Spontaneous emission from a three-level atom

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(Received 24 October 1994)

Quantum interference between decay processes from two upper levels, which are coupled by the same vacuum modes to a lower level, have been investigated and its effects on the spontaneous emission spectrum have been studied. The interference can result in spectral narrowing and a black dark line in the spectrum. The population in the upper levels is not a simple exponential decay due to the interference.

PACS number(s): 42.50.Lc, 32.50.+d, 32.80.−t

I. INTRODUCTION

Recently quantum interference and coherence in a multilevel atomic system have attracted a lot of attention, because they can lead to absorption cancellation [1–9], electromagnetically induced transparency (EIT) [2,7,8], and population inversion without emission [3]. These quantum interference effects may result in a new type of laser system operating without population inversion (LWI) [1–5] and transparent high-index materials [10]. The EIT [7,8] and LWI [11] have been experimentally observed. It is well known that the quantum-noise limit results from the atomic spontaneous emission. The spectrum of spontaneous emission from a two-level atom is Lorentzian with a peak at the atomic transition frequency, and the width of the spectrum depends on the decay rate of the upper level. It was pointed out that the noise of the radiation field from the new-type laser systems (LWI) might be less compared to the noise of a laser light from two-level laser systems [3,12]. In this paper we investigate the spontaneous emission from a three-level atom with two upper levels coupled by the same vacuum modes [13,14] and how the quantum interference affects the spontaneous emission process and its spectrum.

II. BASIC THEORY

Consider a three-level atom with two upper levels $|a_1\rangle$ and $|a_2\rangle$, as shown in Fig. 1. The two upper levels are coupled by the same vacuum modes to the lower level $|b\rangle$. The interaction Hamiltonian of the system composed of the atom and the vacuum modes in the interaction picture can be written as

$$\begin{aligned} V = i\sum_k [&g_k^{(1)} e^{i(\omega_{a_1b}-\omega_k)t} b_k |a_1\rangle\langle b| \\ &+ g_k^{(2)} e^{i(\omega_{a_2b}-\omega_k)t} b_k |a_2\rangle\langle b|] \\ - i\sum_k [&g_k^{(1)} e^{-i(\omega_{a_1b}-\omega_k)t} b_k^\dagger |b\rangle\langle a_1| \\ &+ g_k^{(2)} b_k^\dagger e^{-i(\omega_{a_2b}-\omega_k)t} |b\rangle\langle a_2|] , \end{aligned} \tag{1}$$

where ω_{a_1b}, ω_{a_2b} are the frequency differences between levels $|a_1\rangle$, $|a_2\rangle$ and $|b\rangle$, b_k ($b_k^\dagger$) is the annihilation (creation) operator for the kth vacuum mode with frequency ω_k, and $g_k^{(1,2)}$ are the coupling constants between the kth vacuum mode and the atomic transitions from $|a_1\rangle$ and $|a_2\rangle$ to $|b\rangle$. Here k stands for both momentum and polarization of the vacuum modes, and $\hbar=1$ and real $g_k^{(1,2)}$ have been assumed. This Hamiltonian controls the spontaneous emission of the atom initially in the upper levels. The initial-state vector can be written as

$$|\psi(0)\rangle = A^{(1)}(0)|a_1\rangle|0\rangle + A^{(2)}(0)|a_2\rangle|0\rangle . \tag{2}$$

The evolution of the state vector obeys the Schrödinger equation

$$\frac{d}{dt}|\psi(t)\rangle = -iV|\psi(t)\rangle . \tag{3}$$

The state vector at time t can be written as

$$\begin{aligned} |\psi(t)\rangle = &A^{(1)}(t)|a_1\rangle|0\rangle + A^{(2)}(t)|a_2\rangle|0\rangle \\ &+ \sum_k B_k(t) b_k^\dagger |0\rangle|b\rangle . \end{aligned} \tag{4}$$

Substituting Eq. (4) into (3), we can obtain

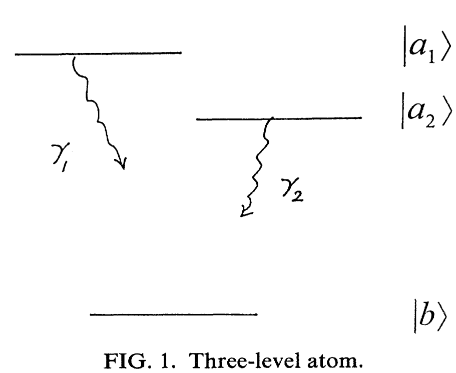

FIG. 1. Three-level atom.

