Electromagnetic field quantization in absorbing dielectrics

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The electromagnetic field is quantized in dielectric media that show both loss and dispersion. The complex dielectric function of the medium is assumed to be a known function and the loss is modeled by Langevin forces in the forms of noise current operators. The noise current correlation function is related to the assumed dielectric function by the fluctuation-dissipation theorem. Field quantization is carried out for the infinite homogeneous dielectric, the semi-infinite dielectric, and the dielectric slab, where the fields in the second and third cases are restricted to propagation perpendicular to the dielectric surfaces. The forms of the vector potential operator are obtained in the different spatial regions for all three geometries, and in each case the required canonical commutation relation for the vector potential and its conjugate generalized momentum operator is verified. The spatial dependence of the vacuum field fluctuations is calculated for the two dielectric geometries that have surfaces.

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I. INTRODUCTION

The growth in experiments on quantum optical processes that take place inside material media has stimulated the development of techniques for the quantization of the electromagnetic field in dielectrics. There have been several presentations of the quantization procedure for various special cases of the general problem. The treatments have mainly considered lossless dielectrics in which the dielectric function $\varepsilon(\mathbf{r},\omega)$ is real, and they cover nondispersive inhomogeneous materials with interfaces, in which $\varepsilon(\mathbf{r},\omega)$ is independent of ω [1–3], dispersive homogeneous materials in which $\varepsilon(\mathbf{r},\omega)$ is independent of $\mathbf{r}$ [4,5], and, more generally, dispersive inhomogeneous materials [6,7]. The quantization has been carried out explicitly for simple geometries of nondispersive media, such as semi-infinite samples [8–10] and slabs [11], and quantum-optical effects at dielectric interfaces have been investigated [12]. The general features of quantum electrodynamics in the presence of dielectrics have also been studied, with detailed applications to processes in specific geometries of material media [13], and with a careful derivation of the multipolar form of the Hamiltonian based on the canonical quantization procedure [14]. These derivations assume a known phenomenological form for the dielectric function, but the theory has also been developed for microscopic models in which the oscillator variables of the dielectric are explicitly included, so that the form of the dielectric function is itself determined by the model; the quantization procedure employs the coupled polariton modes of the interacting field-oscillator system, and results have been obtained for both homogeneous [15] and inhomogeneous media [16].

A dielectric that shows dispersion must inevitably be lossy, in accordance with the Kramers-Kronig relations, and it is intrinsically inconsistent to assume the presence of one property without the other. However, the quantization of the electromagnetic field in lossy, or absorptive, dielectrics is considerably more complicated since the loss inevitably couples the field to a reservoir, whose oscillators act as noise sources. A relatively simple representation of the loss mechanism is provided by a distribution of fictitious optical beam splitters in an otherwise lossless dielectric [17]; these remove a portion of any incident field and also add a corresponding portion of the reservoir noise fields. More fundamental microscopic models have also been employed to quantize the field in terms of polariton and reservoir operators [18–20]; the complex dielectric function in these calculations is again determined by the model, and the results obtained so far apply to homogeneous media. They have been used to calculate the vacuum field fluctuation in absorbing dielectrics [21] and so to find the spontaneous emission rate for an excited atom; the expression obtained agrees with that derived from fluctuation-dissipation theory, where formal quantization of the electromagnetic field is unnecessary. The method has also been extended to obtain the field-field correlation function between different space-time points in terms of the complex permittivity of an arbitrary dielectric material [22]. Another approach to the problem of quantization in lossy dielectrics uses Langevin forces to represent the noise, and this method has been applied to calculation of quantum-optical processes in dielectric slabs [23,24] and of Casimir effects in absorbing media [25].

The aim of the present paper is a more comprehensive treatment of electromagnetic field quantization in dispersive and absorbing dielectrics. The cases of an infinite homogeneous medium, a semi-infinite medium with a flat surface, and a plane parallel slab of material are treated for light beams that propagate perpendicular to the surface with plane parallel wave fronts and a single linear polarization direction. The three geometrical arrangements of dielectric are shown schematically in Fig. 1; brief details of the quantization for the first two have been given previously [26]. The method of Langevin forces is used here, but it is shown where appropriate that the results agree with microscopic model calculations, beam splitter models, and Casimir force calculations.

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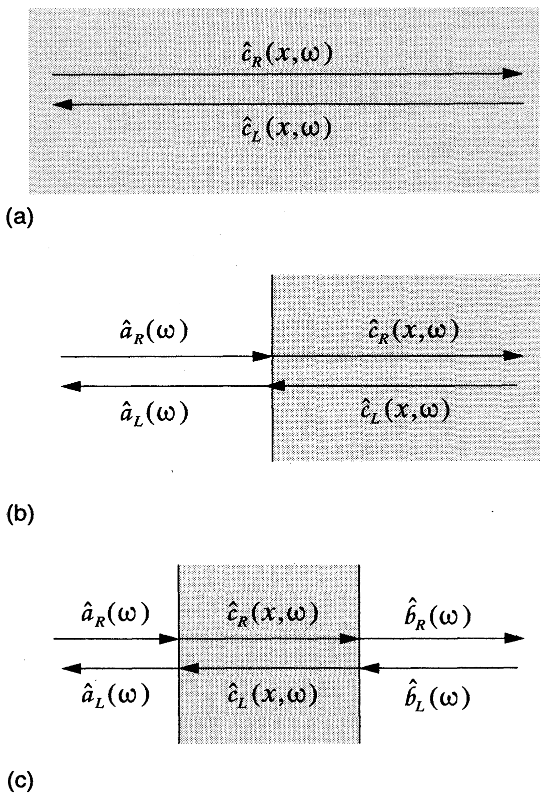

FIG. 1. Representations of the three spatial arrangements of dielectric treated, showing notations for the destruction operators used in the definitions of the vector potential operator, (a) infinite homogeneous dielectric, (b) semi-infinite dielectric, and (c) dielectric slab. Operators denoted by the same symbols are generally given by different expressions in the three geometries.

The Langevin method has the advantages that it is valid for an arbitrary dielectric function and that the avoidance of any specific model for the dielectric material removes the need for complicated diagonalization of material, reservoir, and field variables. The forms of the quantized field expressions in the different spatial regions are determined for each geometrical arrangement of dielectric, and it is demonstrated that the canonical commutation requirements are satisfied. This demonstration provides a rather stringent test of the accuracy of our calculations [9]. The present paper is restricted to the formal quantization of the electromagnetic field and the evaluation of vacuum fluctuation spectra. The resulting operator expressions are easy to use, and they provide all possible information needed to calculate the development of measurable properties of light beams as they propagate through the dielectric samples.

