PHYSICAL REVIEW A VOLUME 52, NUMBER 6 DECEMBER 1995

# Conditions for anomalous resonance fluorescence in a squeezed vacuum

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(Received 21 March 1995; revised manuscript received 4 August 1995)

We discuss the conditions which give rise to anomalous features in the resonance fluorescence of a two-level atom interacting with a resonant classical field and a squeezed vacuum. A simple expression is obtained, which is shown to coincide closely with the conditions for amplification of a probe beam and with the collapse of the atomic system into a pure state. We thereby illustrate striking observable consequences of the evolution of a driven system into a pure state. This situation also provides a method for preparing an atom in a pure state whose nature depends on the squeezing phase and Rabi frequency. The conditions for hole burning are obtained analytically, and the sensitivity of these effects to the two-photon correlations produced by the squeezed vacuum is demonstrated.

PACS number(s): 42.50.Dv, 32.80.−t

## I. INTRODUCTION

A decade ago, the first few experiments announcing the successful observation of "squeezed" light were reported [1–3], thus demonstrating the reduction of the quantum fluctuations in one quadrature of a light beam below the geometric mean of the minimum dictated by the uncertainty principle. Apart from the basic interest in manipulating quantum fluctuations, the study of squeezed light was also motivated by potential applications to telecommunications [4] and high precision measurement, such as gravity wave detection [5]. Squeezed light has been used to enhance sensitivity in saturation spectroscopy [6].

Nowadays, with many laboratories throughout the world capable of producing squeezed sources, the search for novel features in the interaction between atomic systems and squeezed light is a topic of renewed interest. The first prediction of unusual features in this interaction was made by Gardiner [7], who showed that the two quadratures of the polarization of a two-level atom interacting with a squeezed vacuum decay at vastly different rates. The modifications of the resonance fluorescence spectrum of a such a system were studied by Carmichael, Lane, and Walls [8], who showed that for large classical applied field strengths the spectrum is a triplet, as in the absence of the squeezed vacuum [9], but that the central peak of the triplet has a width that depends strongly on the relative phase of the driving field and the squeezed vacuum when the driving field is intense. Resonance fluorescence in the cavity environment has also been considered [10].

While we are concerned only with resonance fluorescence in this paper, we should draw attention to other aspects of the interaction between atoms and squeezed light, including modification of Lamb shifts and other atomic properties, probe absorption spectroscopy, and three-level atom and multiatom interactions. References to this large body of work can be found in reviews [11].

Recently, we have reconsidered the problem of the resonance fluorescence of a two-level atom when it interacts with a broadband squeezed vacuum [12], and shown that spectra may occur which are quite unlike any predicted before for this system. They include hole burning at line center and dispersive profiles. An unusual feature of these "anomalous spectra" is that they occur for only a highly restricted range of values of the system parameters.

Our previous analysis was largely numerical, and one object of this paper is to give an analytic description which accounts for the distinctive features of the spectra. In addition to the additional insight which an analytic description provides, we obtain a simple expression which for relatively intense squeezed fields provides the approximate values of the parameters for which anomalous spectra occur. We show that this condition coincides with other properties of a two-level atom interacting with a coherent applied field in the presence of an intense squeezed vacuum, namely, the condition for the amplification of a probe beam [15] and the condition for the atomic system to collapse into a pure state under the influence of the squeezed vacuum [16–18].

The latter effect is a particularly interesting feature of the squeezed vacuum, and shows that the squeezed field behaves quite differently from a conventional reservoir. Here we show that the evolution of the system into a pure state has striking experimental consequences—namely, a sharp reduction in the overall intensity of resonance fluorescence, and the appearance of anomalous features.

The particular pure state into which the atom evolves depends upon the squeezing phase and the Rabi frequency. Hence we have a means of producing a variety of atomic pure states, which are coherent superpositions of the ground and excited states.

## II. THE RESONANCE FLUORESCENCE SPECTRUM

In the rotating frame, and using a system of units in which $\hbar=1$, the Hamiltonian for a single two-level atom interacting with a monochromatic laser field of frequency $\omega_L$ is given by

$$H=\frac{\delta}{2}(|1\rangle\langle 1|-|0\rangle\langle 0|)+g|1\rangle\langle 0|+g^*|0\rangle\langle 1|, \tag{1}$$

where $|1\rangle$ and $|0\rangle$ are the excited and ground state of the atom, respectively, with energies $E_1$ and $E_0$, $\delta$ is the detuning, $\delta\equiv E_1-E_0-\omega_L$, and $g\equiv(i/2)\Omega e^{i\phi_L}$ is the coupling constant, $\Omega$ being the Rabi frequency and $\phi_L$ the laser phase.

 

The atom also interacts with a broadband squeezed vacuum, whose center frequency coincides with the atomic resonance frequency. Essentially all the reservoir modes with which the atom interacts are squeezed. The squeezed vacuum is characterized by the parameters $N$ and $\xi = \gamma M e^{i\phi}$ ($N, \gamma, M$, and $\phi$ being real) where $N$ and $M$ satisfy the following inequality:

$$M^2 \leqslant N(N+1). \tag{2}$$

$N$ is a measure of the intensity of the squeezed field and $M$ measures the degree of two-photon correlation. The situation of greatest interest is that in which the equality holds in Eq. (2). This corresponds to the maximum degree of two-photon correlations in the squeezed field, and the field then has the greatest nonclassical character. It also corresponds to a minimum uncertainty squeezed state. Another important parameter is the squeezing phase $\Phi$, which is defined as the difference between twice the phase of the driving field and the phase of the squeezed vacuum:

$$\Phi \equiv 2\phi_L - \phi. \tag{3}$$

Some recent reviews of squeezing phenomena are cited in [11].