$$\frac{d}{dt}A^{(1)}(t)=\sum_k g_k^{(1)}e^{i(\omega_{a_1b}-\omega_k)t}B_k(t)\ , \tag{5a}$$

$$\frac{d}{dt}A^{(2)}(t)=\sum_k g_k^{(2)}e^{i(\omega_{a_2b}-\omega_k)t}B_k(t)\ , \tag{5b}$$

$$\frac{d}{dt}B_k(t)=-g_k^{(1)}A^{(1)}(t)e^{-i(\omega_{a_1b}-\omega_k)t} -g_k^{(2)}A^{(2)}(t)e^{i(\omega_{a_2b}-\omega_k)t}\ . \tag{5c}$$

Formally integrating Eq. (5c), and then substituting into Eqs. (5a) and (5b), we find

$$\frac{d}{dt}A^{(1)}(t)=-\frac{\gamma_1}{2}A^{(1)}(t)-\frac{\sqrt{\gamma_1\gamma_2}}{2}A^{(2)}(t)e^{i\omega_{12}t}\ , \tag{6a}$$

$$\frac{d}{dt}A^{(2)}(t)=-\frac{\gamma_2}{2}A^{(2)}(t)-\frac{\sqrt{\gamma_1\gamma_2}}{2}A^{(1)}(t)e^{-i\omega_{12}t}\ , \tag{6b}$$

where ω_{12} is the frequency difference between the two upper levels and is assumed to be much less than ω_{a_1b}, $\gamma_1=2(\pi g^{(1)})^2D(\omega_1)$, and $\gamma_2=2\pi(g^{(2)})^2D(\omega_2)$. Here $g^{(1)},D(\omega_1)$ and $g^{(2)},D(\omega_2)$ are calculated at frequencies ω_{a_1b} and ω_{a_2b}, respectively, and $D(\omega)$ is the mode density. In obtaining Eqs. (6) $\omega_{12}\ll\omega_{a_1b},\omega_{a_2b}$ have been assumed (but not $\omega_{12}\ll\gamma_1,\gamma_2$) and we have assumed that the two dipole moments of the two transitions are parallel to each other (antiparallel will be the same). Here we did not neglect the time-dependent exponential factors ($e^{\pm\omega_{12}t}$), as was done in a previous similar work [6]. From Eqs. (6) we can obtain the equation of motion for the reduced density matrix of the atom [15]. In order to obtain the spontaneous spectrum, however, Eq. (5c) is necessary, which includes the information of the field radiated by the atom. Similar equations can be found in the problem of photoionization [16]. Solving Eq. (6), we obtain the solution for $A^{(1)}(t)$ and $A^{(2)}(t)$,

$$A^{(1)}(t)=(C_1e^{s_1t}+C_2e^{s_2t})e^{-(\gamma_1/2)t}, \tag{7a}$$

$$A^{(2)}(t)=-\frac{2}{\sqrt{\gamma_1\gamma_2}}(S_1C_1e^{S_1t}+S_2C_2e^{S_2t})e^{-(\gamma_1/2+i\omega_{12})t}\ , \tag{7b}$$

where $S_{1,2}$ are two roots of the equation $S^2-\lambda S-0.25\gamma_1\gamma_2=0$,

$$S_{1,2}=\tfrac{1}{2}(\lambda\pm\sqrt{\lambda^2+\gamma_1\gamma_2}), \tag{8a}$$

$$\lambda=\tfrac{1}{2}(\gamma_1-\gamma_2)+i\omega_{12}\ , \tag{8b}$$

$$C_1=\frac{S_2A^{(1)}(0)+0.5\sqrt{\gamma_1\gamma_2}A^{(2)}(0)}{S_2-S_1}, \tag{8c}$$

$$C_2=\frac{S_1A^{(1)}(0)+0.5\sqrt{\gamma_1\gamma_2}A^{(2)}(0)}{S_1-S_2}. \tag{8d}$$

III. EVOLUTION OF THE UPPER-LEVEL POPULATIONS

The populations in the two upper levels are equal to $|A^{(1)}(t)|^2$ and $|A^{(2)}(t)|^2$, respectively, which can be obtained from Eqs. (7).

A. The $\omega_{12}\neq 0$ case

In this case, the population in the two upper levels tends to zero as time goes to infinity. If $|\omega_{12}|$ is larger than γ_1 and γ_2, the last term in Eq. (6a) or (6b) can be neglected, and consequently the population of the upper levels decays to the lower level. For $|\omega_{12}|$ much less than γ_1 and γ_2, assuming $|\omega_{12}|\ll 0.5(\gamma_1+\gamma_2)$, it can be proven (see the Appendix) that the real parts of $S_i-0.5\gamma_j$ $(i,j=1,2)$ are negative. That is to say, from Eqs. (6), no population is in the upper levels at time equal to infinity. Here we can conclude that neglecting the factors ($e^{\pm\omega_{12}t}$) [6] may lead to some error.

