Self-induced modulation and compression of an ultracold atomic cloud in a nonlinear atomic cavity

Weiping Zhang
School of Mathematics, Physics, Computing and Electronics, Macquarie University, New South Wales 2109, Australia and Optical Sciences Center, University of Arizona, Tucson, Arizona 85721

Pierre Meystre and Ewan M. Wright
Optical Sciences Center, University of Arizona, Tucson, Arizona 85721
(Received 27 December 1994)

We consider a sample of ultracold atoms inside an atomic cavity and driven by a laser field. We show that this field acts effectively as a nonlinear "light crystal" for the intracavity atomic wave, which in turn can cause self-modulation and compression of ultracold atomic wave packets. The analogy to the conventional compression technique of optical pulses in a nonlinear optical cavity is discussed.

PACS number(s): 42.50.Vk, 32.80.−t

I. INTRODUCTION

Atom optics, which exploits the wavelike nature of atoms, has led in recent years to a number of developments, including the prediction and demonstration of atom interferometers [1–3], atomic gratings [4,5], atomic mirrors [6,7], and atomic cavities [8–10]. In combination with recent progress in atomic cooling and trapping, this work has recently been extended to include many-atom effects for ultracold atoms. In the ultracold regime, it is best to think of the atoms as a quantum field. The analogy between the matter fields of ultracold atoms and the electromagnetic fields of conventional optics has led to an extension of atom-optics research toward the study of the quantum statistics of ultracold atoms as well as nonlinear atom optics [11–21].

In this paper, we apply the general formalism of nonlinear atom optics to the analysis of a potential geometry which yields modulation and compression of an ultracold atomic cloud. Specifically, we consider an ultracold atomic cloud trapped in a cavity formed by two spatially separated atomic mirrors in the longitudinal direction x of the atomic center-of-mass motion, and a harmonic trap in the transverse direction. In addition, a laser beam perpendicular to the cavity axis induces an effective nonlinear atomic interaction via photon exchange and absorption. A schematic diagram of this arrangement is shown in Fig. 1. The cavity is perpendicular to the direction of the earth's gravitational field, so that its longitudinal modes are not affected by the earth's gravity. The purpose of the harmonic trap in the transverse direction is to prevent the atoms from falling out of the cavity due to gravity. The transverse confinement could be provided by, for example, a magneto-optical trap. The remainder of this paper is organized as follows. Section II reviews the general formalism of nonlinear atom optics and discusses the modes of the atomic cavity under consideration. Section III then analyzes the self-induced modulation and compression of an ultracold atomic cloud in the nonliner atomic cavity. Finally, Sec. IV gives our summary and conclusions.

II. GENERAL FORMALISM

A general quantum field theory of ultracold atoms interacting with a light wave has been recently developed [11–13,15,16,19]. The basic idea of this theory is to treat the ultracold atomic ensemble as an N-component vector quantum field, each component corresponding to one of the electronic states of the atoms. For example, in the case of two-level atoms, this would be a two-component field $\Psi(\vec{r})=\Psi_1(\vec{r})|1\rangle+\Psi_2(\vec{r})|2\rangle$ with $|1\rangle$ and $|2\rangle$ denoting the internal ground state and excited atomic state, and $\Psi_1(\vec{r})$ and $\Psi_2(\vec{r})$ being the corresponding atomic field operators. Further, the ground state $|1\rangle$ is assumed to be the lowest-energy state of the atoms so that it does not decay spontaneously. When the ultracold atomic ensemble interacts with a laser beam, the dynamic evolution of the ensemble can be described either by a vector nonlinear stochastic Schrödinger equation [12,15,16] or by a nonlinear master equation [19]. We adopt the former point of view in the present paper.

