x},\boldsymbol{x}+\boldsymbol{r};t)$, i.e., the one-point coherence function that was studied in [4] for the one-dimensional case. Finally, one learns from Eq. (2) that the normalization condition for the decoherence function simply reads $D(0)=1$, thus determining the constant prefactor in Eq. (11).

III. DISCUSSION

Let us now discuss different experimental conditions, i.e., detectors that, seen from the atom's position, cover various solid angles $\Delta\Omega$ in $\boldsymbol{k}$ space.

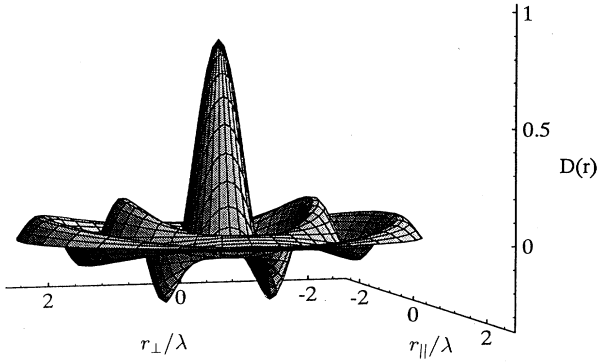

FIG. 1. Dependence of the decoherence function [Eq. (13)] on the parallel and the orthogonal components of the displacement vector $\boldsymbol{r}$ with respect to the dipole axis $\boldsymbol{d}$, $\Delta\Omega = 4\pi$.

A. $\Delta\Omega = 4\pi$

This is the situation one encounters when a fictitious detector is employed that counts any photon, or, more physically, when no measurement is made at all. The integral, Eq. (11), can be split into two angular integrations over ϕ, θ, and one radial integration over k running from 0 to ∞. The k integral can be extended over the whole real axis introducing a negligible error [10]; hence this yields a residual integral and the intermediate result

$$D_{4\pi}(\boldsymbol{r}) = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \, \sin\theta \, |\lambda_{\boldsymbol{k}_0}|^2 \, \frac{\pi}{\gamma_0} \, e^{-\gamma_0 r|\cos\theta|} \, e^{i\omega_0 r\cos\theta} . \tag{12}$$

Neglecting the very smooth envelope $e^{-\gamma_0 r|\cos\theta|}$ and inserting the coupling according to Eq. (7), we finally obtain

$$D_{4\pi}(\boldsymbol{r}) = \frac{3}{2}\frac{1}{2d_\perp^2 - d_{||}^2}\left\{ \mathrm{sinc}(k_0 r) d_\perp^2 + \left[\frac{\sin(k_0 r)}{(k_0 r)^3} - \frac{\cos(k_0 r)}{(k_0 r)^2}\right] \times (2d_{||}^2 - d_\perp^2) \right\} , \tag{13}$$

where $d_{||}$ and $d_\perp$ denote the parallel and the orthogonal components, respectively, of the dipole moment $\boldsymbol{d}$ with respect to $\boldsymbol{r}$ and $\mathrm{sinc}(x) = (\sin x)/x$. It follows from the behavior of the sinc function that the coherence is destroyed for $k_0 r \geq 1$; i.e., when r is of the order of the wavelength λ. This is indeed what one expects from a simple physical argument: The radiation could be collected by a microscope to image the emitter. It is well known from classical theory that in this way one can determine the position of the emitter with an accuracy that is roughly given just by λ. Hence, quantum correlations must be restricted to a spatial region of extension λ.

Moreover, one observes from the result (13) that coherence is damped off faster in the direction of $\boldsymbol{r}$ parallel to the dipole moment than perpendicular to $\boldsymbol{d}$. This feature becomes evident from Fig. 1.

In what follows we will specialize to the case of a randomly orientated dipole; then the second term in curly brackets of Eq. (13) vanishes.

