Atomic anapole moments in the electroweak theory

A. Bhattacharya, M. M. Panja, and B. Talukdar
Department of Physics, Visva-Bharati University, Santiniketan 731 235, India
(Received 30 January 1995; revised manuscript received 10 April 1995)

The role of anapole moments ($\vec{a}$) arising from the P-violating and T-invariant theory of electron-nucleon interactions in the atom is examined. The standard model of the electroweak theory is used to compute numbers for $|\vec{a}|$ for alkali-metal atoms and muonic hydrogen. The physical implications of the results obtained are discussed.

PACS number(s): 32.80.ys

In the standard model (SM) [1] of the electroweak interaction in the neutral current form, there is always a Z_0 channel accompanying the corresponding γ channel. The exchange of massive Z_0 bosons between the electron and the nucleus is responsible for the weak interaction and leads to a zero-range effective Hamiltonian that must be added to the usual Coulomb Hamiltonian. The Z_0 gauge boson can couple like an axial vector either to the electronic current or to the nucleonic current. The first one makes the major contribution to atomic parity nonconservation (PNC) [2]. The corresponding nonrelativistic electron-nucleus potential V_{pv} is given by [3]

$$V_{pv}=\frac{Q_W}{4\sqrt{2}}\frac{G_F}{m}\{\vec{p}\cdot\vec{\sigma},\delta(\vec{r})\}_+, \tag{1}$$

where m, $\frac{1}{2}\vec{\sigma}$, $\vec{p}$, and $\vec{r}$ are, respectively, the mass, spin, momentum, and position of the electron. In the Weinberg-Salam model [1] the effective weak charge

$$Q_W=-[(4\sin^2\theta_W-1)Z+N] \tag{2}$$

with θ_W the Weinberg angle. The quantities N and Z stand for the neutron and proton number in the nucleus. The Fermi constant $G_F=10^{-5}m_p^{-2}$ (m_p is the mass of the proton). The interaction in (1) causes mixing between even and odd parity states. Considering $s_{1/2}$ and $p_{1/2}$ orbitals, the electronic wave function for the mixed state can be written as

$$\Psi=\frac{1}{\sqrt{4\times\pi}}[R_0(r)-i\eta R_1(r)\vec{\sigma}\cdot\hat{\vec{r}}]\chi, \tag{3}$$

where $R_0(r)$ and $R_1(r)$ are the appropriate radial wave functions and χ is the spin function. That the mixing coefficient $i\eta$ is purely imaginary is a result of the T invariance of the weak interaction. The wave function in (3) can be rewritten in the form

$$\Psi=\frac{1}{\sqrt{4\pi}}R_0(r)\left[1-i\theta(r)\frac{\vec{\sigma}\cdot\hat{\vec{r}}}{2}\right]\chi, \quad \theta(r)=2\eta\frac{R_1(r)}{R_0(r)} \tag{4}$$

to exhibit that the admixture is equivalent to a local rotation of the spinor χ around the direction $\hat{\vec{r}}$ [4]. This spin precession (Fig. 1) is a source of current that gives rise to a third family of multipole moments in addition to usual electric and magnetic ones [5]. These are toroidal multipole moments. The toroidal multipole moment of rank one [toroidal dipole moment (TDM)] is often called an *anapole moment* (AM) (Fig. 2). Regarded as a source of parity nonconserving interaction, studies in AM are an interesting curiosity.

In this work we shall be concerned with semileptonic anapoles which arise from the exchange of Z_0 gauge bosons between the atomic electrons and hadrons in the nucleus. As with semileptonic anapoles one can also talk about hadronic and leptonic ones. A single free particle may also acquire TDM from the radiative correction in the electroweak theory. Recently, the scenario for the dynamical origin of different

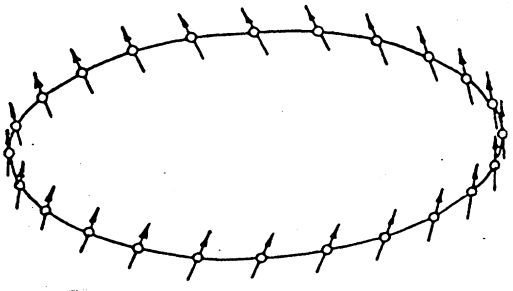

FIG. 1. Precision of the spinor χ resulting from $s_{1/2}$ and $p_{1/2}$ mixing due to the interaction V_{pv} in (4).