II. PRELIMINARIES

The dielectric function $\varepsilon(x,\omega)$ for the three spatial arrangements of material to be considered is equal to unity in the free-space regions and is equal to $\varepsilon(\omega)$ in regions occupied by the dielectric medium. The complex refractive index $n(\omega)$ is related to the material dielectric function $\varepsilon(\omega)$ in the usual way,

$$\varepsilon(\omega)=[n(\omega)]^2, \tag{2.1}$$

where $n(\omega)$ is related to the real refractive index $\eta(\omega)$ and extinction coefficient $\kappa(\omega)$ by

$$n(\omega)=\eta(\omega)+i\kappa(\omega). \tag{2.2}$$

These functions are defined for positive frequencies, but their forms for negative frequencies are obtained from the crossing relations [27]

$$\varepsilon(-\omega)=\varepsilon^*(\omega), \quad n(-\omega)=n^*(\omega),$$
$$\eta(-\omega)=\eta(\omega), \quad \kappa(-\omega)=-\kappa(\omega). \tag{2.3}$$

The form of $\varepsilon(\omega)$ in the upper half of the complex ω plane is restricted by causality considerations to have neither poles nor zeros. It also conforms to the limit [27]

$$\varepsilon(\omega)\to 1 \quad \text{for } \omega\to\infty \text{ in any manner}. \tag{2.4}$$

The function $\varepsilon(\omega)-1$ has the nature of a generalized susceptibility, and its properties outlined above are shared by all functions of similar nature, otherwise known as linear response functions or Green functions [28].

We consider electromagnetic waves that propagate parallel to the x axis with their transverse electric and magnetic vector operators $\hat{E}(x,t)$ and $\hat{B}(x,t)$ parallel to the y and z axes, respectively. The field operators satisfy Maxwell's equations in the forms

$$\frac{\partial \hat{E}(x,t)}{\partial x}=-\frac{\partial \hat{B}(x,t)}{\partial t} \tag{2.5}$$

and

$$-\frac{\partial \hat{B}(x,t)}{\partial x}=\mu_0\frac{\partial \hat{D}(x,t)}{\partial t}+\mu_0\hat{j}(x,t), \tag{2.6}$$

where $\hat{D}(x,t)$ represents the electric displacement. The transverse current operator $\hat{j}(x,t)$ plays the role of a Langevin force associated with the noise sources. The forms of the Maxwell equations given in (2.5) and (2.6) are identical to those used by Lifshitz in his derivation of the Casimir force between two parallel dielectric slabs (see the very clear discussion in Ref. [29]). An alternative and equivalent formulation [19] expresses the current as a time derivative of a noise reservoir polarization, which appears in the displacement term in (2.6). The field operators are separated into positive and negative frequency components in the usual way:

$$\begin{aligned}\hat{E}(x,t)&=\hat{E}^+(x,t)+\hat{E}^-(x,t),\\ \hat{B}(x,t)&=\hat{B}^+(x,t)+\hat{B}^-(x,t),\end{aligned} \tag{2.7}$$

and Fourier transform operators are defined according to

$$\begin{aligned}\hat{E}^+(x,t)&=\frac{1}{\sqrt{2\pi}}\int_0^\infty d\omega\,\hat{\mathscr{E}}^+(x,\omega)e^{-i\omega t},\\ \hat{B}^+(x,t)&=\frac{1}{\sqrt{2\pi}}\int_0^\infty d\omega\,\hat{\mathscr{B}}^+(x,\omega)e^{-i\omega t},\end{aligned} \tag{2.8}$$

with similar separations and transforms for the displacement and noise current operators. The negative frequency components are given by the Hermitian conjugates of the positive frequency operators.

The Fourier transform displacement operator is defined by

$$\hat{\mathscr{D}}^{+}(x,\omega)=\varepsilon_0\varepsilon(x,\omega)\hat{\mathscr{E}}^{+}(x,\omega), \tag{2.9}$$

and the frequency domain Maxwell equations are obtained from (2.5) and (2.6) as

$$\frac{\partial\hat{\mathscr{E}}(x,\omega)}{\partial x}=i\omega\hat{\mathscr{B}}^{+}(x,\omega) \tag{2.10}$$

and

$$-\frac{\partial\hat{\mathscr{B}}(x,\omega)}{\partial x}=-\frac{i\omega\varepsilon(x,\omega)}{c^2}\hat{\mathscr{E}}^{+}(x,\omega)+\mu_0\hat{\mathscr{J}}^{+}(x,\omega). \tag{2.11}$$

It is convenient to express the field operators in terms of a vector potential operator $\hat{A}(x,t)$, which has decompositions similar to (2.7) and (2.8), using the relations

$$\hat{\mathscr{E}}^{+}(x,\omega)=i\omega\hat{\mathscr{A}}^{+}(x,\omega),$$
$$\hat{\mathscr{B}}^{+}(x,\omega)=\frac{\partial\hat{\mathscr{A}}^{+}(x,\omega)}{\partial x}. \tag{2.12}$$

The first Maxwell equation (2.10) is automatically satisfied when these expressions are substituted, while the second Maxwell equation (2.11) gives

$$-\frac{\partial^2\hat{\mathscr{A}}^{+}(x,\omega)}{\partial x^2}-\frac{\omega^2\varepsilon(x,\omega)}{c^2}\hat{\mathscr{A}}^{+}(x,\omega)=\mu_0\hat{\mathscr{J}}^{+}(x,\omega). \tag{2.13}$$

The vector potential operator is given by standard Green-function methods in the form

$$\hat{\mathscr{A}}^{+}(x,\omega)=S\int_{-\infty}^{\infty}dx'\,\mathscr{G}(x,x',\omega)\hat{\mathscr{J}}^{+}(x',\omega), \tag{2.14}$$

where S is an area of quantization in the y-z plane, perpendicular to the direction of propagation, and the Green function is determined by solution of

$$-\left(\frac{\partial^2}{\partial x^2}+\frac{\omega^2\varepsilon(x,\omega)}{c^2}\right)\mathscr{G}(x,x',\omega)=\frac{\mu_0}{S}\delta(x-x'). \tag{2.15}$$

The Fourier-transform Green function defined by

$$\mathscr{G}(x,k,\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dx'\,\mathscr{G}(x,x',\omega)e^{ikx'} \tag{2.16}$$

is easily found for each spatial region, where $\varepsilon(x,\omega)$ has the value unity or $\varepsilon(\omega)$, independent of x. It has the form

$$\mathscr{G}(x,k,\omega)=\frac{\mu_0}{\sqrt{2\pi}S}\frac{e^{ikx}}{k^2-[\omega^2\varepsilon(x,\omega)/c^2]}, \tag{2.17}$$

and this result is used in the following sections to derive the forms of the vector potential operator for the various geometries of absorbing dielectric sample.