The resonance fluorescence spectrum may be calculated by adapting the approach of Mollow [9] to include the effects of a squeezed vacuum [8]. It can be related to the Fourier transform of the atomic correlation function [13,14]:

$$G(\omega) = \mathrm{Re}\left[ \int_0^\infty \langle \sigma_+(0)\sigma_-(t)\rangle e^{i\omega t} dt \right], \tag{4}$$

where $\omega$ is the frequency measured relative to the laser frequency $\omega_L$, and the $\sigma_\pm(t)$ are the usual Pauli spin-$\frac{1}{2}$ operators. From the Laplace transform of the equations of motion for the $\sigma$, the spectrum may be evaluated using the quantum regression theorem [9]. It takes the form

$$G(\omega) = \mathrm{Re}[\Lambda(z=-i\omega)], \tag{5}$$

where [14]

$$\Lambda(z) = \frac{z[(z+2\Gamma)(z+\Gamma-i\delta)+\Omega^2/2]\rho + s(z+\gamma)(z+\Gamma-i\delta+\gamma M e^{-i\Phi})}{z\{(z+2\Gamma)[(z+\Gamma)^2+\delta^2-\gamma^2M^2]+\Omega^2(z+\Gamma+\gamma M\cos\Phi)\}}, \tag{6}$$

with $\Gamma = \gamma(N+\frac{1}{2})$, and

$$\rho = \frac{\gamma N(\Gamma^2-\gamma^2M^2+\delta^2)+\frac{1}{2}\Omega^2(\Gamma+\gamma M\cos\Phi)}{2\Gamma(\Gamma^2-\gamma^2M^2+\delta^2)+\Omega^2(\Gamma+\gamma M\cos\Phi)} \tag{7}$$

and

$$s = \frac{\gamma\Omega^2(\Gamma+i\delta+\gamma M e^{i\Phi})/4}{2\Gamma(\Gamma^2-\gamma^2M^2+\delta^2)+\Omega^2(\Gamma+\gamma M\cos\Phi)} \tag{8}$$

are related to the stationary excited-state population and coherence $\sigma_{10}$, respectively.

By writing $\Lambda(z)$ in the form

$$\Lambda(z) = \frac{b_0 + z b_1(z)}{z[a_0 + z a_1(z)]}, \tag{9}$$

where $a_0$ and $b_0$ are independent of $z$, we may divide the spectrum into the sum of coherent and incoherent terms:

$$\Lambda(z) = \Lambda_{\mathrm{coh}}(z) + \Lambda_{\mathrm{inc}}(z), \tag{10}$$

and

$$\Lambda_{\mathrm{coh}}(z) = \frac{b_0}{a_0 z}, \tag{11}$$

$$\Lambda_{\mathrm{inc}}(z) = \frac{a_0 b_1(z) - b_0 a_1(z)}{a_0[a_0 + z a_1(z)]}. \tag{12}$$

We are only concerned with the incoherent part here.

### III. THE CONDITIONS FOR ANOMALOUS SPECTRA

The anomalous resonance fluorescence spectra take a variety of forms, all quite distinct from what we may call the standard resonance fluorescence spectra in a squeezed vacuum. The latter, first investigated by Carmichael, Lane, and Walls [8], take the form of a single peak with subnatural linewidth in the low intensity limit, $\Omega \ll \gamma$, the width being independent of the squeezing phase $\Phi$. As $\Omega$ increases, there is a threshold value of $\Omega$ above which the spectrum splits into three peaks, which at large values of $\Omega$ are clearly separated. The value of this threshold, and the relative heights and widths of the peaks, depends upon $\Phi$ as well as upon $N$ and $M$.

Recently, we have shown that other, quite different line shapes are possible under special conditions [12]. The anomalous spectra occur in the intermediate intensity range $\Omega \sim \gamma$, where the standard spectra consist of a single line, whose height and width depend markedly on $\Phi$. For $\Phi = 0$, they take the form of sharp holes bored into a broader single peak at line center. As $\Phi$ increases, the character of the spectra gradually changes until for $\Phi = \pi/2$ they have a dispersive profile. Other line shapes are possible. As $\Phi$ increases and approaches $\pi$, the anomalous characteristics of the spectra gradually disappear.

Apart from their unusual profiles, a distinctive feature of the anomalous spectra is that they only occur for a very restricted set of values of the available parameter space. The first consideration is therefore to identify this set. The anomalous features only occur in the presence of the squeezed vacuum with $M \simeq \sqrt{N(N+1)}$ and $\delta \simeq 0$. For simplicity, we henceforth set $\gamma = 1$ so that all parameters are measured relative to this quantity.

To proceed with our analytic investigation, we assume $\delta=0$ and note that the maximum value of $M$ permitted by the inequality (2) may be expressed as

$$M_{\max}=\sqrt{N(N+1)}=\sqrt{\Gamma^2-\tfrac{1}{4}}, \tag{13}$$

or

$$M_{\max}\simeq\Gamma-\frac{1}{8\Gamma}\quad\left(\Gamma\gg\frac{1}{4}\right). \tag{14}$$

In the following we assume that $N$ is large enough to satisfy $\Gamma\gg\frac{1}{4}$. In fact, $N$ does not have to be large for (14) to be a good approximation. For $N=1$, for example, the exact value of $M_{\max}$ is $M_{\max}=\sqrt{2}=1.4142$, while the expansion (14) gives $M_{\max}\simeq1.4167$. Even for $N=0.2$, the error is only 6%.