When the atoms are confined inside a cavity, their interaction time with the laser beam can be significantly longer than the times characterizing their dipole interaction with light, namely, the inverse spontaneous emission rate of the excited states, the inverse atom-field detun-

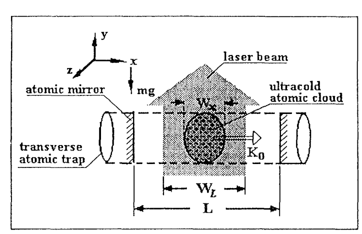

FIG. 1. Schematic diagram of the nonlinear atomic cavity.

ings, and the inverse Rabi frequencies of the various transitions under consideration. Assuming then detunings large enough that the excited states can be adiabatically eliminated, the dynamics of the ultracold atomic ensemble is approximately determined by a reduced Heisenberg equation of motion for the atomic ground-state quantum field operation [12,15,16], which can be taken to be scalar for the case of a nondegenerate ground state. This equation is of the general form

$$i\hbar\frac{\partial\Psi_1}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2m}+V_{\rm ex}(\vec r)+V(\vec r)+V_R(\vec r)\right]\Psi_1 + \int d^3\vec r' Q(\vec r,\vec r')\Psi_1^\dagger(\vec r')\Psi_1(\vec r')\Psi_1(\vec r)\ , \quad (1)$$

where m is the atomic mass, $V_{\rm ex}(\vec r)$ is the external potential,

$$V(\vec r)=\hbar|\Omega(\vec r)|^2(\delta-i\gamma/2)/(4\delta^2+\gamma^2)$$

is the light-induced single-atom potential, $V_R(\vec r)$ is a random potential which accounts for the random scattering of atoms due to spontaneous emission [12,16], and

$$Q(\vec r,\vec r')\approx\frac{\hbar\gamma|\Omega(\vec r)|^2\delta(\delta-i\gamma/2)}{2(\delta^2+\gamma^2/4)^2}\times W(\vec r-\vec r')\cos[k_L(y-y')] \quad (2a)$$

is the two-body dipole-dipole interaction potential, which originates from the photon exchange between ultracold atoms during spontaneous emission. The function

$$W(\vec r-\vec r')=\frac{3}{4}\left[-\sin^2\theta\frac{\cos(k_L|\vec r-\vec r'|)}{k_L|\vec r-\vec r'|}+(1-3\cos^2\theta)\left[\frac{\sin(k_L|\vec r-\vec r'|)}{(k_L|\vec r-\vec r'|)^2}+\frac{\cos(k_L|\vec r-\vec r'|)}{(k_L|\vec r-\vec r'|)^3}\right]\right] \quad (2b)$$

determines the spatial distribution of the dipole-dipole interaction, and θ is the angle between the dipole moment $\vec\mu$ and the relative position vector $\vec r-\vec r'$. In expression (2a), we have denoted the spontaneous emission rate of the excited-state atoms γ, the detuning of the laser beam from the atomic resonance δ, the wave number of the laser k_L, and the Rabi frequency $\Omega(\vec r)=2\vec\mu\cdot\vec E(\vec r)/\hbar$, with $\vec E(\vec r)$ the field strength of the laser beam. Both the single-atom potential $V(\vec r)$ and the two-body interaction potential $Q(\vec r,\vec r')$ are non-Hermitian complex potentials, their complex nature arising from the spontaneous decay of the excited-state atoms. The imaginary parts of the complex potentials and the random potential $V_R(\vec r)$ account for the inelastic scattering of atoms into other incoherent channels [16,22] during the spontaneous emission. The external potential is assumed to have the form $V_{\rm ex}(\vec r)=V_L(x)+V_T(y,z)$. In this paper, we assume that the atomic mirrors have perfect reflection, so that the longitudinal potential has the property $V_L(x)=0$ for $0<x<L$, and $V_L(x)=\infty$ at the two ends of the cavity $x=0$ and $x=L$. The transverse potential has the form $V_T(y,z)=mgy+m\nu^2(y^2+z^2)/2$ with ν denoting the transverse trapping frequency of the atomic cavity, and may be supplied by, e.g., a magneto-optical trap.