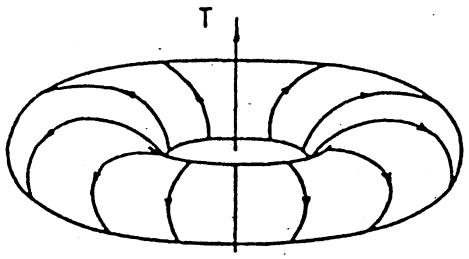

FIG. 2. A toroidal solenoid with poloidal current C giving rise to anapole moment ($\vec{T}$).

types of anapole moments has been expounded by Lewis [6] in the context of leptonic TDM calculations for atoms.

The calculation of semileptonic AM for a general many-electron atom is complicated by summations over infinite classes of Brueckner-Goldstone perturbation theoretic diagrams [7]. In view of this we shall compute values of AM's for the alkali and exotic atoms because we believe that, for these systems, large atomic physics uncertainties can be avoided to a fair degree of accuracy. For example, the structure and spectra of alkali atoms can be understood relatively simply because they are made of closed shells with one valence electron. The situation is similar to the hydrogen atom except that the potential on the valence electron is not purely Coulombic but is shielded by the core electrons resulting in the removal of l degeneracy. The effect of inner screening is well taken care of by the so-called quantum defect theory (QDT) [8].

For alkali-metal atoms we proceed by assuming that the core electrons merely screen the nuclear field and do not participate in the weak interaction with the outer electron. In other words we ignore the leptonic contribution to TDM. This is justified since it has been found that the leptonic TDMs are three orders of magnitude smaller than the universal scale $G_F/c \simeq 4.55\times 10^{-33}$ cm^2 set for toroidal moments [6]. The valence electrons of alkali atoms are s electrons. Under the influence of parity-violating potential in (1), parity mixing occurs between s and p orbitals. In the independent particle model of the atom, the matrix elements of V_{pv} reads

$$\langle n_i s_{1/2}|V_{pv}|np_{1/2}\rangle = -3i\frac{G_F Q}{8\pi m\sqrt{2}}R'_{np}(0)R_{n_i p}(0). \quad (5)$$

Here prime denotes differentiation with respect to r. The values of the radial wave function at the origin are determined by using the Fermi-Segré formula as given in Bouchiat, Bouchiat, and Pottier [9]. The mixing coefficient $\langle n_i s_{1/2}|V_{pv}|np_{1/2}\rangle/(E_{n_i 0}-E_{n1})$ can now be used to construct parity mixed states for the valence electron of alkali atoms.

The anapole moment operator is given by [10]

$$\vec{a} = \frac{\pi}{m}\left\{(\vec{r}\times\vec{\sigma}) - \frac{i}{3}[l^2,\vec{r}]\right\}. \quad (6)$$

The first term in (6) represents the contribution to $\vec{a}$ by the spin current while the second term arises due to the orbital current. The matrix element of $\vec{a}$ between the parity mixed states gives the following expression for the anapole moment $|\vec{a}|$.

$$a = -CQZ^2\sum_{n=n_i}^{n_f}\frac{[1+\delta_0(E_{n_i 0})+\delta_1(E_{n1})]r(n_i 0,n1)}{(E_{n_i 0}-E_{n1})_{\text{in Ry}}(\nu_{n_i 0}\nu_{n1})^{3/2}}, \quad (7)$$

where

$$r(n_i 0,n1) = \int_0^\infty r^3 R_{n_i 0}R_{n1}dr, \quad (8)$$

TABLE I. Values of anapole moments (a) from ^{3}Li to ^{55}Cs. The numbers in brackets stand for powers of 10.