In order to determine the values of $\Omega$ and $\Phi$ which give rise to the anomalous features, we observe from our previous investigations that they occur when the overall intensity of the incoherent spectrum drops sharply to a minimum value. One way to identify the relevant parameter range is therefore to find an expression for the incoherent spectrum and then to establish the conditions which make the intensity at line center a minimum. From Eq. (5), the intensity at line center is given by $\Lambda_{\text{inc}}(0)$. In fact, since approximate expressions will suffice, we consider the minimum in the numerator only of $\Lambda_{\text{inc}}(0)$. As we are assuming that $M$ is close to its maximum value allowed by the inequality (2) we set

$$M=\Gamma-\frac{a}{\Gamma},\quad \text{where } a\geqslant\frac{1}{8}. \tag{15}$$

Setting $a=\frac{1}{8}$ gives $M$ its maximal value. We substitute this expression into Eq. (12), expand in powers of $\Gamma$, and then find the condition for the leading two terms in the expansion to vanish. After some algebra, this leads to the result that the numerator of $\Lambda_{\text{inc}}(0)$ is a minimum when

$$\Omega^2=\frac{2[\Gamma(1-4a)+\sqrt{4a^2+\Gamma(1-8a)}]}{(1+\cos\Phi)(2\Gamma+1)},\quad \Gamma\gg1. \tag{16}$$

If we set $a=\frac{1}{8}$ ($M$ is maximal), this reduces to the very simple condition

$$\Omega=\frac{1}{2\cos(\Phi/2)},\quad \Gamma\gg1. \tag{17}$$

Both (16) and (17) show a very simple dependence on $\Phi$. From the latter we see that $\Omega\rightarrow\frac{1}{2}$ as $\Phi\rightarrow0$, $\Omega=1/\sqrt{2}$ when $\Phi=\pi/2$, and $\Omega\rightarrow\infty$ as $\phi\rightarrow\pi$. Thus for $\Phi\rightarrow\pi$ we do not observe the anomalous spectra.

It transpires that conditions similar to Eq. (17) have arisen in the consideration of other aspects of the interaction of this system with the squeezed vacuum. These concern the absorption of a probe beam and the existence of a pure state for the atom. We shall now briefly discuss these.

In [15] the same system we discuss here was considered, except for the addition of a probe beam. Due to the existence of the squeezed vacuum, this system was shown to exhibit gain without any form of population inversion for appropriate values of the Rabi frequency. The value of the Rabi frequency at which the shift from absorption to amplification of the probe beam occurred was

$$\Omega^2=\frac{2\Gamma^2(\Gamma^2-M^2)}{M(M+\Gamma\cos\Phi)}. \tag{18}$$

We note that for a minimum uncertainty state [i.e., the equality of Eq. (2) holds], we have $\Gamma^2-M^2=\frac{1}{4}$. If, further, the condition $N\gg1$ holds, then $M\simeq N+\frac{1}{2}=\Gamma$, and Eq. (18) tends to the limit (17) for large $N$.

As we discuss below, Eq. (17) is also the condition for the atom to be found approximately in a pure state. It is very surprising that a driven system interacting with a reservoir can achieve a steady state which is a pure state, and this result shows that a squeezed vacuum reservoir is fundamentally different from the reservoirs usually studied. The decay to a pure state of a pair of two-level atoms interacting with a squeezed vacuum (but with no coherent driving field) was first predicted by Palma and Knight [16]. The existence of special values of the parameters from an assembly of two-level atoms interacting with a broadband squeezed vacuum and an external field which lead to decay to a pure state was pointed out by Agarwal and Puri [17], for the case $\Phi=0$ or $\pi$. In our notation, their condition for the single-atom system to be approximately in a pure state is

$$\Omega=\frac{\sqrt{M}}{(\sqrt{N+1}+\sqrt{N})\cos(\Phi/2)}. \tag{19}$$

For $N\gg1$ and $M$ maximal, this reduces to condition (17). Note that the two-photon correlations are essential ($M\neq0$) for the system to be capable of achieving a pure state. For $\Phi=0$, the system is exactly described by a pure state when Eq. (19) is satisfied: for other values of $\Phi$ the pure state is only approximately achieved, and in addition, $N$ must be sufficiently large.

The situation for a single two-level atom was also discussed by Tucci [18] within the context of the entropy, information, and temperatures of the system. The pure-state question received particular attention, and it was pointed out that the pure state achieved for the $\Phi=0$ case for our system is an eigenstate of the $\sigma_x$ Pauli operator.

The parameter employed by Tucci for determining whether the system is in a pure state is the length of the expectation value of the Bloch vector $(\langle\sigma_x\rangle, \langle\sigma_y\rangle, \langle\sigma_z\rangle)$. In Fig. 1 we have plotted the quantity

$$\Sigma=\langle\sigma_x\rangle^2+\langle\sigma_x\rangle^2+\langle\sigma_z\rangle^2 \tag{20}$$

for $N=1$ and $\Phi=0$, $\pi/2$, $3\pi/4$, and $\pi$. When $\Sigma=1$, the system is in a pure state, and when it is zero, the system is in a highly mixed state (the entropy is a maximum). The figure shows that for $\Phi=0$ (solid line) $\Sigma$ acquires the maximum value of unity for $\Omega=\frac{1}{2}$. For $\Phi=\pi/2$ (dot-dashed line), $\Sigma$ acquires its maximum value, which is close to but less than 1, for $\Omega\simeq0.63$. This trend of decreasing maxima occurring at larger values of $\Omega$ continues for $\Phi=3\pi/4$ (dashed line), but for $\Phi=\pi$ (lower solid line) we see that $\Sigma$ has no maximum, and in fact declines monotonically from a low initial value over the range of $\Omega$ shown.