In general, the absorption of the laser beam by the ultracold atoms will result in a spatial variation of the Rabi frequency, which depends on the density distribution $\rho(\vec r)=\Psi^\dagger(\vec r)\Psi(\vec r)$ for the ultracold atomic ensemble. In the regime of weak absorption and in the slowly varying envelope approximation, the correct Rabi frequency due to photon absorption has the form [15,16]

$$|\Omega(\vec r)|^2\approx[\Omega_0F(x,z)]^2\times\left[1-\frac{\gamma^2}{4\delta^2+\gamma^2}\frac{3\lambda_L^2}{2\pi}\times\int_{-\infty}^{y}\Psi^\dagger(x,y',z)\Psi(x,y',z)dy'\right]\ , \quad (3)$$

where Ω_0 is the peak Rabi frequency determined by the initial peak laser intensity , the function $F(x,z)$ describes the transverse profile of the laser beam incident perpendicular to the cavity axis, and the laser wavelength is denoted as λ_L. In principle, Eqs. (1)–(3) completely determine the dynamic evolution of the atomic cloud in the nonlinear atomic cavity. It is, however, difficult in general to obtain a direct solution of these equations. Here we assume for simplicity that the external transverse trapping potential $V_T(y,z)$ is sufficiently strong compared to the two-body interaction that the transverse center-of-mass motion is approximately determined by the single-atom behavior. Under this assumption, we have the eigenvalue problem for the transverse motion of the atoms

$$\left[-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]+mgy+\tfrac{1}{2}m\nu^2(y^2+z^2)\right]\varphi_T=E_T\varphi^T\ . \quad (4)$$

Equation (4) determines the transverse displaced Hermite-Gaussian eigenmodes of the atomic cavity,

$$\varphi_T^{nm}=\frac{(2^{n+m}n!m!)^{-1/2}}{W_T\sqrt{\pi}}\exp\left[-\frac{(y+g/\nu^2)^2+z^2}{2W_T^2}\right]\times H_n\left[\frac{y+g/\nu^2}{W_T}\right]H_m\left[\frac{z}{W_T}\right]\ , \quad (5)$$

with the eigenvalues $E_T^{nm}=(n+m+\frac{1}{2})\hbar\nu-mg^2/2\nu^2$. The width of the ground transverse mode is defined as $W_T=\sqrt{\hbar/m\nu}$. From Eq. (5), we see that the earth's gravity only results in a displacement of the atomic center of mass in the y direction.

III. SELF-INDUCED MODULATION AND COMPRESSION

Our main purpose is to study the nonlinear dynamics of an ultracold atomic cloud loaded in the atomic cavity. To simplify our discussion, we consider a simple case where the transverse profile of the initial atomic cloud matches the transverse ground mode ($m=n=0$) of the atomic cavity given in Eq. (5). In a magneto-optical trap this is the case if we assume that the atoms occupy the ground mode prior to applying the laser field. In addition, to obtain a nonlinear interaction for an extended time, we choose the incident laser beamwidth to fill the atomic cavity so that the profile factor $F(y,z)\approx 1$ in the cavity. Under these assumptions, the three-dimensional problem can be reduced to one dimension by introducing the approximate expression of the atomic quantum field operator

$$\Psi_1(\vec{r},t)=\varphi_T^{00}(y,z)\phi(x,t)\exp(-iE_T^{00}t/\hbar) , \tag{6}$$

this approximation being valid as long as the transverse motion is confined to the ground mode. Substituting Eq. (6) into Eqs. (1)–(3), in the ultracold regime [13,15,16], yields a reduced one-dimensional nonlinear Schrödinger equation for the longitudinal envelope atomic quantum field operator $\phi(x,t)$,

$$i\hbar\frac{\partial\phi}{\partial t}=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V_x(x)+V_0\left[1-i\frac{\gamma}{2\gamma}\right][1-\kappa_a\phi^\dagger(x)\phi(x)-\kappa_d\phi^\dagger(x)\phi(x)]\right]\phi , \tag{7a}$$

where $V_0=\hbar\Omega_0^2\delta/(4\delta^2+\gamma^2)$, the characteristic length associated with the nonlinearity due to photon absorption is

$$\kappa_a=3\lambda_L^2\gamma^2/[(2\pi)^{3/2}(4\delta^2+\gamma^2)W_T] ,$$

and the characteristic length associated with the photon exchange is determined by

$$\kappa_d=\{\gamma\delta\lambda_L^3/[4\pi^4W_T^2(\delta^2+\gamma^2/4)]\}\beta .$$