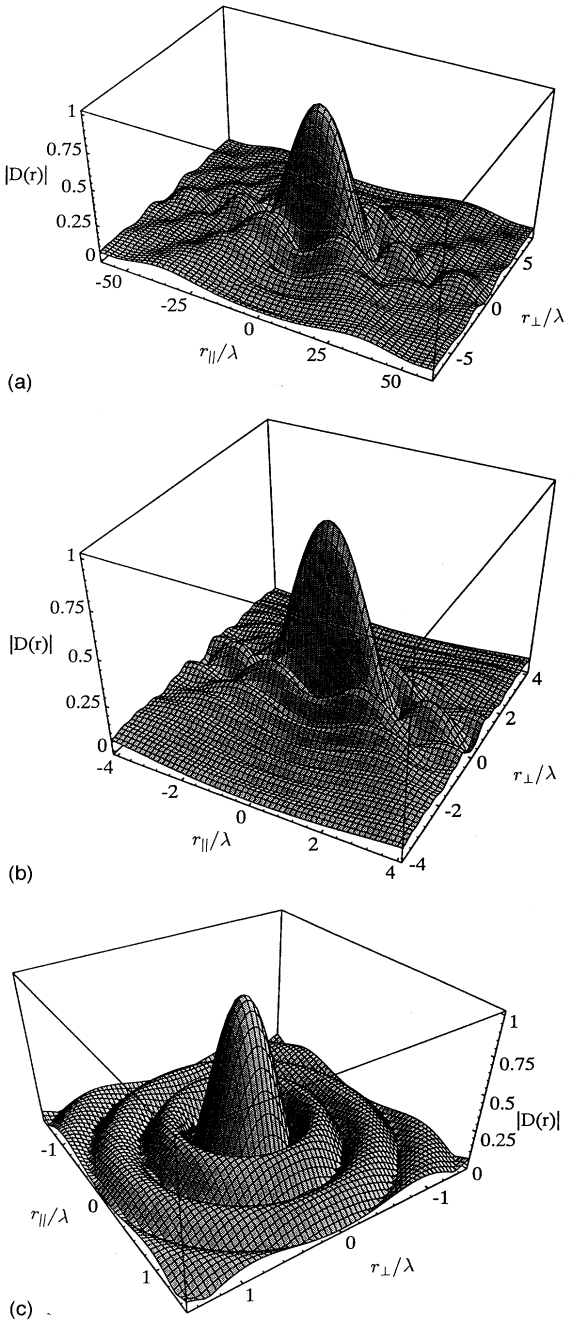

FIG. 2. Modulus of the decoherence function for a randomly oriented dipole [Eq. (18)] dependence on the parallel and the orthogonal components of the displacement vector $\boldsymbol{r}$ with respect to the detector axis. The collecting angle of the detector is assumed to be a polar cap with polar angle $\pi/10$ (a), $\pi/3$ (b), and π (c), respectively. Note the different scaling of the coordinate axes in these plots.

B. $\Delta\Omega < 4\pi$

For a randomly orientated dipole the squared coupling constant (7), $|\lambda_{\boldsymbol{k}}|^2$, reduces to $g_k^2 d^2$. Even with this simplification, the integration over the polar angles in Eq. (11) becomes rather involved for the general case of a fixed detector position with collecting angle $\Delta\Omega$ as well as an arbi-

trary displacement $\boldsymbol{r}$. It is only when we adapt the shape of $\Delta\Omega$, for given $\boldsymbol{r}$, to a polar coordinate system with its polar axis in the $\boldsymbol{r}$ direction, i.e., assume $\Delta\Omega$ to be bound by $\vartheta_1 \leq \vartheta \leq \vartheta_2$, $\varphi_1 \leq \varphi \leq \varphi_2$, that the integration can be done analytically. Neglecting the effect of the finite atomic lifetime γ_0^{-1}, the integrand, as a function of k, can be approximated, up to a constant factor, by a delta function $\delta(k-k_0)$, where $k_0=\omega_0/c$. Then the integral (11) is readily evaluated yielding

$$D_{\Delta\Omega}(\boldsymbol{r})=\frac{\exp(ik_0r\cos\vartheta_1)-\exp(ik_0r\cos\vartheta_2)}{ik_0r(\cos\vartheta_1-\cos\vartheta_2)}$$
$$=\operatorname{sinc}(k_0r[\cos\vartheta_1-\cos\vartheta_2]/2)$$
$$\times\exp\{ik_0r[\cos\vartheta_1+\cos\vartheta_2]/2\}\ . \qquad (14)$$

Note that $D_{\Delta\Omega}(\boldsymbol{r})$ does not depend on $\varphi_2-\varphi_1$ when normalized such that $D(0)=1$. We see from Eq. (14) that, in contrast to the case $\Delta\Omega=4\pi$, the decoherence factor is now a complex function. For the extreme case of a detector covering an infinitesimal solid angle $d\Omega$, Eq. (14) simplifies to

$$D_{d\Omega}(\boldsymbol{r})=\exp[i\boldsymbol{k}_0\cdot\boldsymbol{r}]\ . \qquad (15)$$

This simple result shows that a photon emitted into a precisely defined direction does not decohere the atomic density matrix, but only shifts its phase. Hence it is not the photon's recoil that is primarily responsible for the decoherence effects, but the optical resolving power of the employed detector. The destruction of coherence is described by the modulus of $D(r)$ that follows from Eq. (14) as

$$|D(\boldsymbol{r})|=|\operatorname{sinc}(k_0r[\cos\vartheta_1-\cos\vartheta_2]/2)|$$
$$=\left|\operatorname{sinc}\left(k_0r\sin\frac{\vartheta_1+\vartheta_2}{2}\sin\frac{\vartheta_2-\vartheta_1}{2}\right)\right|\ . \qquad (16)$$

One learns from Eq. (16) that $|D(\boldsymbol{r})|$, for fixed $\vartheta_2-\vartheta_1$, falls off faster the closer the detector's central axis along $(\vartheta_1+\vartheta_2)/2$ comes to the equator of the polar coordinate system at $\vartheta=\pi/2$. Since the polar axis is given by the $\boldsymbol{r}$ direction, this means that the decoherence is strongest in the plane *perpendicular* to the direction of observation. Moreover, it becomes obvious from Eq. (16) that $D(\boldsymbol{r})$ falls off slower, the smaller $\vartheta_2-\vartheta_1$; i.e., the smaller $\Delta\Omega$.