The form of the generalized momentum operator that is conjugate to the vector potential in the quantized field theory is determined by the well-known techniques of Lagrangian mechanics. The Lagrangian densities of systems formed from regions of dielectric that do not fill all of space are the same as that derived for an unbounded dielectric [18], but modified by appropriate restrictions on the spatial extent of the matter that forms the sample. The resulting canonical commutators have the same forms not only in free space and in unbounded dielectric media, but also more generally in the semi-infinite dielectric and the slab considered in the present paper. It is convenient to assume a gauge in which only the transverse part of the vector potential participates in the quantization [30]. This could be the Coulomb gauge, in which the conjugate momentum is $-\varepsilon_0\hat{E}(x,t)$ and the canonical commutation relation is

$$[\hat{A}(x,t),-\varepsilon_0\hat{E}(x',t)]=(i\hbar/S)\delta(x-x'), \tag{2.18}$$

or the multipolar gauge, in which the conjugate momentum is $-\hat{D}(x,t)$ and the canonical commutation relation is

$$[\hat{A}(x,t),-\hat{D}(x',t)]=(i\hbar/S)\delta(x-x'). \tag{2.19}$$

These two commutators are mutually consistent because, in a fully microscopic treatment, $\varepsilon_0\hat{E}$ and $\hat{D}$ differ by a contribution proportional to the variables of the dielectric material, which commute with the vector potential $\hat{A}$.

The field commutation relations are closely related to the commutation properties of the noise current operator, or Langevin force, in (2.13). For noise that is uncorrelated at different positions and different frequencies, the latter commutation relation has the form

$$[\hat{\mathscr{J}}^{+}(x,\omega),\hat{\mathscr{J}}^{-}(x',\omega')]=\alpha(\omega)\delta(x-x')\delta(\omega-\omega'), \tag{2.20}$$

where the real function $\alpha(\omega)$ is determined by the condition that (2.20) should be consistent with (2.18) and (2.19). The noise current operators are also assumed satisfy

$$[\hat{\mathscr{J}}^{+}(x,\omega),\hat{\mathscr{J}}^{+}(x',\omega')]=[\hat{\mathscr{J}}^{-}(x,\omega),\hat{\mathscr{J}}^{-}(x',\omega')]=0. \tag{2.21}$$

III. INFINITE HOMOGENEOUS DIELECTRIC

A. Field quantization

We consider in this section the field quantization in an absorbing dielectric medium that fills all of space. The dielectric function is accordingly

$$\varepsilon(x,\omega)=\varepsilon(\omega)=[n(\omega)]^2 \quad \text{for all } x. \tag{3.1}$$

The coordinate-space Green function is obtained by contour integration of (2.17) as

$$\mathscr{G}(x,x',\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dk\,\mathscr{G}(x,k,\omega)e^{-ikx'}$$
$$=\frac{i}{2\varepsilon_0c\omega n(\omega)S}\exp\left(\frac{i\omega n(\omega)|x-x'|}{c}\right) \tag{3.2}$$

and a particular integral solution of (2.13) for the vector potential operator is obtained with the use of (2.14) as

$$\hat{\mathscr{A}}^{+}(x,\omega)=\frac{i}{2\varepsilon_0 c\omega n(\omega)}\int_{-\infty}^{\infty}dx'\,\exp\left(\frac{i\omega n(\omega)|x-x'|}{c}\right)\times\hat{\mathscr{J}}^{+}(x',\omega). \tag{3.3}$$

It is convenient to separate the vector potential operator into rightwards and leftwards traveling contributions as

$$\hat{\mathscr{A}}^{+}(x,\omega)=\hat{\mathscr{A}}_R^{+}(x,\omega)+\hat{\mathscr{A}}_L^{+}(x,\omega), \tag{3.4}$$

where

$$\hat{\mathscr{A}}_R^{+}(x,\omega)=\frac{i}{2\varepsilon_0 c\omega n(\omega)}\int_{-\infty}^{x}dx'\,\exp\left(\frac{i\omega n(\omega)(x-x')}{c}\right)\times\hat{\mathscr{J}}^{+}(x',\omega) \tag{3.5}$$

and

$$\hat{\mathscr{A}}_L^{+}(x,\omega)=\frac{i}{2\varepsilon_0 c\omega n(\omega)}\int_{x}^{\infty}dx'\,\exp\left(-\frac{i\omega n(\omega)(x-x')}{c}\right)\times\hat{\mathscr{J}}^{+}(x',\omega). \tag{3.6}$$

These rightwards and leftwards vector potential operators have the commutation properties

$$[\hat{\mathscr{A}}_R^{+}(x,\omega),\hat{\mathscr{A}}_R^{-}(x,\omega')]=[\hat{\mathscr{A}}_L^{+}(x,\omega),\hat{\mathscr{A}}_L^{-}(x,\omega')]=\frac{\alpha(\omega)\delta(\omega-\omega')}{8\varepsilon_0^2 c\omega^3|n(\omega)|^2\kappa(\omega)} \tag{3.7}$$

and

$$[\hat{\mathscr{A}}_R^{+}(x,\omega),\hat{\mathscr{A}}_L^{-}(x,\omega')]=[\hat{\mathscr{A}}_L^{+}(x,\omega),\hat{\mathscr{A}}_R^{-}(x,\omega')]=0, \tag{3.8}$$

where (2.20) has been used.