The parameter β has the definition

$$\beta\equiv\left|\int_0^{k_LL}d\xi_x\int_{-\infty}^{\infty}d\xi_y\int_{-\infty}^{\infty}d\xi_z W(\xi)\cos(\xi_y)\exp\left[-\frac{(\xi_y+k_Lg/v^2)^2+\xi_z^2}{(k_LW_T)^2}\right]\right| . \tag{7b}$$

In the derivation of Eq. (7a), we have assumed that the atomic field is slowly varying in the longitudinal direction compared to the optical wavelength λ_L. We point out that the integral in the expression (7b) may be divergent for the dipole-dipole interaction potential defined by Eq. (2b). However, such a divergence can be avoided by noting that the spatial orientation of the atomic dipole moment $\vec{\mu}$ is uncertain for real atomic samples. In this case, the real long-range dipole-dipole interaction potential should be given by integrating over angle θ in Eq. (2b). As a result, we have the function $W(\xi)=-\cos(\xi)/4\xi$ which leads to a finite characteristic length κ_d. In addition, we remark that if we had not made the single-transverse-mode approximation (6), then we would be faced with a three-dimensional nonlinear Schrödinger equation including the transverse potential. This does not pose any conceptual problems but here our intention is to highlight the longitudinal dynamics in Eq. (7a).

To study nonlinear effects in the atomic cavity, we consider a coherent ultracold atomic cloud composed of N bosonic atoms in the macroscopic single quantum state described by the wave function

$$|U_N(t)\rangle=\frac{1}{\sqrt{N!}}\left[\int dx_1\int dx_2\cdots\int dx_N\Phi(x_1,x_2,\ldots,x_N,t)\phi^\dagger(x_1)\phi^\dagger(x_2)\cdots\phi^\dagger(x_N)\right]|0\rangle . \tag{8}$$

For a large atom number N, and in the time-dependent Hartree approximation, the envelope wave function $\Phi(x_1,x_2,\ldots,x_N,t)$ then satisfies the c-number nonlinear Schrödinger equation [23,15]

$$i\hbar\frac{\partial\Phi}{\partial t}=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V_x(x)+V_0\left[1-i\frac{\gamma}{2\delta}\right][1-\kappa(N-1)|\Phi|^2]\right]\Phi , \tag{9}$$

where $\kappa=\kappa_a+\kappa_d$ is the total characteristic length determined by the atomic nonlinearity. The physics implicit in Eq. (9) is quite transparent. The imaginary part of the linear potential accounts for the spontaneous emission loss of ground-state atoms due to inelastic scattering of atoms into other incoherent channels. The real part of the nonlinear potential is analogous to the Kerr-type nonlinearity of conventional nonlinear optics; finally, the imaginary part of the nonlinear potential is similar to the self-saturation nonlinearity of conventional nonlinear optics. Hence we conclude that the laser beam acts as a nonlinear absorption "crystal" for the cold atomic wave when the effects of spontaneous emission are included. In view of this discussion, it is clear that the proposed nonlinear atomic cavity is analogous to a conventional optical cavity with saturable absorber [24]. Here the laser beam plays the role of the saturation absorption cell for the atoms.

