Reexamining the assumption that elements of reality can be Lorentz invariant

O. Cohen and B. J. Hiley
Physics Department, Birkbeck College, University of London, Malet Street, London WC1E 7HX, England
(Received 23 May 1994)

We examine a gedanken experiment described by Hardy, which purports to prove that the "elements of reality" corresponding to Lorentz-invariant observables cannot themselves be Lorentz invariant without violating quantum mechanics. We consider a number of criticisms of this proof and show that these criticisms are not convincing. We demonstrate that the contradiction, which arises in the gedanken experiment and forms the basis of Hardy's proof, has nothing to do with realism, but is a consequence of the well-known noncovariance of the reduction postulate. A reexamination of the gedanken experiment, using a more appropriate formalism, helps to clarify its implications. We conclude with a brief appraisal of realist interpretations of quantum mechanics in the relativistic domain.

PACS number(s): 03.65.Bz

Bell [1] has constructed a realist quantum-field theory and although this model is not very successful, it raises some interesting questions. In particular, it raises the question as to whether it is possible, in general, to make theories of this nature fully Lorentz invariant. In a recent paper, Hardy [2] has presented a gedanken experiment where the question of Lorentz invariance can be examined in a particularly interesting manner. Using this experiment he claimed to have proved that the "elements of reality" corresponding to Lorentz-invariant observables cannot themselves be Lorentz invariant without violating quantum mechanics. This proof was effected by first assuming Lorentz invariance for the elements of reality referred to in the experiment and then showing that a contradiction necessarily followed from this assumption.

More recently there have been a number of criticisms of Hardy's proof [3,4]. In this paper we discuss these criticisms, and Hardy's reply [5], and show that they lead to some confusion. A further criticism by Vaidman [6] leads to the conclusion that the gedanken experiment does not lead to any contradiction, because it involves a pre- and postselected quantum system, for which, it is claimed, the "product rule" does not apply. Unfortunately, as we show in a separate paper [7], Vaidman's analysis is not valid because it makes incorrect use of the formula of Aharonov, Bergmann, and Lebowitz [8,9].

We will also show that Hardy's analysis of the gedanken experiment leads to the same contradiction even if we reformulate it in terms of standard quantum mechanics [10] and avoid any mention of realism. One difficulty is that Hardy uses a formalism that is not suitable for the relativistic case. When the gedanken experiment is analyzed using a more appropriate formalism, the source of the contradiction and its implications are made explicit.

The apparatus used in the gedanken experiment consists of two Mach-Zehnder interferometers that are arranged so that their paths overlap (see Fig. 1). An electron-positron pair is created at S and one of the particles is fed into each interferometer. Both interferometers have detectors attached to their outputs, labeled $C+, D+$ for the positron and $C-, D-$ for the electron. If the interferometer paths did not overlap, there would be zero probability of a detection at either $D+$ or $D-$ because of destructive interference within each interferometer. However, the overlap is such that there is a probability $\frac{1}{4}$ for the particles to meet at a point P and annihilate. The possibility of this annihilation alters the final detection probabilities, so that there is an overall probability of $\frac{1}{16}$ for the positron and the electron to be detected at $D+$ and $D-$, respectively.

Consider two sets of Lorentz observers $\{L_1\}$ and $\{L_2\}$ such that for the set $\{L_1\}$, the positron is detected at $C+$ or $D+$ before the electron arrives at BS2$-$, while for the set $\{L_2\}$, the electron is detected at $C-$ or $D-$ before the positron arrives at BS2$+$. From Hardy's analysis the appropriate wave function for each L_1 immediately before the detection of a positron at $C+$ or $D+$ is

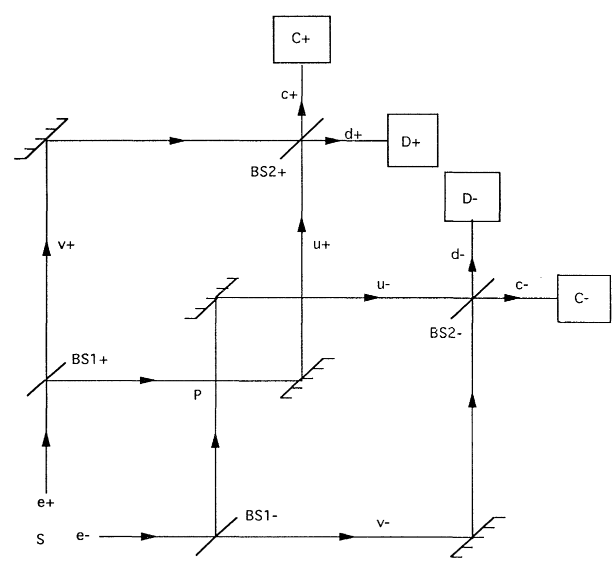

FIG. 1. Double Mach-Zehnder interferometer for positrons and electrons.