However, during the time evolution one of the upper-level populations may increase first, even if initially there is no population in it. Let the atom initially be in level $|a_1\rangle$. The evolution of the populations in both upper levels is shown in Fig. 2. The population in level $|a_2\rangle$, which is initially zero, increases from zero to a maximum (about 0.1) and then declines to zero, while the population in level $|a_1\rangle$ monotonically decreases to zero. The population in an initially empty level could reach a maximum value of a little less than 0.25 under certain conditions (one of them is small ω_{12}). In Fig. 3(a), we plot the evolution of an initially empty upper level with $\omega_{12}=0.2\gamma_1$, $\gamma_2=\gamma_1$. The maximum population in this level is 0.237.

In some situations, the population of an initially empty level (also the other upper level) oscillates for several cycles, and then tends to zero [see Fig. 3(b)]. The decay rate of the total population of the two upper levels not only depends on the decay rate γ_1 and γ_2, but also de-

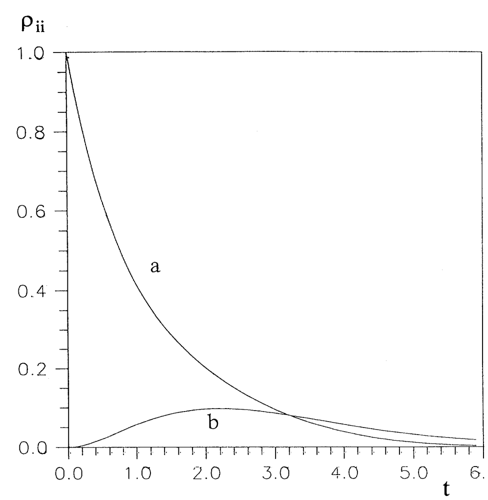

FIG. 2. Time evolution of populations in (a) $|a_1\rangle$ and (b) $|a_2\rangle$ ($\omega_{12}=\gamma_1$ and $\gamma_2=0.5\gamma_1$).

pends on the frequency separation of the two upper levels, and the initial coherence of the two upper levels. In Fig. 4, we plot the total upper-level population as a function of time for different ω_{12} with the same γ_1 and γ_2. Therefore, the decay process from one of the two upper levels or that from the total population decay are no longer simple exponential ones.

The increase of population in one of the two upper levels is due to the interference between the two transitions, which are coupled by the same vacuum modes. We may consider the population increase results from the interaction between the corresponding transition (in the above example, between level $|a_2\rangle$ and the lower level) and the radiation field emitted by the other transition (between level $|a_1\rangle$ and the lower level). However, this is not exactly the situation. There is strong interference between the two decay channels, which will become clear in Sec. IV.

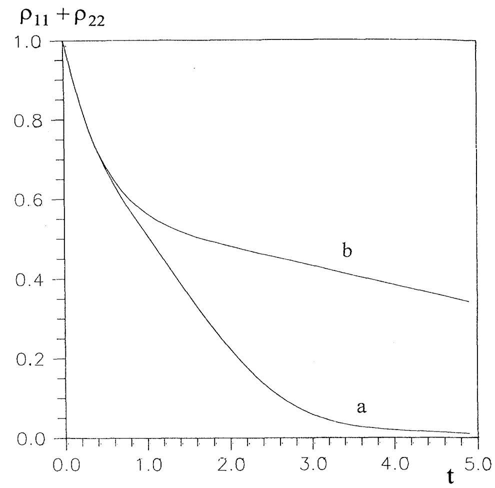

FIG. 4. Time evolution of the total population in the two upper levels with $\gamma_2=\gamma_1$ and (a) $\omega_{12}=2\gamma_1$ and (b) $\omega_{12}=0.5\gamma_1$.