To solve Eq. (9), a direct method is to expand the envelope wave function $\Phi(x,t)$ in terms of the longitudinal modes of the (linear) atomic cavity. For our numerical simulations, we assume that an initial coherent ultracold atomic cloud is released at the center of the atomic cavity with velocity $v_0=\hbar K_0/m$ along the x direction. If the atomic cloud is prepared in a harmonic trap as a pure state, we expect that the initial condition for Eq. (9) has the form

$$\Phi(x,0)=\frac{1}{(\sqrt{\pi}W_x)^{1/2}}\exp\left[-\frac{(x-L/2)^2}{2W_x^2}+iK_0x\right], \quad (10)$$

where W_x is the initial atomic beamwidth in the longitudinal direction. Such a moving Gaussian wave packet will initially excite several longitudinal modes of the atomic cavity around an averaged mode index $n_0 \approx K_0L/\pi$ where n_0, although it does not need to be, is assumed to be an integer for convenience. We can then expand the envelope wave function in terms of the cavity longitudinal modes in the normalized form

$$\Phi(x,t)=\left[\frac{L}{\sqrt{\pi}W_x}\right]^{1/2} \times \sum_{n=-n_0+1}^{\infty} a_n(t)\left[\frac{2}{L}\right]^{1/2}\sin\left[\frac{(n+n_0)\pi x}{L}\right]. \quad (11)$$

Substituting expression (11) into Eq. (9) and using the initial condition (10), we obtain a set of nonlinear coupled equations for the intracavity cold atomic waves,

$$\frac{\partial a_n}{\partial \tau}=-i\left[n+\frac{n^2}{2n_0}+\frac{n_0}{2}\right]a_n-i\varepsilon\left[1-i\frac{\gamma}{2\delta}\right]a_n +i\varepsilon\left[1-i\frac{\gamma}{2\delta}\right]\chi\left[\sum_{pq}a^*_{n-p+q}a_pa_q +\tfrac{1}{2}\sum_{pq}a^*_{p+q-n}a_pa_q\right], \quad (12)$$

with the initial condition

$$a_n(0)=\left[\frac{2\sqrt{\pi}W_x}{L^2}\right]^{1/2} \times\int_0^L dx\,\Phi(x,0)\sin\left[\frac{(n+n_0)\pi x}{L}\right]. \quad (13)$$

Here $\tau=n_0\hbar\pi^2t/mL^2$ is a dimensionless time, $\varepsilon=V_0mL^2/n_0\hbar^2\pi^2$ is the normalized potential parameter, and

$$\chi=\kappa(N-1)/\sqrt{\pi}W_x =[\gamma^2/(4\delta^2+\gamma^2)]\rho_0(3W_T\lambda_L^2/2\sqrt{2\pi}+\delta\beta\lambda_L^3/\pi^3\gamma) \quad (14)$$

the dimensionless nonlinear characteristic length. In the expression of the dimensionless nonlinear characteristic length, we used the peak density of the ultracold atomic cloud $\rho_0=(N-1)/\pi^{3/2}W_xW_T^2$. Equation (12) is the basis for our study of the intracavity dynamics of an ultracold atomic cloud of bosonic atoms. Three typical cases are considered in our simulations. In the first case, the laser field is turned off and the atomic cloud experiences a linear evolution in the atomic cavity. The result is shown in Fig. 2. We see that the initial atomic cloud moves from the center of the cavity to the right-end mirror and then experiences repeated reflections between the two cavity mirrors. At every reflection period π, the spatial distribution of the atomic cloud repeats its initial shape.

