# Generalized Jaynes-Cummings model with random telegraph noise

S. V. Lawande,[1] Amitabh Joshi,[2,*] and Q. V. Lawande[1]
[1]*Theoretical Physics Division, Bhabha Atomic Research Centre, Trombay, Bombay 400 085, India*
[2]*Department of Mathematics, University of Manchester Institute of Science and Technology, P.O. Box 88, Manchester M60 1QD, United Kingdom*
(Received 4 November 1994)

The Jaynes-Cummings model of a two-level atom interacting with a single quantized mode of radiation field is generalized to include the effects of stochastic fluctuations in the atom-field coupling. The coupling coefficient is assumed to fluctuate either in phase or in frequency. The fluctuations are modeled by binary random telegraph processes. Invoking the Markovian property of the telegraph noise, we derive in these two cases an equation for the density operator averaged over the distribution of fluctuations. The solution of this equation is used to study the resulting decoherence effects in the dynamical behavior of the atom and the statistical properties of the field.

PACS number(s): 42.50.Ar, 42.50.Lc, 42.50.Ct

## I. INTRODUCTION

The simplest model of atom-field interaction first proposed by Jaynes and Cummings [1] describing the interaction of a two-level atom with a single quantized mode of an electromagnetic field in a lossless cavity continues to receive a great deal of theoretical [2–7] as well as experimental attention [7,8]. Theoretical interest in this model arises from the fact that it is one of the few exactly solvable models in quantum optics showing several quantum-mechanical features. The Jaynes-Cummings model (JCM) predicts several interesting effects such as vacuum field Rabi oscillations [2], collapses and revivals of Rabi oscillations in the coherent field [3–7], photon antibunching [9], squeezing of the cavity field [2,10,11], and chaos [12]. The model is also useful in studying the emission spectra of two-level atoms in a cavity [13]. It is also known [14] that the predictions of the model depend sensitively on the statistics of the field in the cavity.

Recent advances in high-$Q$ cavities have made it possible to verify the predictions of JCM in the optical as well as in the microwave regime. Experimental observation of collapses and revivals has been reported by Rempe, Walther, and Klein [15]. Rabi oscillations have been experimentally observed by Gentile, Hughey, and Kleppner [16]. The creation of a single-mode two-photon maser has also been reported [17].

The analytical simplicity and the experimental realization of the model have inspired several extensions and generalizations of the original JCM in many directions. The model has been extended to include the effects of finite cavity damping [18] and blackbody photons [19]. The influence of the atomic spontaneous emission decay on the collapse-revival phenomenon has also been investigated [20]. The JCM has also been modified to treat an atom undergoing a two-photon transition in an ideal cavity [21]. It has been extended further to study the behavior of multiatom systems [22] as well as multilevel systems [23]. The effects due to spatial structure of the cavity field mode [24] and those due to the Kerr-like medium [25] on the behavior of the JCM have also been considered recently.

In a recent paper [26], the standard one-photon as well as the two-photon JCM's were extended to include the transient effects arising from a time-dependent atom-field coupling coefficient. In subsequent papers [27,28], we attempted to study the effects due to stochastic time dependence of the atom-field coupling coefficient on the behavior of a single two-level atom interacting with a single mode of the electromagnetic field in an ideal cavity. These stochastic fluctuations in the cavity may presumably be inherited from the source of the chosen single-mode coherent field coupled to the cavity. An alternative mechanism for the fluctuations in the atom-field coupling parameter may arise from the following consideration. In the current experiments on cavity quantum electrodynamics, a stabilized beam of Rydberg atoms enters in a superconducting cavity. The flow rate of the atomic beam is well controlled so that the single atom-field interaction is the dominant process. However, any variation in the mechanism of the production of Rydberg atoms due to instabilities either in the atomic vapor production source or in the dye laser system that is responsible for exciting the atoms to the Rydberg states may introduce stochastic fluctuations. It is conceivable that these fluctuations may in turn be acquired by the process of interaction of the electromagnetic field with the atomic beam in the cavity. Also, it has been reported very recently that the atom-field coupling coefficient or the fluctuations of vacuum Rabi frequency play a significant role in observing trapping state dynamics of a one-atom micromaser system [29]. In fact, large fluctuations in the vacuum Rabi frequency $g$ wash out the trapping states.

*Permanent address: Laser and Plasma Technology Division, Bhabha Atomic Research Centre, Trombay, Bombay 400085, India.

 

Experiments do confirm that the fluctuations in $g$ are about 20% and so we are motivated to model such fluctuations in $g$.

In Ref. [28] we assumed that the atom-field coupling coefficient fluctuates both in amplitude as well as in phase and that the fluctuations in the phase and the amplitude are statistically independent. The fluctuations in the phase were described by the well-known Gaussian process of phase diffusion [30], while the amplitude fluctuations were described by a Gaussian distribution for colored noise [31]. Further, we invoked a secular approximation and the theory of multiplicative stochastic processes [32] to derive a master equation for the density operator averaged over the distributions of both phase and amplitude fluctuations. The solution of this master equation was used to study the decoherence in the Rabi oscillations and other statistical properties of the field. Incidently, Moya-Cessa *et al.* [33] have also analyzed the decoherence of Rabi oscillations in the JCM invoking a recent model proposed by Milburn [34] for intrinsic quantum decoherences. The decoherence in the Rabi oscillations arising from phase fluctuations in our model [27] were shown to be qualitatively similar to the above [33], though the mechanisms are different.

Another possible motivation for considering the JCM with stochastic fluctuations is because of its application to study the motion of an ion in a harmonic trap interacting with a standing wave or a traveling wave. This is because in some approximation [35,36] the equations governing the motion of the ion in the trap may be reduced to a form that is similar to the JCM with the field variables replaced by the vibrational modes of the quantized center-of-mass motion of the ion. The coupling coefficient now involves the amplitude and the phase of the standing wave and it is natural to consider the fluctuations in both amplitude and phase in this case.

In the present paper, we consider an alternative model for introducing stochasticity in the JCM based on the so-called jump processes. Such models were first introduced in quantum optics by Burshtein [37] to treat the noisy laser-atom interactions. The simplest example of such a model is the two-state random telegraph. While models based on Gaussian noise have been generally popular in quantum optics, random telegraph models have also been studied considerably by several authors [38–41]. Models of Gaussian noise can be handled analytically using the so-called cumulant approximations only when extremely short coherence times are involved [32]. In contrast, all random telegraph models, whether associated with phase, frequency, or amplitude fluctuations, lead to equations for average response that have exact algebraic solutions [38]. We apply here the model of a two-state telegraph to treat the phase and frequency fluctuations in the JCM. As in the previous paper [28], these fluctuations can be associated with the coupling coefficient in the JCM. We show that exact algebraic equations for the matrix elements of the average density operator can be obtained in both cases. These algebraic equations are solved to study the atomic response as well as the statistical properties of the random field.

The organization of the paper is as follows. In Sec. II we present the basic formulation of the stochastic JCM. After introducing the physical model and the basic properties of the phase and frequency telegraph noise, we consider the atomic dynamics. Exact equations are derived for the relevant density matrix averaged over the fluctuations in phase and frequency. The formal analytical solution of these equations are presented. Approximate analytical limits of these solutions are also derived. These solutions are subsequently used in Sec. III to discuss the dynamical behavior of the atom and the statistical properties of the field. Finally, some concluding remarks are added in Sec. IV.