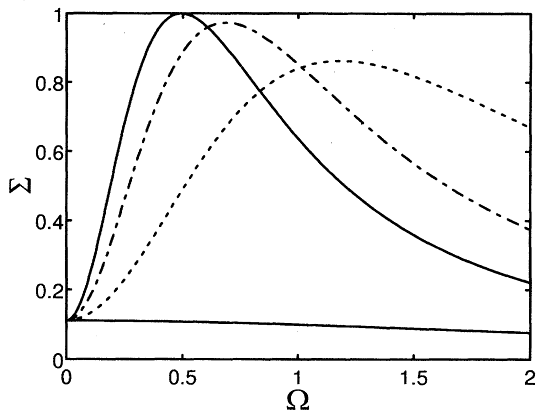


FIG. 1. The quantity $\Sigma = \langle \sigma_x \rangle^2 + \langle \sigma_y \rangle^2 + \langle \sigma_z \rangle^2$ as a function of $\Omega$ for $N=1$ and $\Phi = 0$ (solid line), $\Phi = \pi/2$ (dot-dashed line), $\Phi = 3\pi/4$ (dotted line), and $\Phi = \pi$ (lower solid line). $\Sigma = 1$ corresponds to a pure state.

Increasing the value of $N$ causes the maxima for the $\Phi = \pi/2$ and $3\pi/4$ cases to approach closer to the value one, while the overall magnitude in the $\Phi = \pi$ case decreases even further.

Figure 2 shows the actual components of $\Sigma$ as a function of $\Phi$ for $\Omega = 0.63$ and $N=1$. (This value of $\Omega$ corresponds with the maximum of the dot-dashed line in Fig. 1.) We see that $\langle \sigma_x \rangle$ has a dispersive shape, while $\langle \sigma_y \rangle$ and $\langle \sigma_z \rangle$ show a minimum at $\Phi = \pi$. It will be observed that all three variables $\langle \sigma_x \rangle$, $\langle \sigma_y \rangle$, and $\langle \sigma_z \rangle$ have small absolute values at $\Phi = \pi$ and so $\Sigma$ is small at this point. Also, $\langle \sigma_x \rangle$ and $\langle \sigma_y \rangle$ have values close to $0.7 \simeq 1/\sqrt{2}$ for $\Phi = \pi/2$, so we expect $\Sigma$ to be close to the value one at this point.

These features are confirmed by Fig. 3, which shows $\Sigma$ as a function of $\Phi$ for $N=1$ and $\Omega = 0.63$. The graph has a maximum value (close to one) at $\Phi = \pi/2$, and a minimum (close to zero) at $\Phi = \pi$. Thus the system is well approximated by a pure state at $\Phi = \pi/2$, $\Omega = 0.7$, and $N=1$, where we have shown that dispersive anomalous resonance fluorescence spectra arise [12].

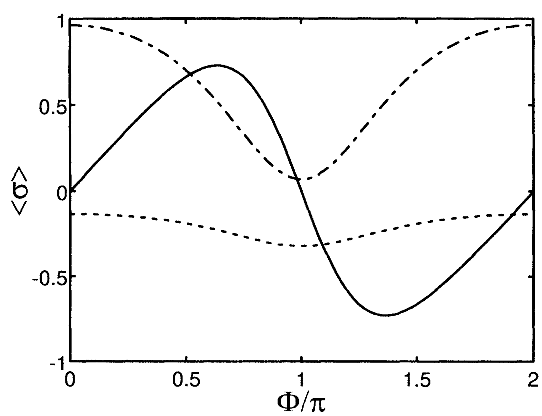


FIG. 2. The components $\langle \sigma_x \rangle$ (solid line), $\langle \sigma_y \rangle$ (dot-dashed line), and $\langle \sigma_z \rangle$ (dotted line) of $\Sigma$ as a function of $\Phi/\pi$ for $N=1$ and $\Omega = 0.63$ (corresponding to the maximum in the dot-dashed curve of Fig. 1).

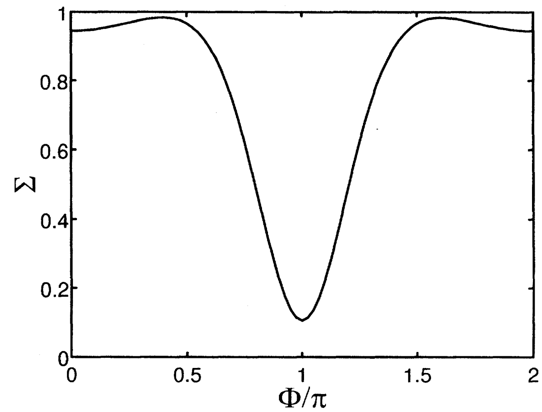


FIG. 3. $\Sigma$ as a function of $\Phi/\pi$ for $N=1$ and $\Omega = 0.63$ (corresponding to the maximum in the dot-dashed curve of Fig. 1).

If we increase the value of $N$ to $N=10$, for example, keeping the other parameters the same, we obtain a qualitatively similar plot. The major difference is that the maxima are much closer to the value one and the minimum much closer to the value zero.