When the laser field is turned on, the motion of the atomic cloud is affected by its interaction with light. In terms of Eq. (12), this interaction leads to a loss and a nonlinearity for the intracavity atomic wave. The magnitude of the loss and the nonlinearity of atoms can be controlled by choosing both the laser and the atomic parameters appropriately. In the example shown here, we choose a laser detuning large enough to reduce the random scattering events of atoms into other incoherent channels due to spontaneous emission and permit a negligible loss of ground-state atoms in the cavity. In such an off-resonance regime, the dipole-dipole interaction and the atomic absorption are weakened as well. However, in terms of expression (14), one can still obtain a considerable nonlinearity by choosing the appropriate peak density and the transverse width of the atomic cloud. In this case, the laser field just acts as a "nonlinear crystal" with a Kerr-type nonlinearity for the intracavity atomic wave. Such an atomic nonlinearity results in the self-phase modulation of the intracavity atomic wave, which causes the mixing of the cavity eigenmodes. In Fig. 3, we see that due to nonlinear multimode mixing the spatial distribution of the atomic cloud in the cavity no longer repeats its initial shape at every reflection period π. This is further illustrated in Figs. 4(a) and 4(b), where we plot the spatial distributions of the atomic cloud for a sequence of normalized times separated by the interval π. Figure 4(a) shows that instead of the periodic recurrence of Fig. 2, we now have a situation where the atomic cloud, after several reflections, exhibits a multipeak structure. The formation of such a multipeak atomic spatial pattern is due to the self-modulation of the atomic wave in the pres-

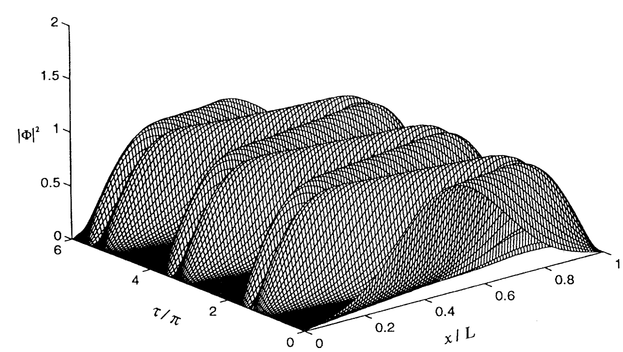

FIG. 2. Dynamic evolution of a coherent ultracold atomic cloud in a (laser-free) linear atomic cavity. The longitudinal width of the initial atomic clouds is chosen as $W_x=L/4\sqrt{2}$.

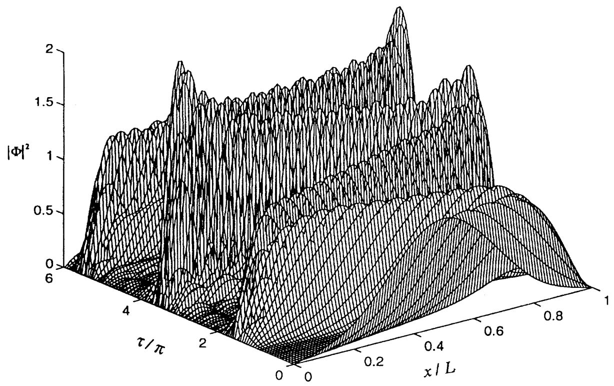

FIG. 3. Dynamic evolution of a coherent ultracold atomic cloud in a laser-driven nonlinear atomic cavity. The laser detuning is chosen to be large enough to give negligible spontaneous emission. The other parameters chosen for the simulation are $W_x = L/4\sqrt{2}, n_0 = 100, \chi = 0.3$, and $\varepsilon = 1$.

ence of the "light crystal." This atomic self-modulation is analogous to self-phase modulation of light waves in a nonlinear optical crystal [25]. Further calculations show that the nonlinear self-modulation behavior in the atomic cavity is almost periodic. In Fig. 4(b), which extends Fig. 4(a) to longer times, we see that the multipeak structure due to self-modulation eventually disappears, to reappear again after some time. This "revival" is similar to the modulation instability of conventional nonlinear optics [26].

Since some loss of ground-state atoms to other incoherent channels always occurs due to spontaneous emission, we finally simulate the intracavity atomic dynamics including the effects of spontaneous emission. The results are plotted in Fig. 5, which displays the compression of the spatial distribution of the atomic cloud. The results are plotted in Fig. 5, which displays the compression of the spatial distribution of the atomic cloud. This compression is induced by the combined effects of self-modulation and the intracavity atomic loss. When spontaneous emission is not negligible, the atoms in the cavity are partially scattered into incoherent channels by random photon recoil. The incoherently scattered atoms account for the atomic cavity losses. As we have seen in Eqs. (9) and (12), for an ultracold atomic sample the atomic cavity losses depend nonlinearly on