## II. BASIC FORMULATION

### A. Physical model

We consider the interaction of a single two-level atom with a single mode of radiation field. The atom is characterized by spin-$\frac{1}{2}$ angular-momentum operators $S_{\pm}, S_z$, while the field is described by the annihilation and the creation operators $a$ and $a^{\dagger}$, respectively. For simplicity we assume that the field is in resonance with the atomic transition frequency $\omega_0$. In the usual rotating-wave approximation, the Hamiltonian of the system takes the form

$$H=\omega_0 S_z+\omega_0 a^{\dagger}a+[g^*(t)S_+a+g(t)S_-a^{\dagger}] , \tag{2.1}$$

where the coupling $g(t)$ between the atom and the field is assumed to be time dependent. Transient effects arising from various forms of deterministic modulation of the coupling coefficient $g(t)$ have been already studied in a recent paper [26]. Moreover, it is easy to introduce stochasticity in the problem through the coupling coefficient $g(t)$, irrespective of its origin. This has been illustrated in recent papers [27,28], where the stochastic fluctuations in the atom-field coupling were treated. In these papers the coupling coefficient was assumed to fluctuate in phase and/or in amplitude. The phase fluctuations were described by a Wiener-Levy (phase-diffusion) process and the amplitude fluctuations by colored Gaussian noise. An alternative model that represents noise by means of discrete jump processes was first introduced into quantum optics by Burshtein [37]. A simple example of such a jump process is the two-state random telegraph.

In order to illustrate such a model we assume that

$$g(t)=g_0 e^{-i\phi(t)} , \tag{2.2}$$

where the nonstochastic amplitude $g_0$ is a positive real quantity while the phase $\phi(t)$ is treated as a stochastic variable. There are now two ways in which the random telegraph model can be used in the system. The first is the so-called random-phase telegraph where the $\phi(t)$ itself fluctuates in the manner of a jump. The second case involves writing

$$\phi(t)=\int \mu(t')dt' , \tag{2.3}$$

where $\dot{\phi}=\mu(t)$ is a random telegraph. We may appropriately call this case a random frequency telegraph. The correlation function $\langle g(t)g(t')\rangle$ is different in the two cases. Following the analysis of Eberly, Wodkiewicz, and Shore [40], we have, for a binary phase, telegraph noise where $\phi(t)$ randomly jumps between two possible values (states) $a$ and $-a$, the correlation function

$$\langle g(t)g(t')\rangle = g_0^2\langle \exp\{i[\phi(t)-\phi(t')]\}\rangle = g_0^2[\cos^2 a+\sin^2 a\,\exp(-2|t-t'|/T)]\ , \tag{2.4}$$

where $T$ is the mean dwell time of the telegraph.

In the other case of frequency telegraph $\dot{\phi}=\mu(t)$ obeys the binary telegraph process. In this case, Wodkiewicz, Shore, and Eberly [41] have shown that

$$\langle \mu(t)\mu(t')\rangle = a^2\exp(-2|t-t'|/T) \tag{2.5}$$

and

$$\langle g(t)g(t')\rangle = (g_0^2/2)\{(1/\nu T+1)\exp[-(1/T-\nu)|t-t'|]-(1/\nu T-1)\exp[-(1/T+\nu)|t-t'|]\}\ , \tag{2.6a}$$

where

$$\nu^2=(1/T^2-a^2)\ . \tag{2.6b}$$

The correlations (2.4) and (2.6a) are central to the derivation of the appropriate master equation.

### B. Atomic dynamics

We now proceed to study the dynamics in the presence of telegrapher noise. For this purpose, we derive the relevant equation of motion from the basic Liouville–von Neumann equation for the density operator. It is convenient to use the interaction representation and write the evolution equation as ($\hbar=1$)

$$i\partial\rho/\partial t=[H_1,\rho]\ , \tag{2.7a}$$

where $H_1$ denotes the interaction Hamiltonian

$$H_1=g_0\{\exp[i\phi(t)]S_+a+\exp[-i\phi(t)]S_-a^\dagger\}\ . \tag{2.7b}$$

We now introduce the states

$$|\Psi_m^+\rangle=|m,\tfrac{1}{2}\rangle\ ,\quad |\Psi_m^-\rangle=|m+1,-\tfrac{1}{2}\rangle,\quad |m,\pm\tfrac{1}{2}\rangle\equiv|m\rangle|\pm\tfrac{1}{2}\rangle\ . \tag{2.8}$$

These states are degenerate in the absence of the interaction. The interaction causes a transition between them. It is convenient to derive from (2.7) the equation of motion for the diagonal and the off-diagonal elements of the operator $\rho$. These equations read as

$$\frac{d}{dt}\rho_{mn}^{\pm,\pm}(t)=i[\alpha_n e^{\mp i\phi}\rho_{mn}^{\pm,\mp}(t)-\alpha_m e^{\pm i\phi}\rho_{mn}^{\mp,\pm}(t)]\ , \tag{2.9}$$

$$\frac{d}{dt}\rho_{mn}^{\pm,\mp}(t)=ie^{\pm i\phi}[\alpha_n\rho_{mn}^{\pm,\pm}(t)-\alpha_m\rho_{mn}^{\mp,\mp}(t)]\ , \tag{2.10}$$

where the notation used is

$$\rho_{mn}^{\pm,\mp}(t)=\langle\Psi_m^{\pm}|\rho(t)|\Psi_n^{\mp}\rangle\ , \tag{2.11}$$

$$\alpha_m=g_0\sqrt{m+1}\ . \tag{2.12}$$

These equations have to be solved taking into account the stochastic nature of the phase variable $\phi(t)$. For this purpose, it is convenient to first solve Eq. (2.10) formally and insert the solution in Eq. (2.9). We thereby obtain

$$\frac{d}{dt}\rho_{mn}^{++}(t)=-\int_0^t dt'\{\alpha_n e^{-i[\phi(t)-\phi(t')]}[\alpha_n\rho_{mn}^{++}(t')-\alpha_m\rho_{mn}^{--}(t')]+\alpha_m e^{i[\phi(t)-\phi(t')]}[\alpha_m\rho_{mn}^{++}(t')-\alpha_n\rho_{mn}^{--}(t')]\}\ , \tag{2.13}$$

$$\frac{d}{dt}\rho_{mn}^{--}(t)=\int_0^t dt'\{\alpha_m e^{-i[\phi(t)-\phi(t')]}[\alpha_n\rho_{mn}^{++}(t')-\alpha_m\rho_{mn}^{--}(t')]+\alpha_n e^{i[\phi(t)-\phi(t')]}[\alpha_m\rho_{mn}^{++}(t')-\alpha_n\rho_{mn}^{--}(t')]\}\ . \tag{2.14}$$