$$|\psi_{L_1}\rangle = \frac{1}{\sqrt{6}}\{|c^+\rangle[-|u^-\rangle + 2i|v^-\rangle] + i|d^+\rangle|u^-\rangle\} , \tag{1}$$

while the wave function for each L_2 immediately before the detection of the electron at $C-$ or $D-$ is

$$|\psi_{L_2}\rangle = \frac{1}{\sqrt{6}}\{[-|u^+\rangle + 2i|v^+\rangle]|c^-\rangle + i|u^+\rangle|d^-\rangle\} . \tag{2}$$

Thus for the combination of detections at $D+$ and $D-$, respectively, one set of Lorentz observes, $\{L_1\}$, on detecting the positron at $D+$, will be able to predict with certainty that the electron must be in the state $|u-\rangle$. This means that, according to Hardy's sufficient condition [11], "electron in state $|u-\rangle$" constitutes an element of reality for each L_1. There is another set of Lorentz observers, $\{L_2\}$, that, on detecting the electron at $D-$, can predict with certainty that the positron must be in the state $|u+\rangle$. So, for each L_2, "positron in state $|u+\rangle$" constitutes an element of reality. If L_1 and L_2 adopt a realist perspective and assume that the positron and the electron follow trajectories, then L_1 can infer that the electron must have passed through P, while L_2 can infer that the positron must have passed through P. But we know that it is impossible to have both the positron and the electron at P together since in that case they would annihilate and therefore could not be detected at $D+$ and $D-$, respectively. Hence there is an apparent contradiction, indicating that the elements of reality referred to cannot be Lorentz invariant.

Berndl and Goldstein [3], Clifton and Niemann [4], and, in response to [3], Hardy [5] have attempted to resolve this contradiction by using a number of different arguments, which raise some interesting points. Their main arguments can be summed up as follows.

(i) From a relativistic perspective, a prediction with certainty, yielding an element of reality, may be applicable only to certain Lorentz observers. For other Lorentz observers, this prediction may correspond to a retrodiction and these observers will not necessarily be able to infer a definite outcome (i.e., with probability one) from their retrodiction.

(ii) It is questionable whether those predictions with certainty that are *not verifiable* in the given experimental setup can still legitimately be labeled as elements of reality.

(iii) The possibility of nonlocal influences further brings into question the significance of some of the predictions with certainty, in particular those predictions that could only be confirmed by measurements performed in a region spacelike separated from the region in which the measurements on which the predictions are based are carried out.

These arguments are compounded, however, by the introduction of the concept of contextuality by the abovementioned authors and by their suggestion that the contradiction arising in the gedanken experiment can be resolved by taking into account contextual aspects. Before we examine these arguments further, let us recall how contextuality arises in realist interpretations of quantum mechanics.

Several so-called "no-go" theorems [13] have purported to show that the results of quantum mechanics cannot be derived using "hidden variables." The proofs of these theorems require an assumption that the hidden variables be identified with the eigenvalues of observables, these values being simply revealed in experiments. However, Bell [14] has shown that *contextual* hidden variable theories are not ruled out by the no-go theorems. In these theories, the value taken by a hidden variable in the context of a particular measurement may differ from the value taken by it in the context of another measurement that is incompatible with the first one. This suggests that, physically, the context should be identified with the apparatus used in an experiment. As a consequence, we would expect the contextual properties of the hidden variables in a particular experiment to become relevant only if the apparatus is replaced or modified in some way. However, in Hardy's gedanken experiment there is only one single experimental arrangement, which is common to all the Lorentz observers; hence it is not immediately apparent in what sense contextual features could be significant here. (Of course hidden variables cannot normally be predicted with certainty and hence they do not in general satisfy the sufficient condition for elements of reality [11]. Nevertheless, we can think of those elements of reality that do satisfy this condition as comprising a subset of the set of hidden variables.)