It is advantageous for the calculations that follow, despite the accompanying proliferation of symbols, to introduce boson-type operators as replacements for the operators defined above. We thus define a modified Langevin force operator by

$$\hat{f}(x,\omega)=\hat{\mathscr{J}}^{+}(x,\omega)/\sqrt{\alpha(\omega)}, \tag{3.9}$$

whose commutation relation from (2.20) has the boson form

$$[\hat{f}(x,\omega),\hat{f}^{\dagger}(x',\omega')]=\delta(x-x')\delta(\omega-\omega'). \tag{3.10}$$

This local commutation property of the force operators also holds for the bounded media considered in Secs. IV and V. New operators for the vector potential are introduced by

$$\hat{\mathscr{A}}_I^{+}(x,\omega)=\left(\frac{\alpha(\omega)}{8\varepsilon_0^2 c\omega^3[n(\omega)]^2\kappa(\omega)}\right)^{1/2}\hat{a}_I(x,\omega),\quad I=R,L, \tag{3.11}$$

where the $\hat{a}(x,\omega)$ operators also have boson-type commutation relations when they are evaluated at the same positions,

$$[\hat{a}_R(x,\omega),\hat{a}_R^{\dagger}(x,\omega')]=[\hat{a}_L(x,\omega),\hat{a}_L^{\dagger}(x,\omega')]=\delta(\omega-\omega'), \tag{3.12}$$

$$[\hat{a}_R(x,\omega),\hat{a}_L^{\dagger}(x,\omega')]=[\hat{a}_L(x,\omega),\hat{a}_R^{\dagger}(x,\omega')]=0.$$

The notation for these rightwards and leftwards operators is illustrated in Fig. 1(a). They reproduce the vector potential commutation relations (3.7) and (3.8), and they convert (3.5) and (3.6) to

$$\hat{a}_R(x,\omega)=i\left(\frac{2\omega\kappa(\omega)}{c}\right)^{1/2}\int_{-\infty}^{x}dx'\,\exp\left(\frac{i\omega n(\omega)(x-x')}{c}\right)\times\hat{f}(x',\omega) \tag{3.13}$$

and

$$\hat{a}_L(x,\omega)=i\left(\frac{2\omega\kappa(\omega)}{c}\right)^{1/2}\int_{x}^{\infty}dx'\,\exp\left(-\frac{i\omega n(\omega)(x-x')}{c}\right)\times\hat{f}(x',\omega). \tag{3.14}$$

It will be seen that these operators provide very convenient representations of the electromagnetic field variables.

B. Canonical commutation relation

In order to evaluate the canonical commutation relation (2.18), with the electric field operator obtained from (2.12), we need the commutation relations of the new operators $\hat{a}(x,\omega)$ at different positions. These are readily obtained from (3.10), (3.13), and (3.14), and the results are

$$[\hat{a}_R(x,\omega),\hat{a}_R^{\dagger}(x',\omega')]=[\hat{a}_L(x',\omega'),\hat{a}_L^{\dagger}(x,\omega)]=\delta(\omega-\omega')\exp\{[i\omega\eta(\omega)(x-x')-\omega\kappa(\omega)|x-x'|]/c\} \tag{3.15}$$

and

$$[\hat{a}_R(x,\omega),\hat{a}_L^{\dagger}(x',\omega')]=[\hat{a}_L(x',\omega'),\hat{a}_R^{\dagger}(x,\omega)]=\delta(\omega-\omega')\theta(x-x')\frac{2\kappa(\omega)}{\eta(\omega)}\sin\left(\frac{\omega\eta(\omega)(x-x')}{c}\right)\times\exp\left(-\frac{\omega\kappa(\omega)(x-x')}{c}\right), \tag{3.16}$$

where $\theta(x-x')$ is the unit step function. It is seen that, apart from a phase factor in (3.15), the $\hat{a}_R$ and $\hat{a}_L$ operators obey independent boson commutation relations in the absence of dielectric loss, when $\kappa(\omega)=0$. However, the presence of loss produces a decay of the like-operator commutators (3.15) with spatial separation and a coupling of the leftwards and rightwards operators in the commutator (3.16). The coupling is caused by the interaction of these operators with common noise sources associated with the absorption. A step function occurs in (3.16) because the two kinds of waves only sample the same noise sources when they are evaluated at positions such that they have passed through a common region of the dielectric. The physical requirements on the quantization procedure are, however, expressed not by the commutation properties of these subsidiary operators but by the canonical commutation relation (2.18), and with the use of (3.15) and (3.16) this can be put in the form

$$[\hat{A}(x,t),-\varepsilon_0\hat{E}(x',t)] = i\int_0^\infty d\omega \frac{\alpha(\omega)}{8\pi\varepsilon_0 c\omega^2\eta(\omega)\kappa(\omega)} \times\left\{\frac{\exp[i\omega n(\omega)|x-x'|/c]}{n(\omega)} + \frac{\exp[-i\omega n^*(\omega)|x-x'|/c]}{n^*(\omega)}\right\}. \quad (3.17)$$

The nature of the function $\alpha(\omega)$, which was introduced in the commutation relation (2.20), is determined by the condition that the right-hand sides of (2.18) and (3.17) should be identical. Consider the choice

$$\alpha(\omega)=4\varepsilon_0\hbar\omega^2\eta(\omega)\kappa(\omega)/S. \quad (3.18)$$

Then with use of the crossing relations (2.3), (3.17) becomes

$$[\hat{A}(x,t),-\varepsilon_0\hat{E}(x',t)] = \frac{i\hbar}{2\pi cS}\int_{-\infty}^{\infty} d\omega \frac{\exp[i\omega n(\omega)|x-x'|/c]}{n(\omega)}, \quad (3.19)$$

and with an application of the standard rules for contour integration, this can be written

$$[\hat{A}(x,t),-\varepsilon_0\hat{E}(x',t)] = \frac{\hbar}{2\pi^2 S}\int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} dk\, \omega \frac{\exp[ik(x-x')]}{k^2c^2-\omega^2[n(\omega)]^2}, \quad (3.20)$$

where there are simple poles in the upper and lower halves of the complex k plane at $k=\pm\omega n(\omega)/c$. It is now possible to perform the integration over ω; the ω integrand is itself a linear response function, closely related to the Green function in (2.17), with no poles in the upper half of the complex ω plane. The integral along the real ω axis in (3.20) is therefore the negative of the integral around the semicircle at infinity in the upper half plane, so putting $\omega=\rho\exp(i\vartheta)$, we find [19]

$$\begin{aligned}[\hat{A}(x,t),-\varepsilon_0\hat{E}(x',t)] &= -\frac{i\hbar}{2\pi^2 S}\lim_{\rho\to\infty}\int_0^\pi d\vartheta \int_{-\infty}^{\infty} dk \rho^2 e^{2i\vartheta} \times \frac{\exp[ik(x-x')]}{k^2c^2-\rho^2e^{2i\vartheta}[n(\rho e^{i\vartheta})]^2} \\ &= \frac{i\hbar}{2\pi S}\int_{-\infty}^{\infty} dk \exp[ik(x-x')] \\ &= \frac{i\hbar}{S}\delta(x-x'), \end{aligned} \quad (3.21)$$

where the property of the complex refractive index obtained from (2.1) and (2.4), and a standard representation of the delta function have been used. The required form (2.18) of the canonical commutator is thus obtained from the quantization procedure outlined above, and the choice of $\alpha(\omega)$ made in (3.18) is justified. The alternative form (2.19) of canonical commutation relation is also satisfied, as it has the same integral representation as (3.20) but with an additional factor of $\varepsilon(\omega)$ in the integrand; the end result is exactly the same as in (3.21) because $\varepsilon(\omega)$ has no poles in the upper half plane and also satisfies (2.4).