In describing these equations, we have assumed that $\rho_{mn}^{\pm,\mp}(0)=0$. The reason for this will be made clear later. One can develop similar equations for the evolution of $\rho_{mn}^{\pm,\mp}(t)$, but we will not require them in the present paper. The idea behind recasting Eq. (2.9) in the above form is to make use of the Markovian property of the telegrapher noise. According to this, since $t'<t$, the part involving the exponential factor in Eqs. (2.13) and (2.14) gets decoupled from the other part, which depends on $t'$ [39,40]. This allows us to take the average over the stochastic distribution of the telegrapher noise. Using the symbol $\langle\ \rangle$ to denote such an average and writing

$$c(t-t')=\langle e^{\pm i[\phi(t)-\phi(t')]}\rangle\ , \tag{2.15}$$

we obtain from Eqs. (2.13) and (2.14) the following equations for $\langle\rho_{mn}^{\pm,\mp}(t)\rangle$:

$$\frac{d}{dt}\langle\rho_{mn}^{++}(t)\rangle=-\int_0^t dt'c(t-t')\{(\alpha_m^2+\alpha_n^2)\langle\rho_{mn}^{++}(t')\rangle-2\alpha_m\alpha_n\langle\rho_{mn}^{--}(t')\rangle\}\ , \tag{2.16}$$

$$\frac{d}{dt}\langle\rho_{mn}^{--}(t)\rangle=\int_0^t dt'c(t-t')\{2\alpha_m\alpha_n\langle\rho_{mn}^{++}(t')\rangle-(\alpha_m^2+\alpha_n^2)\langle\rho_{mn}^{--}(t')\rangle\}\ . \tag{2.17}$$

For convenience we may introduce two quantities

$$F_{mn}^{\pm}(t)=\tfrac{1}{2}\{\langle\rho_{mn}^{++}(t)\rangle\pm\langle\rho_{mn}^{--}(t)\rangle\} \tag{2.18}$$

and rewrite (2.16) and (2.17) in the form

$$\frac{d}{dt}F^{\pm}_{mn}(t)=-\Omega^{\mp}_{mn}\int_0^t dt'c(t-t')F^{\pm}_{mn}(t') , \quad (2.19)$$

where

$$(\Omega^{\mp}_{mn})^2=(\alpha_m\mp\alpha_n)^2=g_0^2(\sqrt{m+1}\mp\sqrt{n+1})^2 . \quad (2.20)$$

Equation (2.19) may be readily solved by using Laplace transform techniques. The form of the correlation function $c(t)$ depends on the telegrapher process considered. In the subsequent subsection we obtain explicitly the solution of Eq. (2.19) in the two cases of telegraph noise and apply it to discuss the behavior of atomic observables and the field statistics presented in Sec. III.

### C. Solutions of the equation of motion

#### 1. Phase telegraph

The correlation function $C(t-t')$ for the phase telegraph is given by [see Eq. (2.4)]

$$C(t-t')=\cos^2 a+\sin^2 a e^{-2|t-t'|/T} . \quad (2.21)$$

Inserting Eq. (2.21) in Eq. (2.19) and taking the Laplace transform with respect to $t$, we obtain

$$\tilde{F}^{\pm}_{mn}=\frac{z(z+2/T)F^{\pm}_{mn}(0)}{z^3+(2/T)z^2+(\Omega^{\mp}_{mn})^2 z+(2/T)(\Omega^{\mp}_{mn})^2\cos^2 a} , \quad (2.22)$$

where $\tilde{F}^{\pm}_{mn}(z)$ is the Laplace transform of $F^{\pm}_{mn}(t)$. When $n=m$, $\Omega^{-}_{mn}(0)=0$ and we have directly from Eq. (2.19)

$$F^{+}_{mn}(t)=F^{+}_{mn}(0) . \quad (2.23)$$

On the other hand, when $n\neq m$, inversion of Eq. (2.22) yields the general solution

$$F^{\pm}_{mn}(t)=F^{\pm}_{mn}(0)\left\{\sum_{j=1}^{3}\frac{\lambda_j(\lambda_j+2/T)}{\prod_{k\neq j}(\lambda_j-\lambda_k)}\exp(\lambda_j t)\right\} , \quad (2.24)$$

where $\lambda_j$ are the roots of the cubic equation

$$\lambda^3+(2/T)\lambda^2+(\Omega^{\mp}_{mn})^2\lambda+(2/T)(\Omega^{\mp}_{mn})^2\cos^2 a=0 . \quad (2.25)$$

This equation has one negative real and a pair of complex-conjugate roots.

We may obtain the approximate solution in the two limits of large and small $T$. When $T$ is very large ($g_0 T\gg 1$), the real approximate roots of the cubic equation (2.25) are

$$\lambda_1=-(2/T)\cos^2 a , \quad (2.26a)$$

$$\lambda_{2,3}=\pm i(\Omega^{\mp}_{mn})-\frac{\sin^2 a}{T} \quad (2.26b)$$

and the solution for $F^{\pm}(t)$ reads as

$$F^{\pm}_{mn}(t)=\frac{F^{\pm}_{mn}(0)}{D_1}\left[-(\sin^2 2a)e^{-\gamma_1 t}+\left[[(4\cos^4 a+\sin^4 a)+(\Omega^{\mp}_{mn}T)^2]\cos\Omega^{\mp}_{mn}t+\frac{\sin^2 a}{\Omega^{\mp}T}[(\Omega^{\mp}_{mn}T)^2-(3\cos^2 a-1)(\cos^2+1)]\sin(\Omega^{\mp}_{mn}t)\right]e^{-\gamma_2 t}\right] , \quad (2.27)$$

where

$$\gamma_1=(2/T)\cos^2 a,\quad \gamma_2=\frac{\sin^2 a}{T} , \quad (2.28a)$$

$$D_1=(\Omega^{\mp}_{mn}T)^2+(3\cos^2 a-1)^2 . \quad (2.28b)$$

In the opposite case of small $T$ ($g_0 T\ll 1$), the approximate roots of the cubic equation (2.25) are

$$\lambda_1=-(2/T)\left[1-\frac{(\Omega^{\mp}_{mn}T)^2}{4}\sin^2 a\right] , \quad (2.29a)$$

$$\lambda_{2,3}=\pm i\Omega^{\mp}_{mn}\cos a-(\Omega^{\mp}_{mn}T)^2\frac{\sin^2 a}{4} \quad (2.29b)$$

and the solution for $F^{\pm}_{mn}(t)$ takes the form

$$F^{\pm}_{mn}(t)=\frac{F^{\pm}_{mn}(0)}{D_2}\left[-(\Omega^{\mp}_{mn}T\sin a)^2 e^{-\gamma_3 t}+\left[[(1-3\sin^2 a)(\Omega^{\mp}_{mn}T)^2+4]\cos(\Omega^{\mp}_{mn}t\cos a)-\frac{\sin^2 a}{\cos a}(\Omega^{\mp}_{mn}T)\sin(\Omega^{\mp}_{mn}t\cos a)\right]e^{-\gamma_4 t}\right] , \quad (2.30)$$

where

$$\gamma_3=(2/T)\left[1-\frac{(\Omega_{mn}^{\mp}T)^2}{4}\sin^2 a\right], \quad (2.31a)$$

$$\gamma_4=(\Omega_{nm}^{\mp}T)^2\frac{\sin^2 a}{4}, \quad (2.31b)$$

$$D_2=(1-4\sin^2 a)(\Omega_{mn}^{\mp}T)^2+4 . \quad (2.31c)$$

The effect of phase telegraph noise is manifest in the solutions (2.27) and (2.30) in terms of the modified amplitude and frequency of Rabi oscillations and the decay coefficients $\gamma_i$.

In the particular case, when $a=0$ or $\pi$, the cubic equation (2.25) has the exact roots

$$\lambda_1=-2/T, \quad \lambda_{2,3}=\pm i\Omega_{mn}^{\mp} . \quad (2.32)$$

In this case the exact solution for $F_{mn}^{\pm}(t)$ reads as

$$F_{mn}^{\pm}(t)=F_{mn}^{\pm}(0)\cos(\Omega_{mn}^{\mp}t) \quad (2.33)$$

irrespective of the value of the dwell time $T$. This result is to be expected as the correlation function of Eq. (2.21) is unity in this case. In this special case, the telegraph noise does not introduce damping of Rabi oscillations. Note also that the special solution (2.33) is recovered from both Eqs. (2.27) and (2.30) when the parameter $a$ is set equal to 0 or $\pi$.