We now return to the arguments (i), (ii), and (iii). Consider first how argument (i) relates to the gedanken experiment. L_1's prediction yielding the element of reality electron in the state $|u-\rangle$ (according to the sufficient condition [11]) can only be made *after* the positron has been detected at $D+$. This precludes the possibility of L_1 making any prediction that might yield the element of reality positron in the state $|u+\rangle$. Similarly, L_2's prediction yielding the element of reality positron in the state $|u+\rangle$ can only be made *after* the electron has been detected at $D-$, thus precluding the possibility of L_2 making any prediction that might yield the element of reality electron in the state $|u-\rangle$. L_1 can only retrodict what the earlier state of the positron might have been had the appropriate measurement been carried out and, equally, L_2 can only retrodict what the earlier state of the electron might have been following an earlier (counterfactual) measurement. These retrodictions cannot be made with certainty; they do not yield outcomes with probability one. However, it would be wrong to infer from this relativistic prediction-retrodiction ambiguity that the aforementioned elements of reality are context dependent. It would be more accurate to describe them as "observer specific." A change of Lorentz observer does not normally imply a change of context; simply introducing a second Lorentz observer or Lorentz boosting the original observer does not in general influence the system being observed. The Lorentz observers can be at an arbitrary spatial separation from the observed system and we would not expect the Lorentz observers' wave functions to be entangled with that of the observed system, so nonlocal influences between observers and the observed system are ruled out.

Now consider arguments (ii) and (iii). It is true that the predictions yielding the elements of reality electron in state $|u-\rangle$ and positron in state $|u+\rangle$ are not verifiable by means of the experimental setup described in [2]. Furthermore, alternative experimental setups designed to verify either of these predictions separately would not be suitable for verifying *both* predictions simultaneously [3]. However, the sufficient condition [11] does not require that predictions be verifiable in order to qualify as elements of reality. This condition is formulated in terms of the predictability of results of hypothetical measurements. According to this formulation, it is irrelevant whether such measurements are, or indeed can be, carried out. Similarly, the possibility of nonlocal influences does not affect the ability of L_1 and L_2 to make predictions with certainty about the states of the electron and the positron, respectively. The sufficient condition [11] would have to be modified in some way if predictions that could be verified only by measurements made in a spacelike separated region were deemed to be unreliable because of possible nonlocal influences.

As far as contextuality is concerned, the suggestion that arguments (ii) and (iii) indicate that the elements of reality discussed in [2] are context dependent can be seen to be incorrect. Quantum-mechanical probabilities in general are independent of context. (The idea that quantum-mechanical probabilities could be context dependent in a *retrodictive* sense arose in the work of Albert, Aharonov, and D'Amato [15], but their analysis has been shown to be erroneous by Bub and Brown [16] and Sharp and Shanks [17].) Hence, if the value taken by a particular element of reality can be predicted with probability one, then, since this probability is not context dependent, it follows that this same element of reality can be predicted with certainty to have the same value in *all* contexts. In other words, the elements of reality satisfying the sufficient condition [11] are not context dependent. (Of course, if there are elements of reality not satisfying this condition, then these elements of reality might be context dependent; but such elements of reality do not enter the argument leading to the contradiction in [2].) Clearly then, the contextual argument does not provide a convincing resolution of the contradiction arising in Hardy's gedanken experiment.

Clifton and Niemann [4] further suggest that, from an orthodox perspective, the contradiction derived by Hardy can be accounted for by the noncovariance of position operators. For example, it has been shown that a spinning particle that is localized for one Lorentz observer will not be localized for any other Lorentz observer [18]. However, as we have already remarked, in Hardy's argument the relevant inferences about particle positions are counterfactual and do not involve measurements and so position operators do not enter the discussion. Thus the apparent noncovariance of Hardy's elements of reality is essentially unrelated to the noncovariance of position operators.