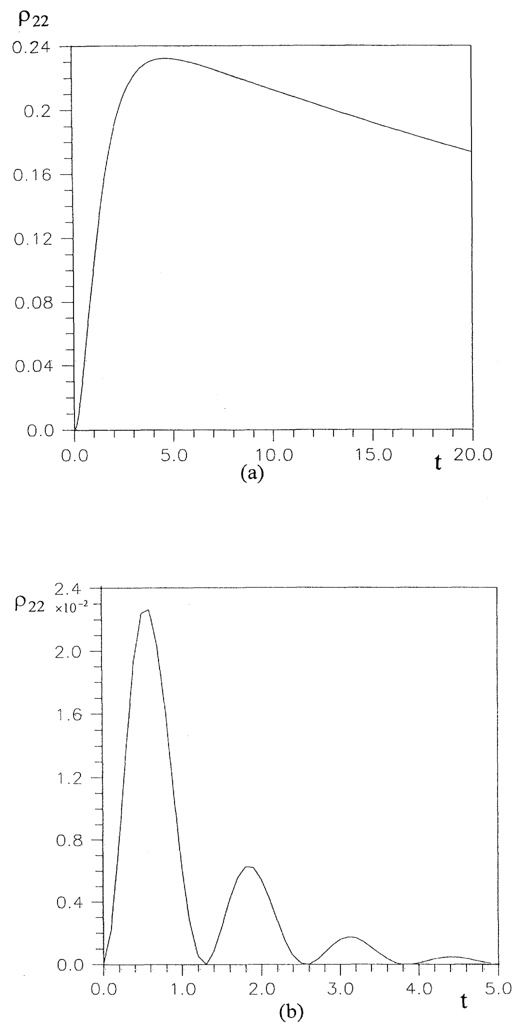

FIG. 3. (a) Temporary population in $|a_2\rangle$ reaches a maximum of 0.237 ($\omega_{12}=0.2\gamma_1$, $\gamma_2=\gamma_1$). (b) Oscillation of the population in $|a_2\rangle$ ($\omega_{12}=5\gamma_1$, $\gamma_2=\gamma_1$).

B. The $\omega_{12}=0$ case

In this case, Eqs. (7) reduce to

$$A^{(1)}(t)=C_1+C_2e^{-\frac{1}{2}(\gamma_1+\gamma_2)t}, \tag{9a}$$

$$A^{(2)}(t)=-\left[\frac{\gamma_1}{\gamma_2}\right]^{1/2}C_1+\left[\frac{\gamma_2}{\gamma_1}\right]^{1/2}C_2e^{-\frac{1}{2}(\gamma_1+\gamma_2)t}. \tag{9b}$$

It is clear from the above equations that the upper-level populations may not totally decay to a lower level when time goes to infinity, if $C_1\neq 0$. That is to say, some population may be trapped in the upper levels [17,18]. For example, if the atom is initially in level $|a_1\rangle$, we have $C_1=\gamma_2/(\gamma_1+\gamma_2)$. The population trapped in the upper levels will be $\gamma_2/(\gamma_2+\gamma_1)$. Some population initially in level $|a_1\rangle$ is transferred to level $|a_2\rangle$ and stays there. The amount of transferred population is $\gamma_1\gamma_2/(\gamma_1+\gamma_2)^2$. Some part of the population $\gamma_1/(\gamma_1+\gamma_2)$ goes to the lower level.

As discussed above, there is no population trapping for $\omega_{12}\neq 0$. However, if the atoms are coupled by a coherent field to another level, the population trapping can still be realized for the $\omega_{12}\neq 0$ case.

IV. DARK LINES AND SPECTRAL NARROWING

The spontaneous spectrum of the atom $S(\omega)$ is the Fourier transform of

$$\langle E^-(t+\tau)E^+(t)\rangle_{t=\infty}=\left\langle\psi(t)\left|\sum_{k,k'}b_k^\dagger b_{k'}e^{i\omega_k(t+\tau)}e^{-i\omega_{k'}t}\right|\psi(t)\right\rangle_{t=\infty}. \tag{10}$$

Substituting Eq. (4) into (10) we have

$$\langle E^-(t+\tau)E^+(t)\rangle_{t=\infty} = \sum_k B_k^*(\infty)B_k(\infty)e^{i\omega_k\tau} = \int_{-\infty}^{\infty} d\omega\, D(\omega)B_k^*(\infty)B_k(\infty)e^{i\omega\tau} . \quad (11)$$

From Eq. (11) we find

$$S(\omega_k)=\gamma|B_k(\infty)|^2/2\pi g^2 . \quad (12)$$

The spontaneous spectrum is proportional to $|B_k(\infty)|^2$. Substituting Eqs. (7) into (5c), and then integrating Eq. (5c), we can obtain

$$B_k(\infty)=\frac{g_k^{(1)}C_1(1-2S_1/\gamma_1)}{S_1-\gamma_1/2-i(0.5\omega_{12}-\delta_k)} + \frac{g_k^{(1)}C_2(1-2S_2/\gamma_1)}{S_2-\gamma_1/2-i(0.5\omega_{12}-\delta_k)} , \quad (13)$$

where $\delta_k=\omega_k-0.5(\omega_{a_1}+\omega_{a_2})+\omega_b$ is the detuning of the kth vacuum mode with respect to the central frequency (from the middle point of the two upper levels to the lower level). The spontaneous spectrum can be obtained by taking the absolute square of Eq. (10), which not only depends on the square of each term in the above equation, but also on their interference terms. The interference results in some very interesting features.