On the other hand, when $a=\pi/2$, the correlation function $C(t-t')=\exp(-2|t-t'|/T)$. In this case also the cubic equation (2.25) is exactly solvable with the roots

$$\lambda_1=0, \quad \lambda_{2,3}=-(1/T)\pm[1/T^2-(\Omega_{mn}^{\mp})^2]^{1/2} \quad (2.34)$$

and the solution take the form

$$F_{mn}^{\pm}(t)=F_{mn}^{\pm}(0)e^{-t/T}\left[\cosh(t\sqrt{1/T^2-\Omega_{mn}^{\mp 2}})+\sinh(t\sqrt{1/T^2-\Omega_{mn}^{\mp 2}})/(T\sqrt{1/T^2-\Omega_{mn}^{\mp 2}})\right] . \quad (2.35)$$

For large $T$, this reduces to the approximate form

$$F_{mn}^{\pm}(t)=F_{mn}^{\pm}(0)e^{-t/T}[\cos(\Omega_{mn}^{\mp}t)+\sinh(\Omega_{mn}^{\mp}t)/\Omega_{mn}^{\mp}T] , \quad (2.36)$$

which may also be obtained from (2.27) by letting $a=\pi/2$ therein. On the other hand, in the opposite limit of small $T$, we have a purely decaying solution

$$F_{mn}^{\pm}(t)=F_{mn}^{\pm}(0)e^{-t/T}\times[(1+T^2\Omega_{mn}^{\mp 2}/4)e^{-\Omega_{mn}^{\mp 2}Tt/2}-(T^2\Omega_{mn}^{\mp 2}/4)e^{-2t(1-\Omega_{mn}^{\mp 2}T^2/4)/T}] , \quad (2.37)$$

which may also be derived from (2.30) by letting $a=\pi/2$ therein.

### 2. *Frequency telegraph*

In the case of the frequency telegraph, the correlation function $C(t-t')$ has the form [see Eq. (2.6a)]

$$C(t-t')=\frac{1}{2}\left[\frac{1}{\nu T}+1\right]e^{-(1/T-\nu)|t-t'|}-\frac{1}{2}\left[\frac{1}{\nu T}-1\right]e^{-(1/T+\nu)|t-t'|} , \quad (2.38)$$

where $\nu^2=(1/T^2-a^2)$. Inserting Eq. (2.38) in Eq. (2.19) and taking the Laplace transform with respect to $T$, we obtain

$$\tilde{F}_{mn}^{\pm}(z)=\frac{\left[z^2+\frac{2z}{T}+a^2\right]F_{mn}^{\pm}(0)}{z^3+2z^2/T+(\Omega_{mn}^{\mp 2}+a^2)z+(2/T)\Omega_{mn}^{\mp 2}} . \quad (2.39)$$

As before, when $n=m$, $\Omega_{mn}^{-}(0)=0$ and it is clear from Eq. (2.19) that $F_{mn}^{+}(t)=F_{mn}^{+}(0)$, as in Eq. (2.23). When $n\neq m$, the inversion of Eq. (2.39) yields the general solution of the form (2.24), where $\lambda_j$ are now the roots of the cubic equation

$$\lambda^3+\frac{2}{T}\lambda^2+(\Omega_{mn}^{\mp 2}+a^2)\lambda+\frac{2}{T}\Omega_{mn}^{\mp 2}=0 . \quad (2.40)$$

This equation has one negative real root and a pair of complex-conjugate roots. As before, when $T$ is large ($g_0T\gg 1$), the approximate roots of the cubic equation (2.40) are given by

$$\lambda_1=-\frac{2}{T}\left[\frac{\Omega_{mn}^{\mp}}{\Gamma_{mn}^{\mp}}\right]^2, \quad (2.41a)$$

$$\lambda_{2,3}=\pm i\Gamma_{mn}^{\mp}-\frac{1}{T}\left[\frac{a}{\Gamma_{mn}^{\mp}}\right]^2, \quad (2.41b)$$

$$(\Gamma_{mn}^{\pm})^2=(\Omega_{mn}^{\pm})^2+a^2 . \quad (2.41c)$$

Note that the quantity $a$ has the dimensions of frequency. The appropriate solution for $F_{mn}^{\pm}(t)$ takes the form

$$F_{mn}^{\pm}(t)=\frac{F_{mn}^{\pm}(0)}{D_1'}\left\{a^2\left[1-4\frac{\Omega_{mn}^{\mp 2}}{\Gamma_{mn}^{\mp 4}T^2}\right]e^{-\gamma_1't}+\left[\left[\Omega_{mn}^{\mp 2}+\frac{4\Omega_{mn}^{\mp 4}+a^4}{\Gamma_{mn}^{\mp 4}T^2}\right]\cos(\Gamma_{mn}^{\mp}t)+\frac{3(\Omega_{mn}^{\mp}a)^2}{(\Gamma_{mn}^{\mp})^3T}\sin(\Gamma_{mn}^{\mp}t)\right]e^{-\gamma_2't}\right\}, \quad (2.42)$$

where

$$\gamma_1'=\frac{2}{T}(\Omega_{mn}^{\mp}/\Gamma_{mn}^{\mp})^2, \quad \gamma_2'=\frac{1}{T}(a/\Gamma_{mn}^{\mp})^2 , \quad (2.43a)$$

$$D_1'=(\Gamma_{mn}^{\mp})^2+(2\Omega_{mn}^{\mp 2}-a^2)^2/(\Gamma_{mn}^{\mp})^4T^2 . \quad (2.43b)$$

On the other hand, when $T$ is small ($g_0T\ll 1$) the approximate roots of the cubic equation are given by

$$\lambda_1=-2/T, \quad \lambda_{2,3}=\pm i\Omega_{mn}^{\mp}-\frac{a^2T}{4} \quad (2.44)$$

and the approximate expression for $F_{mn}^{\pm}(t)$ reads as

$$F_{mn}^{\pm}(t)=\frac{1}{D_2'}(a^2T^2e^{-\gamma_3't}+\{[(\Omega_{mn}^{\mp}T)^2-2(aT)^2+4]\cos(\Omega_{mn}^{\mp}t)-(a^2T/8\Omega_{mn}^{\mp})[2(\Omega_{mn}^{\mp}T)^2-(aT)^2+8]\sin(\Omega_{mn}^{\mp}t)\}e^{-\gamma_4't})\ , \tag{2.45}$$

where

$$\gamma_3'=-2/T,\quad \gamma_4'=-\frac{a^2T}{4}\ , \tag{2.46a}$$

$$D_2'=[(\Omega_{mn}^{\mp})^2-a^2]T^2+4\ . \tag{2.46b}$$

Once again the effects of the random frequency telegraph noise is evident in the solutions (2.42) and (2.45) by the modified amplitude and frequency of Rabi oscillations and the damping coefficients $\gamma_i'$.

Furthermore, when $a=0$, the correlation function $C(t-t')=1$ and $F_{mn}^{\pm}(t)$ reduces to the form (2.33). As a check, one may also verify that the approximate solutions (2.42) and (2.45) reduce to this form when $a$ is set equal to zero.