Atom	n_i	n_f	$\lvert\vec{a}\rvert$(cm^2)
Li	2	6	1.49[−34]
Na	3	7	6.53[−33]
K	4	8	5.79[−32]
Rb	5	10	4.55[−31]
Cs	6	10	2.59[−30]

$$Q = 0.08Z - N \text{ and } C = Ba_0^2 \text{ with } B = \frac{2}{9\sqrt{2}\pi}Gm^2\alpha^3. \quad (9)$$

Here a_0 is the Bohr radius and $E_{nl} = -1/\nu_{nl}^2$, the binding energy in Ry with l degeneracy removed. The quantity $\delta_l(E_n)$ is given in terms of the interpolated quantum defect $\mu_{nl}(E_n) = n - \nu_{nl}$ and average potential acting on the valence electron [9,11]. In writing (7) we have used

$$\langle\vec{a}\rangle = ea\vec{J} \text{ and } \sin^2\theta_W = 0.23. \quad (10)$$

Equation (10) clearly shows that the anapole moment has the direction of the total angular momentum J and its magnitude is given in (7). Based on (7) we have computed values of $|\vec{a}|$ for alkali-metal atoms from ^{3}Li to ^{55}Cs. The energy levels and quantum defects characterizing the expression have been given by Ham [12]. The first term in (7) gives the most dominant contribution but is not infinite because of the removal of l degeneracy by inner screening. In computing the results in Table I we have restricted the sum in accordance with the availability of values for energy levels and quantum defects. Looking at this table we see that, beginning from ^{3}Li, as we go to higher alkalis the values for $|\vec{a}|$ tend to increase approximately linearly in the logarithmic scale. The result for ^{3}Li falls below the universal scale for toroidal moments by one order of magnitude and the numbers become significant from ^{11}Na onward. More interestingly, $|\vec{a}|$ for ^{11}Na compares quite well with the free-electron anapole moment 5×10^{-33} cm^2 [6]. Apenko and Lozovik [13] have computed the anapole moment in deuterium for the $2s_{1/2}$ electron. Here the degeneracy between $2s_{1/2}$ and $2p_{1/2}$ states is not removed by screening. Instead, the energy difference $E_{2s_{1/2}} - E_{2p_{1/2}} = 4.34\times 10^{-6}$ eV is due to the Lamb shift. This is extremely small and the contribution to $|\vec{a}|$ may be thought of as arising entirely due to the first term in the sum like that in (7). The value obtained, $a = 2\times 10^{-30}$ cm^2, is of the same order of magnitude as our result for ^{55}Cs.

We have begun by noting that the most dominant contribution to atomic PNC comes from Z_0 exchange between the electron and the nucleus and assume that the associated neutral current interaction induces an electronic anapole moment. Therefore, one would like to see how the electronic TDM compares with the well established nuclear AM, which has been a subject of experimental interest [14]. The atomic anapole moment $a \sim e\alpha GZ^2 Q_W$. Its energy of interaction with a current density ej will be $\sim\alpha^2 GZ^2 Q_W j$. The interaction of the same current with the nuclear AM [4]

$\sim GZ^2\kappa_a j$ with κ_a a dimensionless constant which for heavy nuclei with odd Z lies between 0.3 and 0.4. Thus the atomic AM is suppressed compared to that of nuclear AM by a factor $\alpha^2 Q_W/\kappa_a$. Ideally, the atomic TDM is expected to interact magnetically with the nucleus and thus contribute to the total P-odd electron-nucleon spin dependent effect leading to hyperfine dependence of the weak interaction. But because of its extreme smallness this dependence will be difficult to observe and for all practical purposes may not be distinguishable from the internal nuclear AM. The calculation presented by us is nonrelativistic. For high Z atoms such as Rb and Cs, one might attempt to incorporate the effect of relativity and try to improve on the results presented by us.