Let us reexamine Hardy's argument in more detail. An essential step in arriving at the contradiction involves an analysis of the collapse of the entangled two-particle Schrödinger wave function. Hardy argues that for an observer L_1, the collapse occurs across the constant time hyperplane defined by the time of detection of the positron at $D+$ as measured by the observer L_1 while for another observer L_2, the collapse occurs along a constant time hyperplane defined by the time of detection of the electron at $D-$ as measured by that observer. Thus for L_1 the wave function given by Eq. (1) collapses to $|d+\rangle|u-\rangle$, while for L_2 the wave function given by Eq. (2) becomes $|u+\rangle|d-\rangle$. In other words, L_1 argues that since $D+$ registered, the state of the electron before it arrived at BS2-, but after it passed P, was $|u-\rangle$, while L_2 argues that since $D-$ registered, the state of the positron before it arrived at BS2+ and after it passed P was $|u+\rangle$. Now a detection at $D+$ and $D-$ for the *same* electron-positron pair implies that, if we combine the conclusions of L_1 and L_2 for this pair, then the positron must be in the state $|u+\rangle$ and the electron must be in the state $|u-\rangle$. But this combined state is impossible according to standard quantum mechanics, because at no stage in its evolution does the wave function describing the system contain a $|u+\rangle|u-\rangle$ term, since this term, according to Hardy, produces annihilation quanta. So the contradiction remains even if we stick to the standard interpretation of quantum mechanics [10], avoiding any mention of trajectories or elements of reality. Hence we see that the contradiction has its roots in the noncovariant nature of the collapse formalism and has nothing to do with how we choose to interpret quantum mechanics.

So far our discussion has been restricted to the nonrelativistic Schrödinger formalism; however, the noncovariant feature of the collapse is still present in the Dirac theory and in quantum electrodynamics. Discussions of this difficulty with noncovariance have a long history going back to the pioneering work of Landau and Peierls [19]. As a consequence of this analysis, Berestetskii, Lifshitz, and Pitaevskii [20] write, ". . . we reach the conclusion that the entire formalism of [nonrelativistic] quantum mechanics becomes insufficient in the relativistic case. The wave functions $\psi(q)$, in their original sense as carriers of unobservable information, cannot appear in the formalism of a consistent relativistic theory." A similar sentiment has been expressed by Dirac [21]. He points out that for typical Hamiltonians used in field theory "it is not possible to get a solution of Schrödinger's equation for which the state vector stays in Hilbert space" and that, as a consequence of this, we must abandon the notion of a wave function in such a theory. But here we are not dealing with such Hamiltonians and the wave function used in the description of Hardy's experiment can be written down. This leaves open the question as to whether it is possible to find a description of the collapse process that *is* Lorentz invariant.

One notable attempt to provide such a description has been made by Hellwig and Kraus [22]. They have suggested that the state reduction does not occur instantaneously, but takes place along the *backward* light cone of the measurement event. This model does indeed satisfy the formal requirements of Lorentz invariance (since the light cone transforms into itself under a Lorentz transformation), although the idea of some form of propagation backward in time in *all* frames may be physically unappealing to some. Nevertheless, it does have the advantage of yielding the correct quantum-mechanical probabilities for measurements of *local* observables, although it does not give the correct probabilities for *nonlocal* observables of the type discussed by Aharonov and Albert [23]. They have shown that the measurement of certain nonlocal observables is possible for relativistic quantum systems without violating causality (despite the long-standing rejection of such a possibility by Landau and Peierls [19]) and that, because of this, Hellwig and Kraus's prescription for the reduction process must be deemed unsatisfactory.

A reexamination of the noncovariance of the standard state vector collapse formalism carried out by Aharonov and Albert [24] shows that, if relativistic considerations are taken into account, the state of a quantum-mechanical system cannot, in general, be described by a function of space-time, but necessitates the introduction of a *functional* on the set of spacelike hypersurfaces. This leads to the description originally formulated by Tomonaga [25] and Schwinger [26] when quantum electrodynamics was being developed. Here the ordinary Schrödinger wave function $\psi(\mathbf{x},t)$ is replaced by the functional $\Psi(t_\sigma(\mathbf{x}))$, where σ is the set of spacelike hypersurfaces through the space-time point $\mathbf{x}$. The functional-based formalism enables Aharonov and Albert to introduce a fully-Lorentz-invariant prescription for describing a measurement event at $\mathbf{x}$. Immediately after such a measurement, the state of the observed system is given by a functional that encompasses all the possible collapsed wave functions relating to the set of spacelike hypersurfaces containing $\mathbf{x}$.