A. Dark lines

For a two-level atom, its spontaneous spectrum is Lorentzian and peaked at its transition frequency due to the population transfer from its upper level to the lower level. In the three-level atom case, the population initially in one upper level (say, $|a_1\rangle$) is partly transferred to another upper level (say, $|a_2\rangle$) during the time evolution. It is expected that the spontaneous spectrum of the three-level atom will differ from its counterpart of a two-level atom due to the transferred population in $|a_2\rangle$. This population in $|a_2\rangle$ will eventually decay to the lower level. Therefore, a major difference will be the weight of the frequency components around the transition frequency from $|a_2\rangle$ to lower ($\omega_{a_2}-\omega_b$) in the spontaneous emission spectrum. The weight of these components might be larger for the three-level atom than that for a two-level atom. In Fig. 5, we plot two spontaneous emission spectra as functions of the detuning δ_k, one for the three-level atom and one for a two-level atom. The three-level atom is initially in upper level $|a_1\rangle$. Comparing the two curves, we find that some components of the three-level spectrum at the neighborhood of $\omega_{a_2}-\omega_b$ are much larger than their counterpart of a two-level spectrum. If we simply added the two spontaneous decay processes together, we might conclude that the spectrum would have two peaks at the two transition frequencies. However, the spontaneous emission spectrum of the three-level atom is not a simple two-peak distribution peaked at the two transitions from levels $|a_1\rangle$ and $|a_2\rangle$ to the lower level. There is strong interference between the two processes. Therefore, the spectrum of the three-

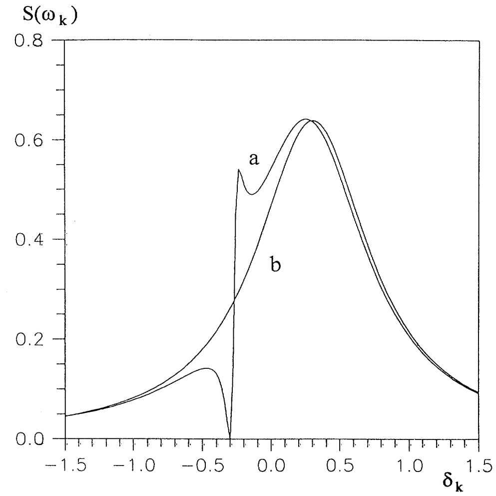

FIG. 5. Spontaneous emission spectrum for (a) the three-level atom with $\omega_{12}=0.6\gamma_1$ and $\gamma_2=0.1\gamma_1$, and (b) a two-level atom.

level atom is not a two-peak one with the two peaks located on the two sides of the central frequency. The interference leads to a dark line in the spontaneous spectrum. In Fig. 5, we plot the spontaneous spectra for a three-level atom and a two-level atom, where it can be seen very clearly that there is a dark line in the spectrum of the three-level atom at the frequency of the transition from $|a_2\rangle$ to the lower level. The dark line results from the interference. In fact, it can be proven that $B_k(\infty)=0$ at ω_k is equal to the frequency of one transition, when the atom is initially in the upper level of the other transition.

Assume the atom is initially in level $|a_1\rangle$ and consider $B_k(\infty)$ at $\omega_k=\omega_{a_2b}$ ($\delta_k=-0.5\omega_{12}$). Substituting $\delta_k=-0.5\omega_{12}$ in Eq. (13), we can obtain

$$B_k(\infty)=\frac{g_k^{(1)}C_1(1-2S_1/\gamma_1)}{S_1-0.5\gamma_1-i\omega_{12}}+\frac{g_k^{(1)}C_2(1-2S_2/\gamma_1)}{S_2-0.5\gamma_1-i\omega_{12}} = \frac{4g_k^{(1)}C_1S_1(-S_2-0.5\gamma_2)/\gamma_1\gamma_2}{S_1-0.5\gamma_1-i\omega_{12}} + \frac{4g_k^{(1)}C_2S_2(-S_1-0.5\gamma_2)/\gamma_1\gamma_2}{S_2-0.5\gamma_1-i\omega_{12}} = \frac{4g_k^{(1)}(S_1C_1+S_2C_2)}{\gamma_1\gamma_2} . \quad (14)$$

In obtaining the above equation

$$S_1S_2=-\frac{\gamma_1\gamma_2}{4} , \quad (15a)$$

$$S_1+S_2=0.5(\gamma_1-\gamma_2)+i\omega_{12} \quad (15b)$$

have been used. Because the atom is initially in $|a_1\rangle$, we have $A^{(1)}(0)=1$ and $A^{(2)}(0)=0$. From Eqs. (8c) and (8d) we can find $S_1C_1+S_2C_2=0$, which yields $B_k(\infty)=0$. From Eq. (12) we get $S(\omega_k=\omega_{a_1b})=0$. This tells us that not only is there a dark line in the spontaneous emission spectrum, but also that the center of the dark line is absolutely black, independent of γ_1, γ_2, and ω_{12}, as long as the two coupling constants are not equal to zero (if one of them is zero, the three-level atom reduces to a two-level one). In addition, we can see in Fig. 5 that there was an attempt to build a peak at the dark line position due to the transferred population.