In the more practical case of a detector with a collecting angle, the form of which does not depend on the direction of $\boldsymbol{r}$, we will choose the central axis of $\Delta\Omega$ as the polar axis and the detector to be a polar cap, $0\leq\varphi\leq2\pi$, $0\leq\vartheta\leq\vartheta_1$. Then, characterizing the direction of $\boldsymbol{r}$ by the polar angles ϕ and Θ, we have to calculate the integral

$$D(\boldsymbol{r})=\text{const}\int_0^{\vartheta_1}\sin\vartheta d\vartheta\int_0^{2\pi}d\varphi\exp\{ik_0r[\cos(\varphi-\phi)$$
$$\times\sin\vartheta\sin\Theta+\cos\vartheta\cos\Theta]\}\ . \qquad (17)$$

The integral over φ yields the Bessel function I_0; i.e., we arrive at the result

$$D(\boldsymbol{r})=\text{const}\int_0^{\vartheta_1}\sin\vartheta\ d\vartheta\ I_0(k_0r\sin\vartheta\sin\phi)$$
$$\times\exp(ik_0r\cos\vartheta\cos\Theta)\ . \qquad (18)$$

This integral has been evaluated numerically. The results are depicted in Fig. 2. One recognizes from Fig. 2(a) that the decoherence is much stronger in the direction of $\boldsymbol{r}$ orthogonal to the detector axis, compared to the parallel orientation. Moreover, it is obvious that with growing collecting angle $\Delta\Omega$ the decoherence effect becomes stronger, as predicted also by the analytic solution (16).

In summary we have used a recent extension of the Weisskopf-Wigner theory of spontaneous emission to derive an explicit expression for the decoherence function. In position representation this decoherence function simply is a factor changing the density matrix of the atomic center-of-mass wave function. We have specified the underlying approximations and studied various observational conditions.

ACKNOWLEDGMENTS

Ole Steuernagel thanks John Vaccaro and Arkadiusz Orłowski for instructive criticism.

[1] M. Sigel, C. S. Adams, and J. Mlynek, in *Frontiers in Laser Spectroscopy*, Proceedings of the International School of Physics "Enrico Fermi," Course CXX, Varenna, 1992, edited by T. W. Hänsch and M. Inguscio (North-Holland, Amsterdam, 1994).

[2] T. Sleator, O. Carnal, A. Faulstich, and J. Mlynek, in *Quantum Measurements in Optics*, edited by P. Tombesi and D. F. Walls (Plenum Press, New York, 1994).

[3] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer, Berlin, 1994), Chap. 17.

[4] T. Pfau, S. Spälter, Ch. Kurtsiefer, C. R. Ekstrom, and J. Mlynek, Phys. Rev. Lett. **73**, 1223 (1994).

[5] L. Mandel, J. Opt. **10**, 51 (1979).

[6] A relationship of the form (2) has already been derived for photon scattering. See C. Cohen-Tannoudji, F. Bardou, and A. Aspect, in *Proceedings of the 10th International Conference on Laser Spectroscopy*, edited by M. Ducloy, E. Giacobino, and G. Camy (World Scientific, Singapore, 1992).

[7] K. Rzążewski and W. Żakowicz, J. Phys. B **25**, L319 (1992).

[8] V. Weisskopf and E. Wigner, Z. Phys. **63**, 54 (1930); **65**, 18 (1930).

[9] The denominator of Eq. (6) can be factorized into $c^2(k-k_-)(k-k_+)$. Eventually an integral over all positive k is to be performed; see Eq. (11). The two poles contribute, but $k_+\approx-2Mc/\hbar$ is very far off resonance and has to be discarded on physical grounds, whereas $k_-\equiv(k_0-i\gamma_0/c)/(1-p\cos\Theta/Mc)$ remains, with Θ as the angle between $\boldsymbol{p}$ and $\boldsymbol{k}$. With our assumption $v\ll c$ we therefore end up with the denominator $2Mc^2(\omega-\omega_0+i\gamma_0)/\hbar$ in Eq. (6) and hence with Eq. (9) as given above.

[10] O. Steuernagel (unpublished).