The final form of the vector potential operator in a homogeneous medium in Fourier space, obtained from (3.11) and (3.18), is

$$\mathscr{A}_I^+(x,\omega) = \left(\frac{\hbar\eta(\omega)}{2\varepsilon_0 c\omega[n(\omega)]^2 S}\right)^{1/2} \hat{a}_I(x,\omega), \quad I=R,L, \quad (3.22)$$

and the time-dependent form obtained with the use of (2.8) and (3.4) is

$$\hat{A}^+(x,t) = \int_0^\infty d\omega\left(\frac{\hbar\eta(\omega)}{4\pi\varepsilon_0 c\omega[n(\omega)]^2 S}\right)^{1/2} \times[\hat{a}_R(x,\omega)+\hat{a}_L(x,\omega)]e^{-i\omega t}. \quad (3.23)$$

Corresponding expressions for the electric and magnetic field operators are derived with the use of (2.12). The latter requires the results

$$\frac{\partial\hat{a}_R(x,\omega)}{\partial x} = i\frac{\omega n(\omega)}{c}\hat{a}_R(x,\omega) + i\left(\frac{2\omega\kappa(\omega)}{c}\right)^{1/2}\hat{f}(x,\omega) \quad (3.24)$$

and

$$\frac{\partial\hat{a}_L(x,\omega)}{\partial x} = -i\frac{\omega n(\omega)}{c}\hat{a}_L(x,\omega) - i\left(\frac{2\omega\kappa(\omega)}{c}\right)^{1/2}\hat{f}(x,\omega), \quad (3.25)$$

which are readily obtained from (3.13) and (3.14). The field operators are thus

$$\hat{E}^+(x,t) = i\int_0^\infty d\omega\left(\frac{\hbar\omega\eta(\omega)}{4\pi\varepsilon_0 c[n(\omega)]^2 S}\right)^{1/2} \times[\hat{a}_R(x,\omega)+\hat{a}_L(x,\omega)]e^{-i\omega t} \quad (3.26)$$

and

$$\hat{B}^+(x,t) = i\int_0^\infty d\omega\left(\frac{\hbar\omega\eta(\omega)}{4\pi\varepsilon_0 c^3 S}\right)^{1/2}[\hat{a}_R(x,\omega)-\hat{a}_L(x,\omega)]e^{-i\omega t}. \quad (3.27)$$

It would appear from the derivation of the expression (3.23) for the vector potential operator, from the particular integral (3.3) onwards, that the complete solution should also include a complementary function contribution. However, the destruction operators (3.13) and (3.14) that occur in (3.23) can be written

$$\hat{c}_R(x,\omega)=\hat{c}_R(x_0,\omega)\exp\left(\frac{i\omega n(\omega)(x-x_0)}{c}\right)+i\left(\frac{2\omega\kappa(\omega)}{c}\right)^{1/2}\int_{x_0}^{x}dx'\ \exp\left(\frac{i\omega n(\omega)(x-x')}{c}\right)\times\hat{f}(x',\omega)\quad(x>x_0) \tag{3.28}$$

and

$$\hat{c}_L(x,\omega)=\hat{c}_L(x_0,\omega)\exp\left(-\frac{i\omega n(\omega)(x-x_0)}{c}\right)+i\left(\frac{2\omega\kappa(\omega)}{c}\right)^{1/2}\int_{x}^{x_0}dx'\exp\left(-\frac{i\omega n(\omega)(x-x')}{c}\right)\times\hat{f}(x',\omega)\quad(x_0>x), \tag{3.29}$$

which have the forms of complementary function plus particular integral. We note that in an infinite sample of absorbing dielectric material, unlike the semi-infinite and slab samples, there are no contributions to the solution from waves incident from outside the dielectric. The forms derived above are therefore complete and the degrees of freedom are specified by the continuum of noise currents. The Hilbert space consists of the electromagnetic vacuum state, denoted $|0\rangle$, and the excited states produced by application of combinations of the $\hat{f}^{\dagger}(x,\omega)$.

C. Vacuum field fluctuations

A straightforward application of the quantized field operator is provided by the derivation of the electric field fluctuation in the vacuum state of the electromagnetic field. Consider first the field-field correlation function in Fourier space, obtained from (2.12) as

$$\langle 0|\hat{\mathscr{E}}(x,\omega)\hat{\mathscr{E}}(x',\omega')|0\rangle=\omega\omega'\langle 0|\hat{\mathscr{A}}^{+}(x,\omega)\hat{\mathscr{A}}^{-}(x',\omega')|0\rangle \tag{3.30}$$

when account is taken of the vanishing of terms in which destruction operators act directly on the vacuum state $|0\rangle$. Thus with the use of the form of the vector potential operator from (3.22) and the commutation relations (3.15) and (3.16), the field-field correlation function becomes

$$\langle 0|\hat{\mathscr{E}}(x,\omega)\hat{\mathscr{E}}(x',\omega')|0\rangle=\frac{\hbar\omega}{2\varepsilon_0 cS}\left\{\frac{\exp[i\omega n(\omega)|x-x'|/c]}{n(\omega)}+\frac{\exp[-i\omega n^{*}(\omega)|x-x'|/c]}{n^{*}(\omega)}\right\}\delta(\omega-\omega'). \tag{3.31}$$

The correlation function can also be expressed in terms of the coordinate space Green function (3.2) as

$$\langle 0|\hat{\mathscr{E}}(x,\omega)\hat{\mathscr{E}}(x',\omega')|0\rangle=2\hbar\omega^2\ \mathrm{Im}[\mathscr{G}(x,x',\omega)]\delta(\omega-\omega'). \tag{3.32}$$