The discussion of Aharonov and Albert leading to the introduction of the functional involves "verification" measurements of a nonlocal observable. As already mentioned, this leads Aharonov and Albert to reject the measurement prescription of Hellwig and Kraus and to conclude that the state reduction process must instead be instantaneous. However, once we adopt the notion of an instantaneous collapse, the necessity of the functional description becomes evident even when we consider a system that is subjected to measurements of *local* observables only. Consider, for example, a one-particle system in the state $\sum_i \psi_i(x,t)$. Suppose we perform a measurement in the laboratory frame at (x_1,t_1) and that, as a result, the state of the system collapses instantaneously to $\psi_n(x,t)$. We then ask, "What is the value that the wave function describing the state of this system takes at the point (x_2,t_2), where (x_1,t_1) and (x_2,t_2) are spacelike separated?" According to the standard reduction postulate, there are two possible answers $\sum_i \psi_i(x_2,t_2)$ and $\psi_n(x_2,t_2)$; which answer is obtained depends on the choice of Lorentz frame to which the measurement is referred. The two possible answers are obviously not related by a Lorentz transformation and for this reason the Schrödinger wave function describing the system cannot be a covariant function of space-time. However, the introduction of a functional, which encompasses both wave functions $\sum_i \psi_i(x,t)$ and $\psi_n(x,t)$, when referred to the set of spacelike hyperplanes containing the point (x_2,t_2), enables us to formulate a covariant description. If one continues to use the instantaneous collapse of nonrelativistic quantum mechanics in a relativistic context, then the introduction of the functional becomes essential if ambiguities are to be avoided.

We can now see how Hardy's formulation of his gedanken experiment, which is carried out in terms of wave functions as opposed to wave functionals, inevitably leads to difficulties. The state of the two-particle system immediately after one of the particles passes through a beam splitter BS2 (or reaches a detector) depends on which set of Lorentz frames is used. By introducing the functional-based formalism and employing the prescription for state reduction proposed by Aharonov and Albert, these ambiguities can be avoided. Thus when the positron is detected at $D+$, we should not single out a particular spacelike hypersurface passing through the space-time point at which the detection occurs. In fact, the state of the system, when the positron is detected at $D+$, will collapse to a functional that encompasses $|d+\rangle|u-\rangle$, $(1/\sqrt{2})|d+\rangle(i|c-\rangle-|d-\rangle)$, and $|d+\rangle|d-\rangle$. Similarly, the state of the system, when the electron is detected at $D-$, will collapse to a functional encompassing $|d-\rangle|u+\rangle$, $(1/\sqrt{2})|d-\rangle(i|c+\rangle-|d+\rangle)$, and $|d-\rangle|d+\rangle$. A contradiction will arise if we single out the two wave functions $|d+\rangle|u-\rangle$ and $|d-\rangle|u+\rangle$ and then try to draw inferences from their combination. The functional-based approach shows that we have no justification for doing this. The system is never actually in the state $|d+\rangle|u-\rangle$ or in the state $|d-\rangle|u+\rangle$. The only unambiguous conclusion we can draw from the detection at $D+$ is that the positron ends up in the state $|d+\rangle$, while the only unambiguous conclusion we can draw from the $D-$ detection is that the electron ends up in the state $|d-\rangle$. Taken together, these two detections show that the final state of the two-particle system must be $|d+\rangle|d-\rangle$. The other collapsed states referred to by Hardy have no physical basis; they are the result of applying an inappropriate formalism and it is not surprising that they lead directly to a contradiction.

The functional-based formalism helps to clarify the implications of the gedanken experiment. First, it shows that we cannot, after all, derive a contradiction simply by combining L_1's description of the state of the electron with L_2's description of the state of the positron. Second, it makes explicit the observer-specific properties of the predicted measurement results electron in state $|u-\rangle$ and positron in state $|u+\rangle$, of L_1 and L_2, respectively. Finally, it shows that the predicted outcome of a measurement on a system cannot necessarily be identified with the state of that system prior to the measurement, even if the measurement outcome can be predicted with certainty. For example, when the positron is detected at $D+$, the state of the electron is then given by a functional; but L_1 can still predict with certainty that if an appropriate measurement were carried out on the electron, immediately following, in L_1's frame, the detection of the positron at $D+$, then the state of the electron would be found to be $|u-\rangle$. Hence the use of the functional-based formalism brings into focus the way in which the identification of elements of reality with the predicted results of counterfactual measurements becomes particularly questionable in the relativistic case. (Of course, if the sufficient condition for elements of reality [11] is modified so that predicted results of counterfactual measurements are excluded, then Hardy's proof will no longer work since all the elements of reality he discusses are of this type.)