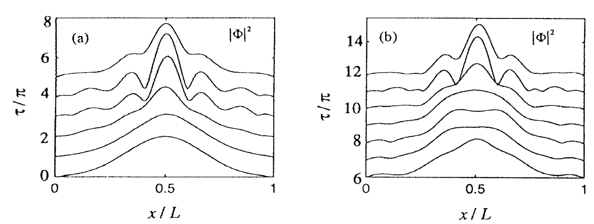

FIG. 4. Self-induced modulation of the ultracold atomic cloud in a nonlinear atomic cavity. The plotted spatial density distributions correspond to a sequence of times varying (a) from $\tau = 0$ to 5π and (b) from $\tau = 6\pi$ to 12π. All other parameters are the same as in Fig. 3.

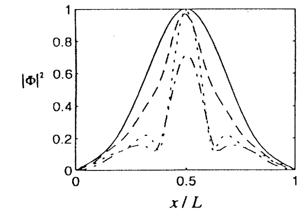

FIG. 5. Self-induced compression of the ultracold atomic cloud in a nonlinear atomic cavity due to the presence of spontaneous emission. The laser and atomic parameters chosen for the simulations are $\delta = 10\gamma, W_x - L/4\sqrt{2}, n_0 = 100, \chi = 0.3$, and $\varepsilon = 1$. The various times are solid line ($\tau = 0$), dashed line ($\tau = \pi$), dotted line ($\tau = 2\pi$), and dash-dotted line ($\tau = 3\pi$).

the atomic sample density. In terms of Eq. (9), the total loss of the atomic cavity has the form

$$\Gamma(x) = V_0\gamma/2\delta\hbar[1 - \kappa(N-1)|\Phi(x)|^2] .$$

Hence the cavity losses depend on the spatial distribution of the atomic cloud. Figure 6 shows the spatial dependence of the total cavity losses at different times. We see that the edge of the atomic cloud experiences a larger loss rate than the center of the cloud. As a result, a dip appears in the spatially dependent cavity loss rate. The shape of the dip changes with time, due to the self-modulation of the atomic density profile. This spatial dip in the loss rate modifies the atomic cloud and narrows its spatial distribution at the center of the atomic cavity. This is similar to passive mode locking in a conventional optical cavity with a nonlinear absorber [24], which can be used to compress a coherent light pulse to generate an ultrashort pulse. The only difference is that the compression of optical pulses is in the time domain whereas the compression of the atomic cloud occurs in the spatial domain.

IV. SUMMARY AND CONCLUSIONS

In conclusion, we have discussed a nonlinear atomic cavity for ultracold atomic clouds. The atomic nonlinearity in the cavity is induced by the interaction of ultracold atoms with an external laser field. To observe the nonlinear atomic effect, the density of the atomic cloud must be high enough to produce the two-body interaction via photon absorption and exchange in the presence of the laser field. As an example, we consider an ultracold cloud of hydrogen atoms ${}^1_1H$. The wavelength for the transition $2S_{1/2} - 3P_{1/2}$ of hydrogen is $\lambda_L = 0.6563$ μm.

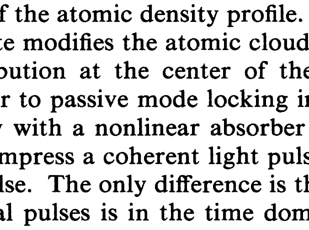

FIG. 6. Spatially dependent loss rate of the nonlinear atomic cavity. The parameters are the same as in Fig. 5.

If the laser frequency is detuned from the atomic resonance by $\delta=10\gamma$, and transverse confinement produces a ground-state wave function with $W_T=100$ μm, then the peak density for $\chi\approx0.3$ as used in our calculations must satisfy the condition $\rho_0\lambda_L^3\approx\frac{4}{3}>1$ or $\rho_0\approx3.5\times10^{12}$ cm^{-3}. Under such a condition, the light wave acts as a nonlinear "crystal" for the intracavity "cold" atomic wave.