The value of the correlation function at a common spatial position determines the power spectrum $\mathscr{S}(x,\omega)$ of the field fluctuations according to

$$\langle 0|\hat{\mathscr{E}}(x,\omega)\hat{\mathscr{E}}(x,\omega')|0\rangle=\mathscr{S}(x,\omega)\delta(\omega-\omega'). \tag{3.33}$$

It follows from (3.31) and (3.32) that the power spectrum is given by

$$\mathscr{S}(x,\omega)=\frac{\hbar\omega}{\varepsilon_0 cS}\frac{\eta(\omega)}{|n(\omega)|^2}=2\hbar\omega^2\ \mathrm{Im}\mathscr{G}(x,x,\omega). \tag{3.34}$$

This expression in terms of the imaginary part of the Green function agrees with the standard form of the fluctuation-dissipation theorem [28] (a difference of 2π from the theorem given in this reference is the result of a different convention for Fourier transforms). The same expression for the field-field correlation function can therefore be obtained either from the quantum-mechanical second-quantization procedure developed here or from the methods of statistical mechanics in which the required Green function is determined classically.

The expression obtained from (3.33) and (3.34) can be compared with the transverse field-field correlation function in a three-dimensional absorbing dielectric [21],

$$\langle 0|\hat{\mathscr{E}}^{T}(\mathbf{r},\omega)\hat{\mathscr{E}}^{T}(\mathbf{r},\omega')|0\rangle=\frac{\hbar\omega^3}{3\pi\varepsilon_0 c^3}\eta(\omega)\delta(\omega-\omega'), \tag{3.35}$$

which is independent of the extinction coefficient $\kappa(\omega)$, in contrast to the one-dimensional correlation function. The three-dimensional field-field correlation function (3.35) determines the rate of spontaneous emission by an excited atom embedded in the absorbing dielectric. The one-dimensional field-field correlation function obtained from (3.33) and (3.34) similarly determines the rate of spontaneous emission by an excited system whose radiation is restricted to electromagnetic field modes that propagate in only a single dimension, for example, an infinite two-dimensional dipole sheet that is coherently excited over its entire area.

The noise-current correlation function is also readily calculated with the use of (3.9), (3.10), and (3.18) in the form

$$\langle 0|\hat{\mathscr{J}}(x,\omega)\hat{\mathscr{J}}(x',\omega')|0\rangle=(2\varepsilon_0\hbar/S)\omega^2\ \mathrm{Im}[\varepsilon(\omega)]\times\delta(x-x')\delta(\omega-\omega'). \tag{3.36}$$

This is another example of a correlation function that can be obtained independently of the quantization procedure by an application of the fluctuation-dissipation theorem [29].

D. Comparisons with previous work

The expression (3.23) for the vector potential operator is precisely the same as the form obtained in Eq. (5.21) of Ref. [19], except for the replacement of $n(\omega)$ by $|n(\omega)|$ in the square-root factor in the integrand of (3.23), which corresponds to an unimportant change in the phase of the destruction operators defined in (3.11). However, there are important differences in the formalisms that lead to the two expressions. The microscopic model of [19] has the advantages that it derives the explicit diagonalization of the coupled electromagnetic field, dielectric oscillator, and reservoir to construct the overall polariton modes of the system, and the form of the dielectric function is derived in terms of the parameters of the model. Much of the derivation in [19] considers a dielectric with a single resonance, but the expressions obtained for the quantized fields are not so restricted, as discussed at the end of Sec. IV in this reference. The microscopic canonical quantization theory thus provides a general self-consistent theory of the dielectric properties and the electromagnetic field operators. On the other hand, the Langevin force calculation presented above has the advantage that it is simpler, in that the variables of the dielectric medium need not be considered explicitly, and it is also general, in that the dielectric function can have any form consistent with causality requirements. However, the dielectric function for a particular application is not provided by the theory, and its form must be obtained from independent calculations or experimental results.

The Langevin force approach has also been used in the calculation of Casimir forces between a pair of absorbing dielectric plates [25]. This work applies to a dielectric with a single resonance and the field is quantized only in the regions of free space around and between the plates. Nevertheless, the Langevin force operator within the dielectric is needed to derive these field operators, and its form can be compared with that derived here from (3.9) and (3.18). The two expressions are found to be in exact agreement after allowances are made for different units and normalization conventions.

The simplest approach of all to the field quantization represents the absorption in the dielectric as being caused by a distribution of beam splitters [17]. Each beam splitter removes a small fraction of the intensity of a propagating light beam and simultaneously couples a small fraction of vacuum field into the beam, in such a way that the input and output mode operators at the beam splitter both satisfy boson commutation relations. A continuously absorbing material is represented by the limit of a dense distribution of feeble beam splitters. The beam splitter model was applied to propagation in the positive direction and the relation between mode operators at two different points is identical to that given in (3.28). The relation between mode operators and the electromagnetic field operator, analogous to (3.26) was not obtained, but the formalism is adequate for calculation of such quantities as the effects of propagation on direct detection noise and on the initial optical squeezing of a light beam [17].

IV. SEMI-INFINITE DIELECTRIC

A. Field quantization

We consider in this section the field quantization in a space that is half occupied by vacuum and half by an absorbing dielectric medium. Brief details of the calculations have been reported previously [26]. The dielectric function is now

$$\varepsilon(x,\omega)=\begin{cases}\varepsilon(\omega)=[n(\omega)]^2 & \text{for } x>0,\\ 1 & \text{for } x<0.\end{cases} \tag{4.1}$$

The vector potential operator is again determined by (2.13) and (2.14), with a Green function obtained by solution of (2.15). The particular integral part of the Fourier-transform Green function is given by (2.17) but there are now also complementary function parts, whose coefficients are determined by the usual electromagnetic boundary conditions at the interface. A straightforward calculation leads to the result

$$\mathscr{G}(x,k,\omega)=\begin{cases}-\dfrac{\mu_0}{\sqrt{2\pi}S}\dfrac{c/\omega}{n(\omega)+1}\dfrac{e^{-i\omega x/c}}{k+[\omega n(\omega)/c]}, & x<0,\\ -\dfrac{\mu_0}{\sqrt{2\pi}S}\left\{\dfrac{(kc/\omega)+1}{n(\omega)+1}\dfrac{e^{i\omega n(\omega)x/c}}{k^2-[\omega n(\omega)/c]^2}-\dfrac{e^{ikx}}{k^2-[\omega n(\omega)/c]^2}\right\}, & x>0,\end{cases} \tag{4.2}$$