The necessity of introducing the functional-based formalism highlights the difficulties that can arise with *property attribution* in the relativistic domain. For example, if we analyze the gedanken experiment from a relativistic perspective, then, immediately after the positron is detected, we cannot attribute a definite wave function to the two-particle system because, as we have seen, this system must be described by a functional. However, if we restrict the analysis to the nonrelativistic domain, then we *can* assign a definite wave function to the system as soon as the positron is detected. In an interesting recent paper ([27] and references therein), Ghirardi and Grassi have argued that the added constraints on attributing properties to quantum systems in the relativistic domain have important implications for the interpretation of the Einstein-Podolsky-Rosen experiment. We will now give a brief summary of Ghirardi and Grassi's argument and show that the introduction of the functional can be seen to support their conclusion.

Suppose two spin-half particles are prepared in the singlet state $(1/\sqrt{2})(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle)$ and then fly apart in opposite directions along the x axis. At (x_1, t_1) the z component of spin of particle 1 is measured, yielding the result $\sigma_z^1 = 1$. Now, in a nonrelativistic treatment of this scenario, we can infer that, for $t < t_1$, the z component of the spin of particle 2 is not well defined, whereas for $t > t_1$ it has the definite value $\sigma_z^2 = -1$. In other words, particle 2 *instantaneously* acquires the definite property "$\sigma_z^2 = -1$" at time t_1, as the result of an event that takes place at an arbitrary spatial separation from it. However, if we look at this situation from a relativistic perspective, then we *cannot* assign a definite z component of spin to particle 2 until its path intersects the future light cone from (x_1, t_1). In the relativistic picture the two-particle system must be described by a functional, which, for any point on the section of particle 2's path in between its intersections with the past and future light cones from (x_1, t_1), encompasses both $(1/\sqrt{2})(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle)$ and $|\uparrow_{1z}\downarrow_{2z}\rangle$. It follows that, in the relativistic case, there is no implication of a *superluminal* influence causing particle 2 to acquire the property $\sigma_z^2 = -1$. This absence of any necessary "spooky action at a distance" in the relativistic case is emphasized by Ghirardi and Grassi.

We now discuss briefly whether, notwithstanding the questions raised by the gedanken experiment, it is still possible to develop a Lorentz-invariant realistic interpretation of quantum mechanics. A well-known interpretation of quantum mechanics involving particle trajectories is Bohm's ontological interpretation [28], in which particles can be nonlocally linked with each other via the quantum potential. The *actual* position and momentum of a particle are beables (a term coined by Bell [1] to contrast with the usual usage of the term "observables" in quantum mechanics), that is, they are assumed to exist independently of measurement or observation. Although this model, when considered at the *observable* level, yields the same Lorentz invariant probabilities as the orthodox approach, Bohm never claimed that his interpretation is Lorentz invariant at the *beable* level. Indeed Bohm and Hiley [29] have suggested that Lorentz invariance may arise as a statistical feature of an underlying stochastic process that is *not* Lorentz invariant.

In fact, if we are considering a system restricted to just *two* nonlocally correlated particles (such as the one described by Hardy), then it *is* possible to construct a realistic model that is effectively covariant at the beable level by assuming that the nonlocal correlations work instantaneously in the center-of-momentum frame only. Models of this type have been proposed by Droz-Vincent [30] and by Vigier [31] and are covariant in the sense that they do not require the introduction of a *global* preferred frame. Unfortunately, however, the center-of-momentum approach becomes problematic if we try to extend it to systems involving three or more particles.

Another set of models, which attempt to give a realistic description of the reduction process, involves the concept of "spontaneous localization." The relativistic linear continuous spontaneous localizations model [32] does not require a preferred Lorentz frame and cannot, even in principle, be used to transmit information faster than light. However, this model also rules out the possibility of ever explaining its stochastic features in terms of a deeper theory.