Figure 4 provides a global view of the variation of $\Sigma$ with $\Omega$ and $\Phi$ for the case $N=1$. It can be seen that as $\Phi$ increases, the maximum in the value of $\Sigma$ occurs at larger values of $\Omega$. The value of the maximum also decreases with the value of $\Phi$. For $\Phi \simeq \pi$, there is no maximum apparent for the range of $\Omega$ considered.

If $N$ is sufficiently large, the analytic approximations (16)–(19) provide an excellent approximation for the value of $\Omega$ for which the anomalous spectra occur, given a value of $\Phi < \pi$. As all three expressions are very close to each other in this limit, we may as well take the simplest, Eq. (17). In Fig. 5 we compare the approximation (17) with the minimum of $\Lambda_{\mathrm{inc}}(0)$ found numerically for the case $N=2$. The agreement is very good, at least up to $\Phi \simeq 0.8\pi$. (For values of $\Phi$ larger than this the anomalous spectra are not particularly interesting anyway.) The agreement improves rapidly as $N$ increases. For $N=10$, for example, the curves are practically indistinguishable.

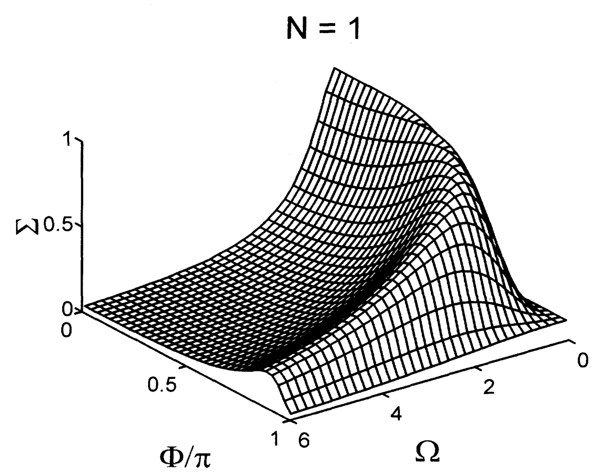


FIG. 4. $\Sigma$ as a function of $\Phi/\pi$ and $\Omega$ for $N=1$.

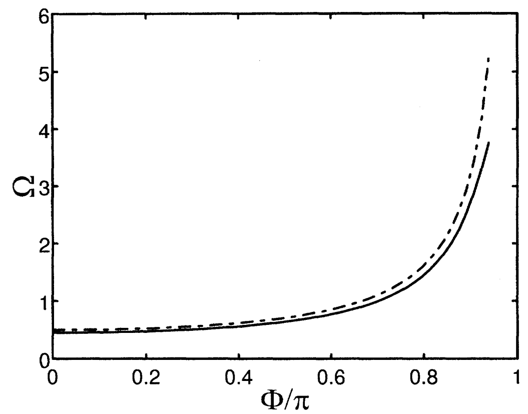


FIG. 5. The position of the minimum in $\Lambda_{\mathrm{inc}}(0)$ evaluated numerically (solid line) and using the approximate expression (17) (dot-dashed line) as a function of the Rabi frequency $\Omega$ and the squeezing phase $\Phi$, for the case $N=2$.

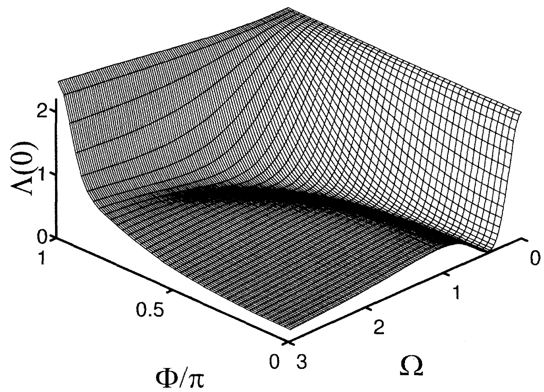


FIG. 7. A three-dimensional plot of the incoherent spectral intensity at line center as a function of the squeezing phase and the Rabi frequency, for $N=1$.

However, for smaller values of $N$ there is a significant disagreement, as we see in Fig. 6 for $N=0.1$. In this regime the three expressions (17)–(19) also disagree among themselves. We have also plotted expression (19) in the figure, and it can be seen that it provides a better estimate than Eq. (17), but both significantly overestimate the value of $\Omega$ which produces the minimum. We have not shown Eq. (18), which in fact has a divergence at $\Phi \simeq 0.69\pi$.

Note that there is a qualitative difference in the behavior of the minimum for large and small values of $N$. For large values of $N$, such as $N=10$, the value of $\Omega$ giving the minimum of $\Lambda_{\mathrm{inc}}(0)$ for a given value of $\Phi$ continues to increase as $\Phi$ increases, as predicted by Eq. (17). For smaller values of $N$, $N \leqslant 1$, the minimum at first moves out as $\Phi$ increases, in qualitative agreement with (17), but then as $\Phi$ continues to increase above a certain value, the minimum moves back towards the origin. This may be seen in Fig. 6.

We should emphasize that, for an arbitrary value of $N$, the value of $\Omega$ which leads to anomalous spectra can be accurately determined from the minimum in $\Lambda_{\mathrm{inc}}(0)$. The analytic expressions give an excellent approximation to this value for $N \gtrsim 2$. Thus the conditions for the anomalous spectra to occur are not in general the same as the conditions for the switch from absorption to amplification of a probe beam, or for the system to collapse into a pure state. However, for sufficiently large values of $N$ all three phenomena occur at the same parameter values.