and the corresponding coordinate-space Green function, with x' assumed positive, is

$$\mathscr{G}(x,x',\omega)=\begin{cases}\dfrac{i}{2\varepsilon_0 c\omega S}T(\omega)\exp\left\{\dfrac{i\omega[n(\omega)x'-x]}{c}\right\}, & x<0,\ x'>0,\\ \dfrac{i}{2\varepsilon_0 c\omega n(\omega)S}\left\{-R(\omega)\exp\left(\dfrac{i\omega n(\omega)(x+x')}{c}\right)+\exp\left(\dfrac{i\omega n(\omega)|x-x'|}{c}\right)\right\}, & x>0,\ x'>0,\end{cases} \tag{4.3}$$

where

$$T(\omega)=\frac{2}{n(\omega)+1},\quad R(\omega)=-\frac{n(\omega)-1}{n(\omega)+1} \tag{4.4}$$

are the usual amplitude transmission and reflection coefficients, respectively, for light incident on the interface from the vacuum. The corresponding coefficients for light incident from the dielectric are, respectively, $n(\omega)T(\omega)$ and $-R(\omega)$. The two kinds of terms in the large brackets of the Green function (4.3) for $x>0$ are typical of interface systems. Thus the first term corresponds to communication between the points x and x' via reflection in the interface, while the second term corresponds to direct communication between the two points. The first term tends to zero in the limit $\omega\kappa(\omega)x/c\gg 1$, while the second term reproduces the form (3.2) for an infinite homogeneous dielectric. The Green functions given in (4.3) determine the electromagnetic field quantization, but in order to calculate the vacuum field fluctuation spectrum, we also need the corresponding expression in the free space to the left of the interface,

$$\mathscr{G}(x,x',\omega)=\frac{i}{2\varepsilon_0 c\omega S}\left\{R(\omega)\exp\left(-\frac{i\omega(x+x')}{c}\right)+\exp\left(\frac{i\omega|x-x'|}{c}\right)\right\},\quad x<0,\ x'<0. \tag{4.5}$$

The expressions (4.3) and (4.5) agree with more general Green functions derived for interfaces in three dimensions [31].

A particular integral solution for the vector potential operator is now obtained by substitution of the Green function into the integrand of (2.14). However, in contrast to the infinite homogeneous dielectric treated in Sec. III, the complete form of the vector potential operator for the semi-infinite dielectric includes a complementary-function part that corresponds to a free field incident from the vacuum at negative x towards the interface at $x=0$. It is convenient to write the vector potential operator in the vacuum as

$$\hat{A}^+(x,t)=\int_0^\infty d\omega\left(\frac{\hbar}{4\pi\varepsilon_0 c\omega S}\right)^{1/2}\times[\hat{a}_R(\omega)e^{i\omega x/c}+\hat{a}_L(\omega)e^{-i\omega x/c}]e^{-i\omega t},\quad x<0, \tag{4.6}$$

where the rightwards-propagating incoming field, corresponding to the contribution with destruction operator $\hat{a}_R(\omega)$ with commutator

$$[\hat{a}_R(\omega),\hat{a}_R^\dagger(\omega')]=\delta(\omega-\omega'), \tag{4.7}$$

has its well-known free-space form [5]. The leftwards propagating field, corresponding to the contribution with destruction operator $\hat{a}_L(\omega)$, has both a complementary function and a particular integral part. The complete form of this operator, determined by the usual electromagnetic boundary conditions and by the Green function (4.3), is

$$\hat{a}_L(\omega)=R(\omega)\hat{a}_R(\omega)+iT(\omega)\left(\frac{2\omega\eta(\omega)\kappa(\omega)}{c}\right)^{1/2}\times\int_0^\infty dx'\exp\left(\frac{i\omega n(\omega)x'}{c}\right)\hat{f}(x',\omega). \tag{4.8}$$

It is straightforward to show that this leftwards operator has the simple commutation relation

$$[\hat{a}_L(\omega),\hat{a}_L^\dagger(\omega')]=\delta(\omega-\omega'). \tag{4.9}$$

The notation for incoming and outgoing field operators is illustrated in Fig. 1(b). Expressions for the electric and magnetic field operators for $x<0$ are very easily obtained from (4.6) with the use of (2.12). The expression for the vector potential operator in the vacuum half-space obtained from (4.6) and (4.8) agrees with that used in calculations of the Casimir force between a pair of absorbing dielectric plates [25] when the plates are coalesced, shifted, and thickened to form a semi-infinite slab, and account is taken of different units and normalization conventions.

It is convenient to retain the form (3.23) for the vector potential operator inside the dielectric. The contribution in $\hat{c}_L(x,\omega)$, which represents a leftwards-traveling wave in the dielectric, is unaffected by the presence of the interface at $x=0$, and its form is still given by (3.14). The contribution in $\hat{c}_R(x,\omega)$, which represents a rightwards-traveling wave in the dielectric, has a complementary-function part determined by the electromagnetic boundary conditions and a particular-integral part determined by the Green function (4.3); the complete expression for the operator is

$$\hat{c}_R(x,\omega)=\left\{\frac{n(\omega)}{[\eta(\omega)]^{1/2}}T(\omega)\hat{a}_R(\omega)+i\left(\frac{2\omega\kappa(\omega)}{c}\right)^{1/2}\times\left[-R(\omega)\int_0^\infty dx'\exp\left(\frac{i\omega n(\omega)x'}{c}\right)\hat{f}(x',\omega)+\int_0^x dx'\exp\left(-\frac{i\omega n(\omega)x'}{c}\right)\hat{f}(x',\omega)\right]\right\}\times\exp\left(\frac{i\omega n(\omega)x}{c}\right). \tag{4.10}$$

This expression reduces to the form (3.13) obtained for the infinite homogeneous dielectric when $\omega\kappa(\omega)x/c\gg 1$, but in general there are additional terms that result from the presence of the interface. It is readily verified that the operator $\hat{c}_R(x,\omega)$ defined by (4.10) continues to satisfy the relation (3.24), and the expressions (3.26) and (3.27) for the electric and magnetic field operators remain valid for $x>0$ inside the dielectric. The degrees of freedom for the semi-infinite dielectric include the incoming field described by $\hat{a}_R(\omega)$ in addition to the continuum of noise currents, and the Hilbert space is correspondingly augmented by the excitations of this field.