## III. RESULTS AND DISCUSSION

We now use the general solution of the preceding section contained in Eqs. (2.27) and (2.30) for phase telegraph noise and in Eqs. (2.42) and (2.45) for frequency telegraph noise to obtain explicit analytic expressions for some physical observables of the system. For this purpose, we assume that initially the field has a photon-number distribution $p_n$ and the atom is in the excited state. Thus the initial density operator has the form

$$\rho(0)=\sum_n p_n|n,\tfrac{1}{2}\rangle\langle n,\tfrac{1}{2}|+\sum_{n\neq m}p_{nm}|n,\tfrac{1}{2}\rangle\langle m,\tfrac{1}{2}|\ ,$$
$$=\sum_n p_n|\Psi_n^+\rangle\langle\Psi_n^+|+\sum_{n\neq m}p_{nm}|\Psi_n^+\rangle\langle\Psi_m^+|\ . \tag{3.1}$$

It is clear from here that $\rho_{mn}^{++}(0)=p_{mn}$ and $\rho_{mn}^{--}(0)=\rho_{mn}^{\pm\mp}(0)=0$ so that

$$F_{mm}^{\pm}(0)=\tfrac{1}{2}p_m,\quad F_{mn}^{\pm}(0)=\tfrac{1}{2}p_{mn}\ . \tag{3.2}$$

The probability $P_e(t)$ of finding the atom to be in the excited state at any time $t$ is

$$P_e(t)=\langle\mathrm{Tr}(S_z+\tfrac{1}{2})\rho(t)\rangle=\langle\rho_{mm}^{++}(t)\rangle\ ,$$
$$=\tfrac{1}{2}+\tfrac{1}{2}\sum_m p_m\chi_m(t)\ . \tag{3.3}$$

In the expression above we have used the fact that $F_{mm}^{+}(t)=F_{mm}^{+}(0)=p_m/2$ and introduced $\chi_m(t)=F_{mm}^{-}(t)/F_{mm}^{-}(0)$. This expression allows us to study the effects of phase fluctuations on the phenomenon of collapses and revivals. We may also study the effects of phase fluctuations on the photon statistics of the field in the cavity. For this purpose, we need the expressions for the mean photon number $n(t)=\langle\langle a^\dagger a\rangle\rangle$ and the quantity $\langle\langle a^{\dagger 2}a^2\rangle\rangle$. These expressions read as

$$n(t)=\langle\langle a^\dagger a\rangle\rangle=\langle\mathrm{Tr}[a^\dagger a\rho(t)]\rangle$$
$$=\tfrac{1}{2}+\sum_{m=0}^{\infty}mp_m-\tfrac{1}{2}\sum_{m=0}^{\infty}p_m\chi_m(t)\ , \tag{3.4}$$

$$\langle\langle a^{\dagger 2}a^2\rangle\rangle=\sum_{m=0}^{\infty}m^2p_m-\sum_{m=0}^{\infty}mp_m\chi_m(t)\ . \tag{3.5}$$

A related quantity of interest is the normalized intensity-intensity correlation

$$g^{(2)}(t)=\langle\langle a^{\dagger 2}a^2\rangle\rangle/(\langle\langle a^\dagger a\rangle\rangle)^2\ , \tag{3.6}$$

which can be obtained from Eqs. (3.4) and (3.5). The function $\chi_m(t)$ contains all the information about the stochastic fluctuations. An approximate analytical form of $\chi_m(t)$ for the case of phase telegraph and frequency telegraph may be obtained from the solutions derived in Sec. II C and is given below.

### A. Phase telegraph

The analytic expression for $\chi_m(t)$ in the case of binary random-phase telegraph reads as

$$\chi_m(t)=Ae^{-\gamma_1t}+[(1-A)\cos(f\Omega_mt)$$
$$+B\sin(f\Omega_mt)]e^{-\gamma_2t}\ , \tag{3.7}$$

where the amplitude factors $A$, $B$, the damping constants $\gamma_1,\gamma_2$, and the frequency factor $f$ depend on the dwell time $T$ of the telegraph. For large $T(g_0T\gg1)$, the factor $f$ is unity and $\Omega_m=2g_0\sqrt{m+1}$, while

$$A=-\sin^2 2a/D\ , \tag{3.8a}$$

$$B=\sin^2a[1-3\cos^2a(3\cos^2a-1)]/(D\Omega_mT)\ , \tag{3.8b}$$

$$D=(\Omega_mT)^2+(3\cos^2a-1)^2\ , \tag{3.8c}$$

$$\gamma_1=2\cos^2a/T,\quad \gamma_2=\sin^2a/T\ . \tag{3.8d}$$

On the other hand, for small $T$ $(g_0T\ll1)$, we have $f=\cos a$ and

$$A=-(\Omega_mT\sin a)^2/D,\quad B=(\Omega_mT)\sin^2a/D\cos a\ , \tag{3.9a}$$

$$D=(\Omega_mT)^2(1-4\sin^2a)+4\ , \tag{3.9b}$$

$$\gamma_1=2[1-(\Omega_mT\sin a)^2/4]/T\ ,$$
$$\gamma_2=(T\Omega_m^2\sin a)/4\ . \tag{3.9c}$$

These approximate analytical expressions are found to agree well with the exact results. The latter are obtained by inserting numerically obtained exact roots of the cubic equation (2.25) in the general form (2.24).

The effects due to phase telegraph noise on the atomic behavior and the photon statistics of the field in the cavity are depicted in Figs. 1–3. In these results the field is assumed to be in a coherent state initially. For such a field

$$p_n=\frac{|\alpha|^2}{n!}e^{-|\alpha|^2},$$
$$p_{nm}=\frac{\alpha^{*n}\alpha^m}{\sqrt{m!}\sqrt{n!}}e^{-|\alpha|^2}, \tag{3.10}$$

where $|\alpha|^2=\bar{n}$, the mean photon number in the field. As $T\rightarrow\infty$ and $a\rightarrow 0$, the expressions for $P_e(t)$, $\langle n(t)\rangle$, and $g^{(2)}(t)$ tend to the corresponding expressions for the JCM without fluctuations. Curve $A$ in Figs. 1–3 represents this case for $|\alpha|^2\equiv\bar{n}=10$. Curves $B(D)$ and $C(E)$ in each figure, on the other hand, show, respectively, the behavior of $P_e(t)$, $n(t)$, and $g^{(2)}(t)$ for a large ($g_0T=10$) and a small ($g_0T=0.1$) value of dwell time $T$ for a fixed value of jump parameter $a=\pi/4$ ($\pi/2$) and $\bar{n}=|\alpha|^2=10$. As expected, physically, the fluctuations affect the oscillatory behavior of the JCM considerably for smaller dwell times of the phase telegraph (see curves $C$ and $E$ in Figs. 1–3). In particular, for $a=\pi/2$ and $g_0T=0.1$, the oscillatory behavior is completely suppressed.