It is believed that, as with conventional atoms, studies of PNC effects in muonic atoms may also be interesting [15,16]. The bound states of these atoms are highly localized owing to the large muon mass and the corresponding Bohr radius is smaller than that of electronic atoms roughly by a factor 1/207. Thus it will be instructive to compare the values for $|\vec{a}|$ for muonic hydrogen and deuterium for the muon in the $2s_{1/2}$ state with the result of Ref. [13]. If we agree to disregard the nuclear recoil and finite nucleon dimension then the formula in (7) can be directly applied for our case study provided the sum is restricted to $n=2$ only and the quantum defect μ_{nl} is set equal to zero. The mass of the electron should be replaced by muonic mass and the Bohr radius should also be appropriately changed. In the case of ordinary hydrogen the energy difference between $2s_{1/2}$ and $2p_{1/2}$ states is due to the Lamb shift. But for muonic hydrogen the main contribution to this energy difference is made by the vacuum polarization. If we use the theoretical value [17] $E_{2s_{1/2}}-E_{2p_{1/2}}=-2.0\times10^{-7}$ eV then our estimate for the muonic anapole moment is 3.26×10^{-27} cm^2. This number is three orders of magnitude larger than the corresponding value of Apenko and Lozovik. For muonic deuterium, there would be a further enhancement in the value of TDM due to an increase in the value of weak charge and we would have $a=4.06\times10^{-26}$ cm^2. In view of these augmented results for $|\vec{a}|$ we hope that as with QED the muonic atom will play a crucial role in the tests of the gauge theories of electroweak interactions.

This work is supported by the Department of Science and Technology, Government of India.

[1] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, in *Elementary Particle Theory*, edited by N. Svartholm (Almquist and Forlag, Stockholm, 1968).
[2] E. D. Commins, Phys. Scr. **T46**, 92 (1993).
[3] M. A. Bouchiat and C. C. Bouchiat, Phys. Lett. **48B**, 111 (1974).
[4] I. B. Khriplovich, *Parity Nonconservation in Atomic Phenomena* (Gordon & Breach, Philadelphia, 1991).
[5] V. M. Dubovik and V. V. Tugushev, Phys. Rep. **187**, 145 (1990).
[6] R. R. Lewis, Phys. Rev. A **48**, 4107 (1993).
[7] S. A. Blundell, W. R. Johnson, and J. Sapirstein, Phys. Rev. Lett **65**, 1411 (1990).
[8] H. Friedrich, *Theoretical Atomic Physics* (Springer-Verlag, New York, 1991).
[9] C. Bouchiat, M. A. Bouchiat, and L. Pottier, in *Atomic Physics* **5**, edited by R. Marrus, M. Prior, and H. Shugart (Plenum Press, New York, 1977).
[10] I. B. Khriplovich and M. E. Pospelov, Z. Phys. D **17**, 81 (1990).
[11] A. Bhattacharya, P. K. Bera, M. M. Panja, and B. Talukdar, Phys. Rev. A **51**, 841 (1995).
[12] F. S. Ham, in *Solid State Physics: Advances in Research and Applications*, edited by F. Seitz and D. Turnbull (Academic Press, New York, 1955), Vol. I.
[13] S. M. Apenko and Yu. E. Lozovik, J. Phys. B **15**, L57 (1982).
[14] M. C. Noecker, B. P. Masterson, and C. E. Wieman, Phys. Rev. Lett. **61**, 310 (1988).
[15] J. Missimer and L. M. Simons, Phys. Rep. **118**, 179 (1985).
[16] P. Langacker, Phys. Lett. B **256**, 277 (1991).
[17] W. Greiner and J. Reinhardt, *Quantum Electrodynamics* (Springer-Verlag, New York, 1992).