In summary, we have predicted the self-induced modulation and compression of the spatial pattern of an ultracold atomic cloud in a nonlinear atomic cavity. The analogy to the conventional nonlinear optical cavity has been established, and on that basis we conclude that, under appropriate conditions, the nonlinear atomic cavity can be employed to generate a short spatially coherent atomic pulse via atomic mode locking.

ACKNOWLEDGMENTS

The work was supported in part by NSF Grant No. PHY92-13762, by the U.S. Office of Naval Research Contract No. N00014-91-J120, and by the Joint Service Optics Program. W.Z. acknowledges the hospitality of the Optical Sciences Center at the University of Arizona during his visit and also the support of the Australian Research Council.

[1] O. Carnal and J. Mlynek, Phys. Rev. Lett. **66**, 2689 (1991).
[2] D. W. Keith, C. R. Ekstrom, Q. A. Tuchette, and D. E. Pritchard, Phys. Rev. Lett. **66**, 2693 (1991).
[3] M. Kasevich and S. Chu, Phys. Rev. Lett. **67**, 181 (1991).
[4] E. Arimondo, A. Bambini, and S. Stenholm, Opt. Commun. **37**, 103 (1981).
[5] J. V. Hajnal and G. I. Opat, Opt. Commun. **71**, 119 (1989).
[6] R. J. Cook and R. K. Hill, Opt. Commun. **43**, 258 (1982).
[7] V. I. Balykin, V. S. Letokhov, Yu. B. Ovchinnikov, and A. I. Sidorov, Phys. Rev. Lett. **60**, 2137 (1988).
[8] V. I. Balykin and V. S. Letokhov, Appl. Phys. B **48**, 517 (1989).
[9] H. Wallis, J. Dalibard, and C. Cohen-Tannoudji, Appl. Phys. B **54**, 407 (1992).
[10] M. Wilkens, E. Goldstein, B. Taylor, and P. Meystre, Phys. Rev. A **47**, 2366 (1993).
[11] Weiping Zhang and D. F. Walls, Quantum Opt. **5**, 9 (1993).
[12] Weiping Zhang, Phys. Lett. A **176**, 53 (1993).
[13] G. Lenz, P. Meystre, and E. M. Wright, Phys. Rev. Lett. **71**, 3271 (1993).
[14] M. Lewenstein and L. You, Phys. Rev. Lett. **71**, 1339 (1993).
[15] Weiping Zhang, D. F. Walls, and B. C. Sanders, Phys. Rev. Lett. **72**, 60 (1994).
[16] Weiping Zhang and D. F. Walls, Phys. Rev. A **49**, 3799 (1994).
[17] J. Javanainen, Phys. Rev. Lett. **72**, 2375 (1994).
[18] J. I. Cirac, M. Lewenstein, and P. Zoller, Phys. Rev. Lett. **72**, 2977 (1994).
[19] (a) G. Lenz and P. Meystre, in *Quantum Optics VI*, edited by J. D. Harvey and D. F. Walls (Springer, Berlin, 1994), p. 68; (b) G. Lenz, P. Meystre, and E. M. Wright, Phys. Rev. A **50**, 1681 (1994).
[20] Weiping Zhang and D. F. Walls, Phys. Rev. A **50**, 4069 (1994).
[21] Weiping Zhang and D. F. Walls, in *Quantum Optics VI* [19], pp. 78–86.
[22] Weiping Zhang and B. C. Sanders, J. Phys. B **27**, 795 (1994).
[23] E. M. Wright, Phys. Rev. A **43**, 3836 (1991).
[24] A. Yariv, *Quantum Electronics* (Wiley, New York, 1967).
[25] Y. R. Shen, *The Principles of Nonlinear Optics* (Wiley, New York, 1984).
[26] G. P. Agrawal, *Nonlinear Fiber Optics* (Academic, San Diego, 1989).