### B. Frequency telegraph

The analytic expression for $\chi_m(t)$ in the case of the binary frequency telegraph may be obtained from the general expressions

$$\chi_m(t)=A'e^{-\beta_1 t}+[(1-A')\cos(\Gamma_m t)+B'\sin(\Gamma_m t)]e^{-\beta_2 t}. \tag{3.11}$$

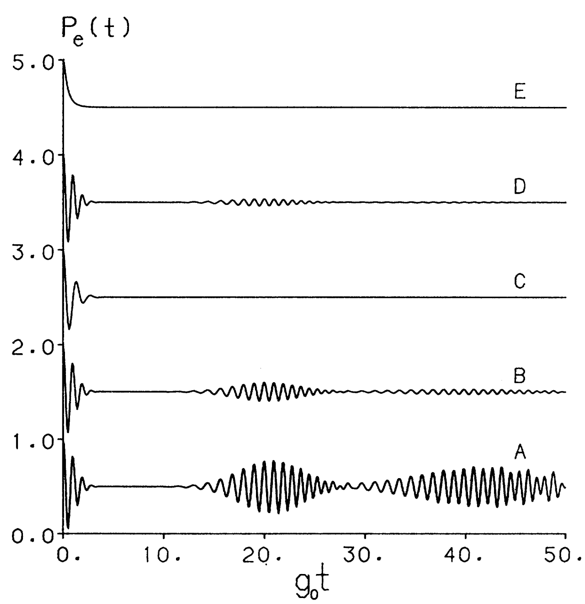


FIG. 1. Excitation probability $P_e(t)$ as a function of time $g_0t$ with phase telegraph noise for the initial coherent field with mean photon number $|\alpha|^2=10$. The curves $A[P_e(t)]$, $B[P_e(t)+1]$, $C[P_e(t)+2]$, $D[P_e(t)+3]$, and $E[P_e(t)+4]$ correspond, respectively, to the parameters $(a,g_0T)=(0,\infty)$, $(\pi/4,10)$, $(\pi/4,0.1)$, $(\pi/2,10)$, and $(\pi/2,0.1)$.

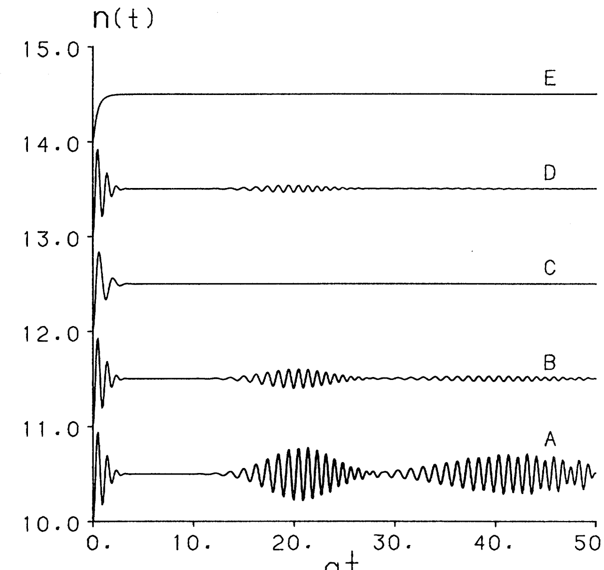


FIG. 2. Photon-number distribution $n(t)$ as a function of time $g_0t$ with phase telegraph noise for the initial coherent field with mean photon number $|\alpha|^2=10$. The curves $A[n(t)]$, $B[n(t)+1]$, $C[n(t)+2]$, $D[n(t)+3]$, and $E[n(t)+4]$ correspond, respectively, to the parameters $(a,g_0T)=(0,\infty)$, $(\pi/4,10)$, $(\pi/4,0.1)$, $(\pi/2,10)$, and $(\pi/2,0.1)$.

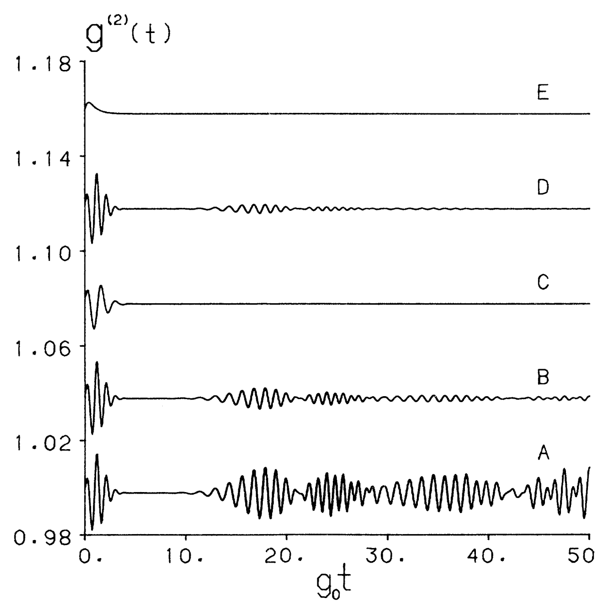


FIG. 3. Intensity-intensity correlation function $g^{(2)}(t)$ as a function of time $g_0t$ with phase telegraph noise for the initial coherent field with mean photon number $|\alpha|^2=10$. The curves $A[g^{(2)}(t)]$, $B[g^{(2)}(t)+0.04]$, $C[g^{(2)}(t)+0.08]$, $D[g^{(2)}(t)+0.12]$, and $E[P_e(t)+0.16]$ correspond, respectively, to the parameters $(a,g_0T)=(0,\infty)$, $(\pi/4,10)$, $(\pi/4,0.1)$, $(\pi/2,10)$, and $(\pi/2,0.1)$.

For large values of $T$ ($g_0 T \ll 1$),

$$\begin{aligned} &\Gamma_m = (\Omega_m^2 + a^2)^{1/2}, \quad \Omega_m = 2g_0\sqrt{m+1} , \\ &A' = a^2[1 - 4(\Omega_m / T\Gamma_m^2)^2]/D' , \\ &B' = (\Omega_m a)^2 / [T\Gamma_m^3 D'] , \\ &D' = \Gamma_m^2 + [2(\Omega_m)^2 - a^2]/(\Gamma_m)^4 T^2 \end{aligned} \tag{3.12}$$

and the damping coefficients are

$$\beta_1 = 2(\Omega_m / \Gamma_m)^2 / T, \quad \beta_2 = (a/\Gamma_m)^2 / T . \tag{3.13}$$

On the other hand, for smaller values of $T$ ($g_0 T \ll 1$), the above parameters take the form

$$\Gamma_m = \Omega_m, A = (aT)^2/D', \quad D' = (\Omega_m^2 - a^2)T^2 + 4 , \tag{3.14a}$$

$$B = a^2 T[2(\Omega_m T)^2 - (aT)^2 + 8]/(8\Omega_m D') , \tag{3.14b}$$

$$\beta_1 = -2/T, \quad \beta_2 = -a^2 T/4 . \tag{3.14c}$$

The effect of the frequency telegraph noise on the excitation probability $P_e(t)$ is pictorially shown in Fig. 4 for a field that is initially in a coherent state with mean photon number $|\alpha|^2 = 10$. Note that as $T \to \infty$ and $a \to 0$, one recovers from (3.11) the usual expression for $P_e(t)$ for the JCM without stochastic fluctuations. This case is shown by curve $A$ in Fig. 4, while curves $B$ ($D$) and $C$ ($E$) represent, respectively, the excitation probability $P_e(t)$ for the dwell time $g_0 T = 10$ and 0.1 for a fixed value of parameter $a = 0.75$ (1.5). The effects are more pronounced for smaller $T$ and larger values of $a$. This is also reflected in the behavior of $n(t)$ and the intensity-