Since the minimum in $\Lambda_{\mathrm{inc}}(0)$ provides the location of the anomalous spectra, we have in Fig. 7 provided a global view of the behavior of this minimum as a function of $\Omega$ and $\Phi$ for $N=10$, showing the position of the minimum and its increasing shallowness as $\Phi=\pi$ is approached. In Fig. 8 we have introduced a detuning of $\delta=0.3$ which shows that the minimum has become much shallower and has shifted towards larger values of $\Omega$.

We now discuss briefly why the anomalous features tend to arise when the system approaches a pure state. The scattered radiation is produced by the fluctuating dipole moment $\langle\sigma_+(t)\rangle=\langle\sigma_x(t)\rangle+i\langle\sigma_y(t)\rangle$. For simplicity, we consider the case $\Phi=0$. Then the time-dependent solutions of the Bloch equations, given the initial expectation values $\langle\sigma_x(0)\rangle$, $\langle\sigma_y(0)\rangle$, and $\langle\sigma_z(0)\rangle$, are

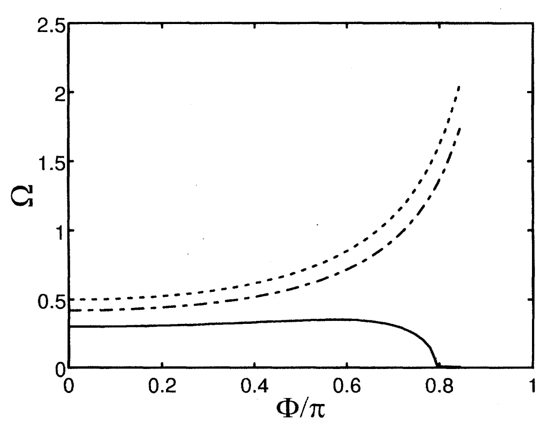


FIG. 6. The position of the minimum in $\Lambda_{\mathrm{inc}}(0)$ evaluated numerically (solid line) and using the approximate expression (19) (dot-dashed line) and (17) (dotted line) as a function of the Rabi frequency $\Omega$ and the squeezing phase $\Phi$, for the case $N=0.1$.

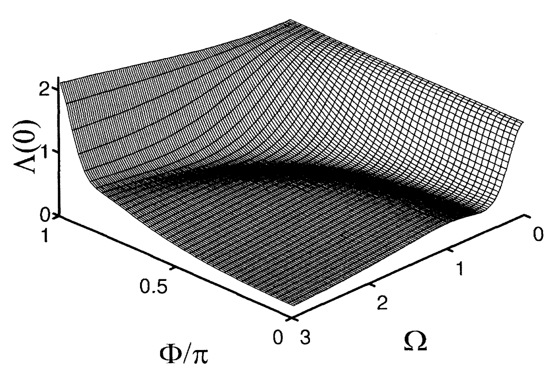


FIG. 8. The same as Fig. 6, but with the nonzero detuning $\delta=0.3$ introduced.

$$\langle\sigma_x(t)\rangle=\langle\sigma_x(0)\rangle\exp(-\gamma_x t), \tag{21}$$

$$\begin{aligned}\langle\sigma_y(t)\rangle=&-\frac{\Omega\gamma}{\gamma_y\gamma_z+\Omega^2}+\exp[-(\gamma_0-\kappa)t]\left[\frac{\Omega\gamma}{2\kappa(\gamma_0-\kappa)}\right.\\&\left.+\frac{\Omega\langle\sigma_z(0)\rangle}{2\kappa}+\frac{\langle\sigma_y(0)\rangle}{2\kappa}\left(\frac{\gamma_x}{2}+\kappa\right)\right]\\&-\exp[-(\gamma_0+\kappa)t]\left[\frac{\Omega\gamma}{2\kappa(\gamma_0+\kappa)}+\frac{\Omega\langle\sigma_z(0)\rangle}{2\kappa}\right.\\&\left.+\frac{\langle\sigma_y(0)\rangle}{2\kappa}\left(\frac{\gamma_x}{2}-\kappa\right)\right],\end{aligned} \tag{22}$$

where

$$\gamma_x=\gamma(N+\tfrac{1}{2}+M), \tag{23}$$

$$\gamma_y=\gamma(N+\tfrac{1}{2}-M), \tag{24}$$

$$\gamma_z=\tfrac{1}{2}(\gamma_x+\gamma_y), \tag{25}$$

$$\gamma_0=\tfrac{1}{2}(\gamma_y+\gamma_z), \tag{26}$$

$$\kappa=(\gamma_x^2/4-\Omega^2)^{1/2}. \tag{27}$$

The incoherent resonance fluorescence spectrum is produced by fluctuations in the dipole moment. The fluctuations are measured by the variances $\langle\Delta\sigma_i\rangle\equiv\langle\sigma_i^2\rangle-\langle\sigma_i\rangle^2$, that is,

$$\langle\Delta\sigma_i\rangle=1-\langle\sigma_i\rangle^2. \tag{28}$$

Now, as suggested by Eq. (21), the variance in $\langle\Delta\sigma_x\rangle$ will give rise to features controlled by $\gamma_x\gg\gamma$. These are broad features, of not much interest. The narrow features, according to Eq. (22), arise from fluctuations in $\sigma_y$ and $\sigma_z$. Since $\langle\sigma_z\rangle$ does not vary greatly from zero over the parameter range of interest, its fluctuations are roughly contrast. Hence the sharp features will be of least consequence when the fluctuations in $\sigma_y$ are a minimum, that is, when $\langle\sigma_y\rangle$ is a maximum. The steady-state value is obtained from Eq. (22) as

$$\langle\sigma_y(\infty)\rangle=\frac{-\Omega\gamma}{\gamma_y\gamma_z+\Omega^2}, \tag{29}$$

which has a maximum when $\Omega=\Omega_0$, where

$$\Omega_0=\sqrt{\gamma_y\gamma_z}. \tag{30}$$

It is easily seen that $\Omega_0\simeq\frac{1}{2}$ when $M$ is maximal and Eq. (15) is employed, with $a=\frac{1}{8}$. We have shown in Eq. (17) that this is the condition for hole burning for $N$ not too small.