The width of the dark line depends on the decay rate of the upper level of the corresponding transition. In Fig. 6, we show two spontaneous emission spectra of the three-level atom initially in $|a_1\rangle$, with the same γ_1 and ω_{12} but different γ_2 ($=0.5\gamma_1$ and $0.05\gamma_1$). It is clear that the larger the decay rate of the corresponding upper level ($|a_2\rangle$ in this example), the wider the width of the dark line will be. As γ_2 (or γ_1) tends to zero, the dark line becomes narrower and narrower (and will finally disappear), and the spectrum becomes closer and closer to a Lorentzian distribution.

B. Spectral narrowing

As mentioned above, the dark line is absolutely black at its center, and its width depends on the decay rate of the corresponding upper level. A larger decay rate results in a wider width. In the above example we use a small value for γ_2 (more precisely, γ_2/γ_1) in order to show clear dark lines. On the other hand, if γ_2 is of the same order or is ever larger than γ_1, the width of the dark line will be big enough to depress one of the two wings of the spectrum. Consequently, the spontaneous emission spectrum can be greatly narrowed. In Fig. 7, we plot the spectra of the three-level atom for different values of the ratios $\gamma_2/\gamma_1=0.01$, 0.5, and 2, where we can see that the width of the dark line increases and the spectrum becomes narrower as the ratio increases. In Fig. 8, we compare the spectrum of the three-level atom with that of a two-level atom. The parameters used for the two-level atom are the same as those used for the three-level atom, except $\gamma_2=0$ (because there is no level $|a_2\rangle$). The width of the spontaneous spectrum for the three-level atom is much narrower than that for a two-level atom.

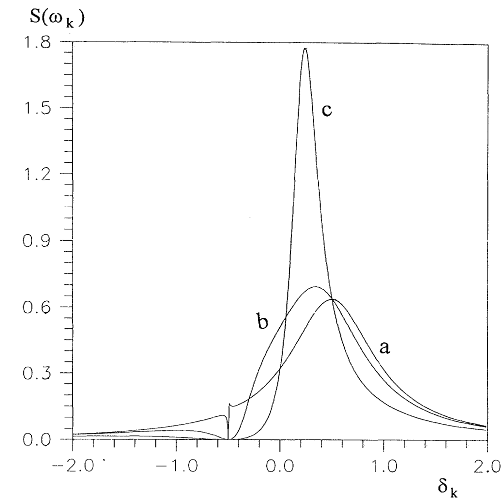

FIG. 7. Spectral narrowing by increasing the ratio $\gamma_2/\gamma_1=$ (a) 0.01, (b) 0.5, and (c) 2, with $\Delta=\gamma_1$.

The spectrum may have three peaks, as shown in Fig. 5, and may have two peaks, as shown in Fig. 8, depending on the parameters γ_1, γ_2, and ω_{12}. From $dS(\omega_k)/d\omega_k=0$ we can obtain a fifth-order polynomial, which may have five real roots corresponding to three peaks, and may have three real roots corresponding to two peaks. One of the roots corresponds to the dark line.

In the above, we assumed that the atom is initially in one of the upper levels. If the atom were initially in a superposition of the two upper levels, we might still have the dark lines. In Fig. 9 we give three spectra for three different combinations of superposition. Each spectrum has a dark line, but at different frequencies.

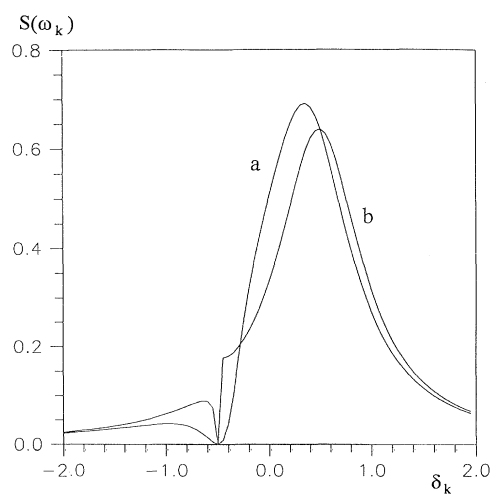

FIG. 6. Dark lines in the spontaneous emission spectrum of the three-level atom with $\omega_{12}=\gamma_1$, and (a) $\gamma_2=0.5\gamma_1$ and (b) $\gamma_2=0.05\gamma_1$.