Another means of avoiding the requirement for a preferred frame altogether would involve abandoning the model based on a unique set of particle trajectories as a representation of the processes taking place at the beable level and replacing it with a model where, in general, the beables are represented by irreducible *distributions* of sets of trajectories. Each Lorentz observer would then "explicate" a single set of trajectories, defined by instantaneous nonlocal correlations in his or her particular frame. The notion of a unique set of particle trajectories would then become an intermediate concept since such unique sets would be neither beables nor observables. The suggestion that distributions of trajectories should be considered irreducible has already been made by Prigogine [33], although for entirely different reasons. In his investigation of the possibility of introducing a microscopic entropy operator, Prigogine concluded that "[it] is the description in terms of bundles of trajectories, or distribution functions, that becomes basic; no further reduction to individual trajectories or wave functions can be performed."

[1] J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987), pp. 171–180.
[2] L. Hardy, Phys. Rev. Lett. **68**, 2981 (1992).
[3] K. Berndl and S. Goldstein, Phys. Rev. Lett. **72**, 780 (1994).
[4] R. Clifton and P. Niemann, Phys. Lett. A **166**, 177 (1992).
[5] L. Hardy, Phys. Rev. Lett. **72**, 781 (1994).
[6] L. Vaidman, Phys. Rev. Lett. **70**, 3369 (1993).
[7] O. Cohen and B. J. Hiley (unpublished).
[8] Y. Aharonov, P. G. Bergmann, and J. L. Lebowitz, Phys. Rev. B **134**, 1410 (1964).
[9] Y. Aharonov and L. Vaidman, J. Phys. A **24**, 2315 (1991).
[10] J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, 1955).
[11] Hardy's sufficient condition for the elements of reality, a slight modification of the one used by Einstein, Podolsky, and Rosen [12], is "If we can predict with certainty (i.e., with probability equal to one) the result of measuring a physical quantity, then there exists an element of reality corresponding to this physical quantity and having a value equal to the measurement result." It follows that quantum-mechanical predictions that satisfy this condition constitute "elements of reality" regardless of whether or not we attribute to them any ontological status.
[12] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[13] A. M. Gleason, J. Math. Mech. **6**, 885 (1957); J. M. Jauch and C. Piron, Helv. Phys. Acta **36**, 827 (1963); S. Kochen and E. P. Specker, J. Math. Mech. **17**, 59 (1967); D. Mermin, Phys. Rev. Lett. **65**, 3373 (1990).
[14] J. S. Bell, Rev. Mod. Phys. **38**, 447 (1966).
[15] D. Z. Albert, Y. Aharonov, and S. D'Amato, Phys. Rev. Lett. **54**, 5 (1985).
[16] J. Bub and H. Brown, Phys. Rev. Lett. **56**, 2337 (1986).
[17] W. D. Sharp and N. Shanks, Philos. Sci. **60**, 488 (1993).
[18] See, for example, T. D. Newton and E. P. Wigner, Rev. Mod. Phys. **21**, 400 (1949).
[19] L. D. Landau and R. Peierls, Z. Phys. **69**, 56 (1931).
[20] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Relativistic Quantum Theory* (Pergamon, Oxford, 1971), p. 3.
[21] P. A. M. Dirac, *Lectures on Quantum Field Theory,* Belfer Graduate School of Science Monograph Series Vol. 3 (Yeshiva University, City, 1966), p. 5.
[22] K. E. Hellwig and K. Kraus, Phys. Rev. D **1**, 566 (1970).
[23] Y. Aharonov and D. Z. Albert, Phys. Rev. D **24**, 359 (1981).
[24] Y. Aharonov and D. Z. Albert, Phys. Rev. D **29**, 228 (1984).
[25] S. Tomonaga, Prog. Theor. Phys. **1**, 1 (1946).
[26] J. Schwinger, Phys. Rev. **76**, 790 (1949).
[27] G. Ghirardi and R. Grassi, Stud. Hist. Philos. Sci. **25**, 397 (1994).
[28] D. Bohm and B. J. Hiley, *The Undivided Universe: An Ontological Interpretation of Quantum Mechanics* (Routledge, London, 1993).
[29] D. Bohm and B. J. Hiley, Found. Phys. **21**, 243 (1991).
[30] P. Droz-Vincent, Phys. Rev. D **19**, 702 (1979).
[31] J.-P. Vigier, in *Open Questions in Quantum Physics,* edited by G. Tarozzi and A. van der Merwe (Reidel, Dordrecht, 1985).
[32] G. Ghirardi *et al.*, Found. Phys. **23**, 341 (1993).
[33] I. Prigogine, *From Being to Becoming* (Freeman, San Francisco, 1980), p. 207.