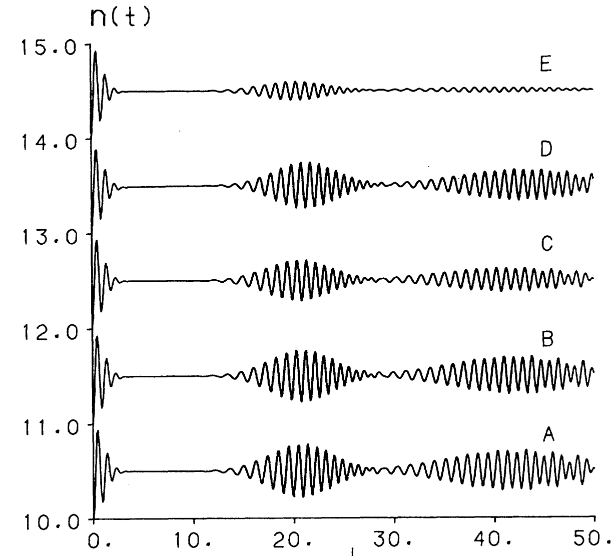


FIG. 5. Photon-number distribution $n(t)$ as a function of time $g_0 t$ with frequency telegraph noise for the initial coherent field with mean photon number $|\alpha|^2 = 10$. The curves $A[n(t)]$, $B[n(t)+1]$, $C[n(t)+2]$, $D[n(t)+3]$, and $E[n(t)+4]$ correspond, respectively, to the parameters $(a, g_0 T) = (0, \infty)$, (0.75,10), (0.75,0.1), (1.5,10), and (1.5,0.1).

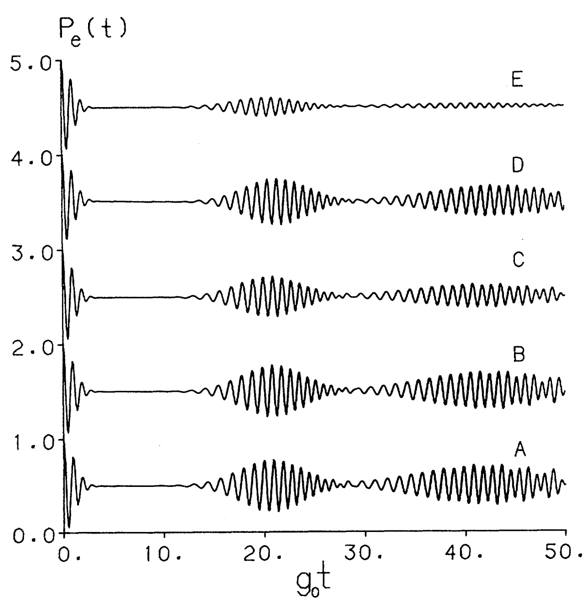


FIG. 4. Excitation probability $P_e(t)$ as a function of time $g_0 t$ with frequency telegraph noise for the initial coherent field with mean photon number $|\alpha|^2 = 10$. The curves $A[P_e(t)]$, $B[P_e(t)+1]$, $C[P_e(t)+2]$, $D[P_e(t)+3]$, and $E[P_e(t)+4]$ correspond, respectively, to the parameters $(a, g_0 T) = (0, \infty)$, (0.75,10), (0.75,0.1), (1.5,10), and (1.5,0.1).

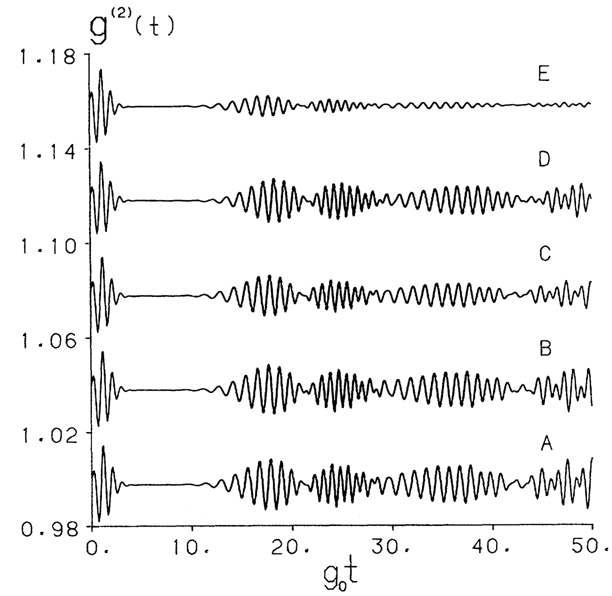


FIG. 6. Intensity-intensity correlation function $g^{(2)}(t)$ as a function of time $g_0 t$ with frequency telegraph noise for the initial coherent field with mean photon number $|\alpha|^2 = 10$. The curves $A[g^{(2)}(t)]$, $B[g^{(2)}(t)+0.04]$, $C[g^{(2)}(t)+0.08]$, $D[g^{(2)}(t)+0.12]$, and $E[P_e(t)+0.16]$ correspond, respectively, to the parameters $(a, g_0 T) = (0, \infty)$, (0.75,10), (0.75,0.1), (1.5,10), and (1.5,0.1).

intensity correlation function $g^{(2)}(t)$ shown in Figs. 5 and 6, respectively.

## IV. CONCLUDING REMARKS

We have considered in this paper yet another mechanism of intrinsic decoherence in the JCM associated with stochastic fluctuations in the atom-field coupling parameter. The mechanism involves a rather basic stochastic model for phase and frequency fluctuations introduced in quantum optics first by Burshtein [37] and subsequently by others [38–41]. This model involves noise in the form of random jump processes. We have used a simple version of such generalized Poisson processes, viz., the two-state random binary telegraph to describe both the phase and the frequency fluctuations in the JCM. An aesthetically pleasing feature of these non-Gaussian models of noise is that they lead to exact equations for atomic observables that may be solved in finite terms. Indeed, this aspect preserves the mathematical tractability of the JCM. The formalism may be suitably extended to study other features of the JCM such as the fluorescent spectrum. Moreover, the approach may also be extended to treat the decoherence effects due to a superposition of $n$ independent random binary telegraphs for phase and/or frequency fluctuations. As has been shown by Wodkiewicz, Shore, and Eberly [39], such a superposition corresponds to a pre-Gaussian noise because in the limit $n \rightarrow \infty$, it converges to a Gaussian stochastic process.

The dephasing mechanism considered here is different from usual dissipation mechanisms such as cavity-field damping and spontaneous emission decay or radiative damping of the system. In these cases one finds that the total energy of the atom plus field system is no longer a constant of motion. Such a kind of dissipation mechanism affects both diagonal as well as off-diagonal elements of the system's density operator. In other words, energy and coherence both undergo relaxation, but the latter decays very fast and thus the collapse-revival phenomenon vanishes quickly, way before any significant change is made in the energy of the system. Finally, as has been remarked earlier, the effects due to stochastic fluctuations in the JCM could be of interest in the micromasers or in the study of the motion of an ion in a harmonic trap interacting with a standing or a traveling wave.


[1] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963); F. W. Cummings, Phys. Rev. **140A**, 1031 (1965).

[2] S. Steinholm, Phys. Rep. C **6**, 1 (1973); P. Meystre, E. Geneux, A. Quattrapani, and A. Faist, Nuovo Cimento B **25**, 21 (1975); P. Meystre and M. S. Zubairy, Phys. Lett. **89A**, 390 (1982).

[3] J. H. Eberly, N. B. Narozhny, and J. J. Sanchez-Mondragon, Phys. Rev. Lett. **44**, 1383 (1980); N. B. Narozhny, J. J. Sanchez-Mondragon, and J. H. Eberly, Phys. Rev. A **23**, 236 (1981); H. I. Yoo, J. J. Sanchez-Mondragon, and J. H. Eberly, *ibid.* **14**, 1383 (1981); J. J. Sanchez-Mondragon, N. B. Narozhny, and J. H. Eberly, Phys. Rev. Lett. **51**, 1383 (1980); Y. I. Yoo and J. H. Eberly, Phys. Rep. **118**, 239 (1985).