## IV. THE SENSITIVITY TO TWO-PHOTON CORRELATIONS

We can also infer the sensitivity of the spectra to the choice of the parameters from Eq. (16). The condition for the radical to be real is

$$a\leq\Gamma\left[1-\left(1-\frac{1}{4\Gamma}\right)^{1/2}\right]\simeq\frac{1}{8}\left(1+\frac{1}{16\Gamma}\right). \tag{31}$$

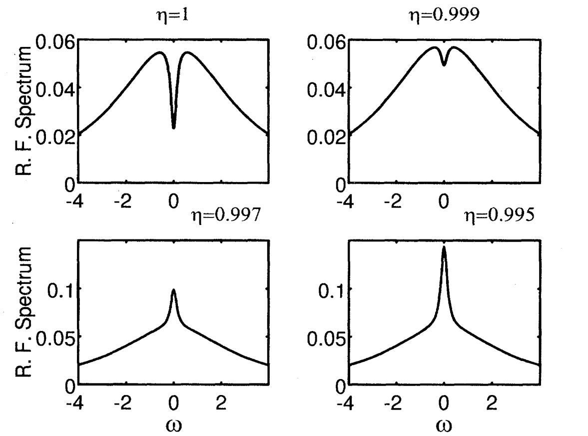


FIG. 9. The incoherent resonance fluorescence spectra for $N=1$ and $\Phi=0$ for different values of the degree of squeezing, $\eta=1$, 0.999, 0.997, and 0.995.

Hence to this level of approximation we can only expect anomalous features for

$$\frac{1}{8}\leq a\leq\frac{1}{8}\left(1+\frac{1}{16\Gamma}\right). \tag{32}$$

Thus there is only a very restricted range of values of $a$ (equivalently $M$) for which the anomalous features are possible, and the available range decreases with $N$. Setting $N=1$ for example, we find that $a$ must lie in the range $0.125\leq a\leq 0.1302$. The value of $M$ corresponding to $a=0.1302$ is $M=1.4132$. Defining the degree of squeezing $\eta$ by

$$\eta=\frac{M}{M_{\max}}, \tag{33}$$

we find $\eta=0.9993$ in this case.

These features are illustrated in Fig. 9, where we show the anomalous spectra for $\Phi=0$, $\delta=0$, and $\Omega=0.424\,26$ with $\eta=1$, 0.999, 0.997, and 0.995, respectively. The hole at line center disappears for a slightly smaller value of $\eta$ than that predicted in the preceding paragraph, but the estimate is nevertheless a useful one. This figure illustrates the vital importance of the two-photon correlations induced by the squeezed vacuum for producing the spectral hole. For $\eta=0.997$ and 0.995 the spectra still have an unusual appearance, caused by the height of the broad peak being of the same order as the height of the narrow peak.

## V. SPECTRAL HOLE BURNING

It is also possible to give an analytic treatment which accounts for the hole burning features of the anomalous spectra for $\Phi=0$. This is presented below.

We consider first the case $\Phi=0$ and exact resonance, $\delta=0$. The widths of the spectral features are determined by the poles of $\Lambda_{\rm inc}(z)$. Again, we set $\gamma=1$ to reduce the complexity of the expressions. For the particularly simple case where $\Phi=0$, the poles occur at

$$z_0 = -(\Gamma + M), \tag{34}$$

$$z_{\pm} = -\frac{(3\Gamma - M)}{2} \pm \left[ \left( \frac{3\Gamma - M}{2} \right)^2 - \Omega^2 \right]^{1/2}. \tag{35}$$

(The poles for the case $\Phi = \pi$ are obtained from the above by reversing the sign of $M$.) For simplicity, we concentrate on the limit where $\Gamma \gg \frac{1}{4}$, where the expansion (14) holds. The approximate positions of the poles are then given by

$$z_0 \simeq -2\Gamma + \frac{1}{8\Gamma}, \tag{36}$$

$$z_+ \simeq -\frac{\Omega^2}{2\Gamma}, \tag{37}$$

$$z_- \simeq -2\Gamma - \frac{1}{8\Gamma}. \tag{38}$$

The incoherent spectrum consists therefore of a sum of three Lorentzian peaks, two with broad peaks with widths given by $z_0$ and $z_-$, and one with a narrow peak given by $z_+$. For $N$ sufficiently large, the broad peaks will form a flat background, and the dominant features will be determined by the narrow peak. We concentrate therefore on calculating the contribution to the incoherent resonance fluorescence spectrum from the pole at $z = z_+$. From Eq. (5), this is given by

$$\mathrm{Re}\left[ R_+ \frac{1}{-i\omega - z_+} \right] = \frac{R_+ z_+}{\omega^2 + z_+^2}, \tag{39}$$

where the residue of $\Lambda_{\text{inc}}(z)$ at the pole $z = z_+$ is given by

$$R_+ = \frac{a_0 b_1(z_+) - b_0 a_1(z_+)}{a_0 (z_+ - z_0)(z_+ - z_-)}$$

and is in fact real in the special cases ($\Phi = \delta = 0$) we are considering.