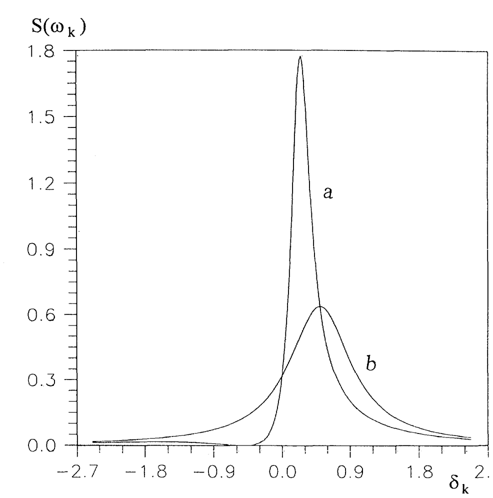

FIG. 8. Spectral narrowing: (a) the spectrum of the three-level atom with $\omega_{12}=\gamma_1$ and $\gamma_2=2\gamma_1$ and (b) the spectrum of a two-level atom with a decay rate γ_1.

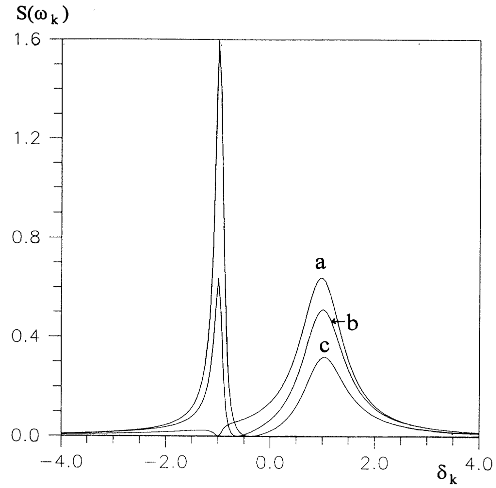

FIG. 9. Spectra for different initial conditions $A^{(2)}(0)/A^{(1)}(0)=$ (a) 0.0, (b) 0.5, and (c) 1.0, with $\gamma_2=0.2\gamma_1$, $\omega_{12}=2\gamma_1$.

V. CONCLUSION

We studied the spontaneous emission spectrum of a three-level atom with two upper levels and compared it with the counterpart of a two-level atom. The additional upper level (the corresponding transition) results in a dark line in the spectrum of the three-level atom due to the interference between the two transitions. The center of the dark line is absolutely black, and the width of the dark line depends on the decay rate of the additional upper level.

These properties can be used to greatly narrow the spontaneous emission spectrum. In real atomic systems, the separation between the two upper levels is quite large. For a large separation, narrowing is small and the dark line is difficult to be measured. There are two ways to overcome the difficulty in making the experimental proof of the narrowing and dark lines of the quantum interference effects achievable. The first one is using a strong coherence field to couple the two upper levels with another level lying above them. With the strong applied field, the dark line and narrowing will still be quite remarkable and the two quantum interference effects can still be observed, even though the separation is several hundreds times the decay rate. The second method is using one upper-level (and one lower-level) atom and a strong coherent field to couple the sole upper level to a third level. This coupling between the upper level and the coherent field creates two dressed upper levels. We plan to publish the details of the two methods in a future work.

ACKNOWLEDGMENTS

This work was supported by HKBU and the HK government.

APPENDIX: FINAL POPULATION ZERO IN THE UPPER LEVELS FOR $\omega_{12}\neq 0$

The two eigenvalues $S_{1,2}$ are

$$\begin{aligned} S_{1,2}&=\tfrac{1}{2}[\tfrac{1}{2}(\gamma_1-\gamma_2)+i\omega_{12}\\ &\quad\pm\sqrt{\tfrac{1}{4}(\gamma_1+\gamma_2)^2-\omega_{12}^2+i\omega_{12}(\gamma_1-\lambda_2)}]\\ &=\tfrac{1}{2}[\tfrac{1}{2}(\gamma_1-\gamma_2)+i\omega_{12}\pm(a+ib)]\ , \end{aligned} \tag{A1}$$

where

$$a=\sqrt{r}\,\cos(\phi/2)\ , \tag{A2}$$

$$b=\sqrt{r}\,\sin(\phi/2)\ , \tag{A3}$$

$$r=\sqrt{[\tfrac{1}{4}(\gamma_1-\gamma_2)^2-\omega_{12}^2]^2+\omega_{12}^2(\gamma_1-\gamma_2)^2}\ , \tag{A4}$$

$$\phi=\arctan\frac{(\gamma_1-\gamma_2)\omega_{12}}{0.25(\gamma_1+\gamma_2)^2+\omega_{12}^2}\ . \tag{A5}$$