[4] P. L. Knight and P. M. Radmore, Phys. Lett. **90A**, 342 (1982); C. Riti and G. Vetri, Opt. Commun. **44**, 115 (1982).

[5] G. Milburn, Opt. Acta **31**, 671 (1984).

[6] G. S. Agarwal, J. Opt. Soc. Am. B **5**, 480 (1985).

[7] S. Haroche, in *New Trends in Atomic Physics,* Proceedings of the Les Houches Summer School of Theoretical Physics, Session XXXVIII, edited by G. Grynberg and R. Stora (North-Holland, Amsterdam, 1984); S. Haroche and J. M. Raimond, Adv. At. Mol. Phys. **20**, 347 (1985).

[8] J. M. Raimond, P. Goy, M. Gross, C. Fabre, and S. Haroche, Phys. Rev. Lett. **49**, 117 (1982); **49**, 1924 (1982); Y. Kaluzhny, P. Goy, M. Gross, J. M. Raimond, and S. Haroche, *ibid.* **51**, 1175 (1983); D. Kleppner, *ibid.* **47**, 233 (1981); D. Meschede, H. Walther, and G. Muller, *ibid.* **54**, 551 (1985).

[9] R. Short and L. Mandel, Phys. Rev. Lett. **51**, 384 (1983).

[10] P. K. Arvinda and H. Guanhui, Physica C **150**, 427 (1988).

[11] J. R. Kuklinski and J. C. Madejczyk, Phys. Rev. A **37**, 317 (1988).

[12] P. W. Milloni, J. R. Ackerhalt, and H. W. Gailbraith, Phys. Rev. Lett. **50**, 966 (1983); R. F. Fox and J. Eidson, Phys. Rev. A **34**, 482 (1986).

[13] J. H. Eberly and K. Wodkiewicz, J. Opt. Soc. Am. **67**, 1252 (1977); J. H. Eberly, C. V. Kunasz, and K. Wodkiewicz, J. Phys. B **13**, 217 (1980).

[14] A. Joshi and R. R. Puri, J. Mod. Opt. **34**, 1421 (1987); **36**, 557 (1989); Phys. Rev. A **42**, 4336 (1990); Opt. Commun. **75**, 189 (1990); A. Joshi and S. V. Lawande, *ibid.* **70**, 21 (1989); R. R. Puri and G. S. Agarwal, Phys. Rev. A **35**, 3433 (1987).

[15] G. Rempe, H. Walther, and N. Klein, Phys. Rev. Lett. **58**, 353 (1987).

[16] T. R. Gentile, B. J. Hughey, and D. Kleppner, Phys. Rev. **40**, 5103 (1989).

[17] M. Brune, J. M. Raimond, P. Goy, L. Davidovich, and S. Haroche, Phys. Rev. Lett. **59**, 1899 (1987).

[18] S. M. Barnett and P. L. Knight, Phys. Rev. A **33**, 2444 (1986); R. R. Puri and G. S. Agarwal, *ibid.* **33**, 3610 (1986).

[19] R. R. Puri and A. Joshi, Opt. Commun. **69**, 369 (1989).

[20] Tran Quang, P. L. Knight, and V. Buzek, Phys. Rev. A **44**, 6092 (1991).

[21] P. Alsing and M. S. Zubairy, J. Opt. Soc. Am. B **4**, 177 (1987); R. R. Puri and R. K. Bullough, *ibid.* B **5**, 2021 (1988); A. Joshi and R. R. Puri, J. Mod. Opt. **36**, 215 (1989).

[22] M. Travis and F. W. Cummings, Phys. Rev. **170**, 379 (1966); S. M. Barnett and P. L. Knight, Opt. Acta **31**, 435 (1984); Z. Deng, Opt. Commun. **54**, 222 (1985); A. Joshi and S. V. Lawande, J. Mod. Opt. **38**, 1407 (1991).

[23] T. M. Makhviladze and L. A. Shelephin, Phys. Rev. A **9**, 538 (1974); N. N. Bogolubov, Jr., F. LeKien, and A. S. Shumovski, Phys. Lett. **101A**, 201 (1984).

[24] R. R. Schlicher, Opt. Commun. **70**, 97 (1989); A. Joshi and

S. V. Lawande, Phys. Rev. A **42**, 1752 (1990).
[25] V. Buzek and I. Jex, Opt. Commun. **78**, 425 (1990); A. Joshi and R. R. Puri, Phys. Rev. A **45**, 5056 (1992).
[26] A. Joshi and S. V. Lawande, Phys. Rev. A **48**, 2276 (1993).
[27] A. Joshi and S. V. Lawande, Phys. Lett. A **184**, 390 (1994).
[28] S. V. Lawande and A. Joshi, Phys. Rev. A **50**, 1692 (1994).
[29] See, for example, Adv. At. Mol. Opt. Phys. Suppl. 2, 57 (1994).
[30] S. Chaturvedi and C. W. Gardiner, J. Phys. B **14**, 1119 (1981); W. Vogel, D. G. Welsch, and K. Wodkiewicz, Phys. Rev. A **28**, 1546 (1983); R. R. Puri and S. S. Hassan, Physica A **120**, 55 (1983).
[31] H. Von Haken, in *Light and Matter Ic,* edited by L. Genzel, Handbuch der Physik Vol. 25/2c (Springer, Berlin, 1970); J. H. Eberly, Phys. Rev. Lett. **37**, 1387 (1976); K. Wodkiewicz, B. W. Shore, and J. H. Eberly, J. Opt. Soc. Am. B **1**, 389 (1984); B. J. Dalton and P. L. Knight, J. Phys. B **12**, 3999 (1982); R. R. Puri and S. V. Lawande, Physica A **120**, 43 (1983).
[32] N. G. Van Kampen, Physica **74**, 215 (1974); **74**, 239 (1974).
[33] H. Moya-Cessa, V. Buzek, M. S. Kim, and P. L. Knight, Phys. Rev. A **48**, 3900 (1993).
[34] G. Milburn, Phys. Rev. A **44**, 5401 (1991).
[35] J. I. Cirac, R. Blatt, P. Zoller, and W. D. Phillips, Phys. Rev. A **46**, 2668 (1992).
[36] C. A. Blockley, D. F. Walls, and H. Risken, Europhys. Lett. **17**, 509 (1992).
[37] A. I. Burshtein, Zh. Eksp. Teor. Fiz. **49**, 1362 (1965) [Sov. Phys. JETP **22**, 939 (1966)]; A. I. Burshtein and V. S. Oseldechick, *ibid.* **51**, 1071 (1966) [*ibid.* **24**, 716 (1967)]; L. D. Zusman and A. I. Burshtein, *ibid.* **61**, 976 (1971) [*ibid.* **34**, 520 (1972)].
[38] P. Zoller and F. Ehlotzky, J. Phys. B **15**, 3023 (1977).
[39] K. Wodkiewicz, B. W. Shore, and J. H. Eberly, J. Opt. Soc. Am. B **1**, 398 (1984).
[40] J. H. Eberly, K. Wodkiewicz, and B. W. Shore, Phys. Rev. A **30**, 2381 (1984).
[41] K. Wodkiewicz, B. W. Shore, and J. H. Eberly, Phys. Rev. A **30**, 2390 (1984).