We evaluate this residue using the approximation (37) for $z_+$. We find

$$R_+ = A/B, \tag{40}$$

where

$$\begin{aligned} A = {} & 16\Gamma^5(4\Omega^2 - 1)^2 + 8\Gamma^4(16\Omega^4 - 1) - 8\Gamma^3\Omega^2(4\Omega^2 - 1)^2 \\ & - 2\Gamma^2\Omega^2(16\Omega^4 - 1) + \Gamma\Omega^2(4\Omega^2 + 1)(2\Omega^4 - 2\Omega^2 + 1) \\ & - \Omega^4(4\Omega^2 + 1), \end{aligned} \tag{41}$$

$$B = 2\Gamma(2\Gamma^2 - \Omega^2)(16\Gamma^2 - 4\Omega^2 - 1)(4\Omega^2 + 1)^2. \tag{42}$$

For fixed $\Gamma$, we regard $R_+$ as a function of $\Omega^2$: $R_+ = R_+(\Omega^2)$. The above expressions are complicated, but we note that a remarkable simplification occurs if we set $\Omega = \frac{1}{2}$. For then the first four terms in the expression for $A$ all vanish, and we have

$$R_+\left(\frac{1}{4}\right) = \frac{4\Gamma - 1}{32(8\Gamma^2 - 1)^2} \simeq \frac{1}{512\Gamma^3} \quad \text{for } \Gamma \gg 1. \tag{43}$$

Note that choosing this particular value for $\Omega$ represents a large decrease in magnitude of the spectrum at line center, as values for $\Omega$ well away from this value result in the leading term in $A$ being of order $\Gamma^5$, not $\Gamma$.

We also note that $R_+$ is positive. However, if we change the value of $\Omega$ by a small amount—to $\Omega^2 = 1/4 - 1/(8\Gamma)$, for example, we find that $R_+$ becomes negative:

$$\begin{aligned} & R_+\left(\frac{1}{4} - \frac{1}{8\Gamma}\right) \\ & = -\frac{256\Gamma^6 - 128\Gamma^5 - 32\Gamma^4 + 48\Gamma^3 - 28\Gamma^2 + 8\Gamma - 1}{2(32\Gamma^3 - 4\Gamma + 1)(16\Gamma^3 - 2\Gamma + 1)(4\Gamma - 1)^2} \end{aligned} \tag{44}$$

$$\simeq -\frac{1}{64\Gamma^2}. \tag{45}$$

If we decrease it by a further small amount, it becomes positive again:

$$R_+\left(\frac{1}{4} - \frac{1}{4\Gamma}\right) = \frac{128\Gamma^5 - 8\Gamma^4 - 10\Gamma^3 + 27\Gamma^2 - 12\Gamma + 3}{8(8\Gamma^3 - \Gamma + 1)(16\Gamma^3 - 2\Gamma + 1)(2\Gamma - 1)^2} \tag{46}$$

$$\simeq \frac{1}{32\Gamma^3}. \tag{47}$$

Thus for a small set of values somewhere in the range

$$\frac{1}{4} > \Omega^2 > \frac{1}{4} - \frac{1}{4\Gamma} \tag{48}$$

the residue $R_+$ becomes negative. Since this residue contributes a narrow linewidth peak, the result of it becoming negative is to produce a sharp hole bored into the broader features of the resonance fluorescence spectrum at line center.

Finally, we turn briefly to the case $\Phi = \pi/2$, which has been discussed analytically in the first of Ref. [12]. The point we wish to emphasize here is that the origin of the anomalous features differs from the $\Phi = 0$ situation. For $\Phi = \pi/2$, even for values of $\Omega$ well away from that which minimizes $\Lambda_{\text{inc}}(0)$, the factors which give rise to the anomalous features are already present—the problem is that they are normally masked by other contributions whose amplitude is much larger. As shown in [12], the essential contribution of the squeezed vacuum is to enable us, by choosing the value of $\Omega$ appropriately, to sufficiently suppress the amplitude of the masking contributions to enable the anomalous features to be revealed. This occurs for $\Omega^2 \simeq \frac{1}{2}$ for $N \gg 1$.

## VI. CONCLUSIONS

We have considered the incoherent part of the resonance fluorescence spectrum of a two-level atom interacting with a squeezed vacuum, and have given analytic expressions for the parameter values which permit the anomalous spectra to arise. For $N \gg 1$, this coincides with the conditions for amplification of a weak probe beam, and for the system to be well approximated by a pure state. For $\Phi = 0$, we have also obtained analytic expressions which explain the hole burning at line center, and the sensitivity of the anomalous features to variations from the optimum values of the parameters. We have shown that as $\Phi \rightarrow \pi$, the anomalous features disappear.

The anomalous features arise when the average fluctuations in the dipole moment are a minimum. This corresponds closely with the condition for the system to be in a pure state. We have shown that the occurrence of the anomalous spectra provides a striking experimental manifestation of the atomic system evolving into a pure state. This system provides a possible method for preparing a two-level atom in a variety of pure states, which are coherent superpositions of the ground and excited states, by suitably choosing the squeezing phase and Rabi frequency.

## ACKNOWLEDGMENTS

We wish to acknowledge helpful conservations with T. A. B. Kennedy and Z. Ficek. This research was supported by a NATO Collaborative Research Grant, and by grants from the United Kingdom EPSRC and the EC. P.Z. thanks Queen's University for financial support.

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