For $|\omega_{12}|\ll 0.5(\gamma_1+\gamma_2)$, we can write

$$\phi\approx\frac{(\gamma_1-\gamma_2)x}{0.5(\gamma_1+\gamma_2)}\ , \tag{A6}$$

$$\cos(\phi/2)\approx 1-0.5\left[\frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\right]^2x^2\ , \tag{A7}$$

$$a=\tfrac{1}{2}(\gamma_1+\gamma_2)\left[1-\frac{1}{2}x^2+\frac{1}{2}\left[\frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\right]^2x^2\right]\ , \tag{A8}$$

where

$$x=\frac{2\omega_{12}}{\gamma_1+\gamma_2}\ . \tag{A9}$$

Substituting Eqs. (A6)–(A8) into (A1), we can obtain

$$\mathrm{Re}(S_1-0.5\gamma_1)=-0.5\left[1-\left[\frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\right]^2\right]x^2\ , \tag{A10}$$

$$\begin{aligned}\mathrm{Re}(S_2-0.5\gamma_1)&=-0.5(\gamma_1+\gamma_2)\\ &\quad-0.25(\gamma_1+\gamma_2)\left[1-\left[\frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\right]^2\right]x^2\ .\end{aligned} \tag{A11}$$

These two equations tell us that both the real parts of $S_1-0.5\gamma_1$ and $S_2-0.5\gamma_1$ are negative. Similarly, $\mathrm{Re}(S_i-0.5\gamma_2)$ $(i=1,2)$ are also negative.

[1] O. Kocharovskaya and Ya. I. Khanin, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 581 (1988) [JETP Lett. **48**, 630 (1988)].

[2] S. E. Harris, Phys. Rev. Lett. **62**, 1033 (1989).

[3] M. O. Scully, S. Y. Zhu, and A. Gavrielides, Phys. Rev. Lett. **62**, 2813 (1989).

[4] A. Lyras, X. Tang, P. Lambropoulos, and J. Zhang, Phys. Rev. A **40**, 4131 (1989).

[5] O. Kocharovskaya and P. Mandel, Phys. Rev. A **42**, 523

(1990).
[6] A. Imamoglu, Phys. Rev. A **40**, 2835 (1989).
[7] S. E. Harris, J. E. Field, and A. Imamoglu, Phys. Rev. Lett. **64**, 1107 (1990); K. J. Boller, A. Imamoglu, and S. E. Harris, *ibid.* **66**, 2593 (1991).
[8] K. Hakuta, L. Marmet, and B. P. Stoicheff, Phys. Rev. Lett. **66**, 596 (1991).
[9] P. Mandel and O. Kocharovskaya, Phys. Rev. A **47**, 5003 (1993).
[10] M. O. Scully, Phys. Rev. Lett. **67**, 1855 (1991); M. O. Scully and S. Y. Zhu, Opt. Commun. **87**, 134 (1992); M. Fleischhauer. *et al.*, Phys. Rev. A **46**, 1468 (1992).
[11] J. Y. Gao, Opt. Commun. **93**, 323 (1992); E. Fry *et al.*, Phys. Rev. Lett. **70**, 3235 (1993); A. Nottelmann, C. Peters, and W. Lange, *ibid.* **70**, 1783 (1993); W. E. van der Veer *et al.*, *ibid.* **70**, 3243 (1993).
[12] M. Fleishhauer, U. Rathe, and M. O. Scully, Phys. Rev. A **46**, 5856 (1992).
[13] J. E. Field and A. Imamoglu, Phys. Rev. A **48**, 2486 (1993).
[14] D. Agassi, Phys. Rev. A **30**, 2449 (1984).
[15] G. S. Agarwal, *Progress in Optics,* edited by E. Wolf (North-Holland, Amsterdam, 1973), Vol. XI; A. Z. Devdoriani *et al.*, Sov. Phys. JETP **71**, 909 (1976).
[16] P. L. Knight, Opt. Commun. **32**, 261 (1980).
[17] P. M. Radmore and P. L. Knight, J. Phys. B **15**, 561 (1982).
[18] G. S. Agarwal, in *Quantum Optics,* edited by Editor(s), Springer Tracts in Modern Physics Vol. 70 (Springer, Berlin, 1974).