The forms of the Green function for the semi-infinite dielectric given in (4.2) and (4.3) are useful in themselves for calculations of the electromagnetic vacuum-field fluctuations, but if these are not required, the vector potential operators can be calculated more directly by a straightforward application of the boundary conditions [26]. The problem is essentially that of finding the relation between input and output operators at the interface [32]. Thus with the form of $\hat{A}^+(x,t)$ given in (4.6) for $x<0$ and the electric and magnetic field operators given in (3.26) and (3.27) for $x>0$, the boundary conditions at $x=0$ give

$$\hat{a}_R(\omega)+\hat{a}_L(\omega)=\frac{[\eta(\omega)]^{1/2}}{n(\omega)}[\hat{c}_R(0,\omega)+\hat{c}_L(0,\omega)] \tag{4.11}$$

and

$$\hat{a}_R(\omega)-\hat{a}_L(\omega)=[\eta(\omega)]^{1/2}[\hat{c}_R(0,\omega)-\hat{c}_L(0,\omega)], \tag{4.12}$$

where a minor change from Eq. (7) in [26] results from a different choice of phase in the definition of the destruction operator (see Sec. III D). These equations can be solved for the output operators $\hat{a}_L(\omega)$ and $\hat{c}_R(0,\omega)$ in terms of the input operators $\hat{a}_R(\omega)$ and $\hat{c}_L(0,\omega)$, regarded as known. The general forms (4.8) and (4.10) of the unknown operators are then easily rederived with the use of (3.14) and (3.28). Note that the input operator $\hat{c}_L(x,\omega)$, which is expressed entirely in terms of the noise operators for the absorbing dielectric according to (3.14), replaces a free wave incident from $+\infty$ in the case of a nonabsorbing dielectric [10].

B. Canonical commutation relation

The evaluation of the canonical commutation relation (2.18) is more complicated for the semi-infinite dielectric as the field operators themselves have more complex structures and there are two spatial regions to consider. Thus in the vacuum, the basic operator commutation relations are given by (4.7) and (4.9), with a cross commutator obtained by the use of (4.8) as

$$[\hat{a}_L(\omega),\hat{a}_R^\dagger(\omega')]=\delta(\omega-\omega')R(\omega). \tag{4.13}$$

The commutation relations of the operators in the dielectric are obtained from (3.10), (3.14), (4.7), and (4.10) as

$$\begin{aligned}[\hat{c}_R(x,\omega),\hat{c}_R^\dagger(x',\omega')] &= \delta(\omega-\omega')\left\{ \exp\left(\frac{i\omega\eta(\omega)(x-x')-\omega\kappa(\omega)|x-x'|}{c}\right)\right. \\ &\quad + i\frac{\kappa(\omega)}{\eta(\omega)}\left[R(\omega)\exp\left(\frac{i\omega n(\omega)(x+x')}{c}\right)\right. \\ &\quad \left.\left. -R^*(\omega)\exp\left(-\frac{i\omega n^*(\omega)(x+x')}{c}\right)\right]\right\},\end{aligned} \tag{4.14}$$

the corresponding relation for $\hat{c}_L(x,\omega)$ is the same as in (3.15), and

$$\begin{aligned}[\hat{c}_R(x,\omega),\hat{c}_L^\dagger(x',\omega')]=[\hat{c}_L(x',\omega'),\hat{c}_R^\dagger(x,\omega)]^* &= \delta(\omega-\omega')\left\{-R(\omega)\exp\left(\frac{i\omega n(\omega)(x+x')}{c}\right)\right. \\ &\quad \left. +\theta(x-x')\frac{2\kappa(\omega)}{\eta(\omega)}\sin\left(\frac{\omega\eta(\omega)(x-x')}{c}\right)\exp\left(-\frac{\omega\kappa(\omega)(x-x')}{c}\right)\right\}.\end{aligned} \tag{4.15}$$

These commutation relations reduce to the forms (3.15) and (3.16) appropriate to an infinite dielectric medium for points far from the interface where $\omega\kappa(\omega)x/c \gg 1$. The operators for the incoming waves on opposite sides of the interface commute,

$$[\hat{a}_R(\omega),\hat{c}_L^\dagger(x',\omega')]=0, \tag{4.16}$$

but the other pairs of operators in the vacuum and dielectric half-spaces have the nonzero commutators

$$[\hat{a}_R(\omega),\hat{c}_R^\dagger(x',\omega')]=\delta(\omega-\omega')\frac{n^*(\omega)T^*(\omega)}{[\eta(\omega)]^{1/2}} \times\exp\left(-\frac{i\omega n^*(\omega)x'}{c}\right), \tag{4.17}$$

$$[\hat{a}_L(\omega),\hat{c}_L^\dagger(x',\omega')]=\delta(\omega-\omega') \times[\eta(\omega)]^{1/2}T(\omega)\exp[i\omega n(\omega)x'/c], \tag{4.18}$$

and

$$[\hat{a}_L(\omega),\hat{c}_R^\dagger(x',\omega')]=-i\delta(\omega-\omega')\frac{\kappa(\omega)T(\omega)}{[\eta(\omega)]^{1/2}} \times\exp\left(\frac{i\omega n(\omega)x'}{c}\right), \tag{4.19}$$

where the forms of the transmission and reflection coefficients given in (4.4) have been used in simplifying the last result.

The canonical commutation relations (2.18) and (2.19) can now be evaluated with use of the expressions (3.23), (3.26), and (4.6) for the vector potential and electric field operators, together with the commutators given above. There are four cases to check, with both positions x and x' either in the vacuum or in the dielectric. Consider first the canonical commutator (2.18) with both positions in the dielectric. It is straightforward to show that

$$\begin{aligned}[\hat{A}(x,t),-\varepsilon_0\hat{E}(x',t)] &= \frac{i\hbar}{2\pi cS}\int_{-\infty}^{\infty}d\omega\left\{-R(\omega)\frac{\exp[i\omega n(\omega)(x+x')/c]}{n(\omega)}\right. \\ &\quad \left.+\frac{\exp[i\omega n(\omega)|x-x'|/c]}{n(\omega)}\right\}, \quad x>0,\ x'>0.\end{aligned} \tag{4.20}$$

The integrals can be evaluated by the same methods as used in (3.20) and (3.21), and indeed the second term in the large bracket of (4.20) reproduces the same integral as (3.19), which gives the desired result for the canonical commutator. For the first term in the large bracket, we use the property that $R(\omega)$ defined in (4.4) is itself a linear response function, which determines the reflected field produced by an incident field. Thus, as mentioned after (2.3), causality requires that the function can have no poles in the upper half of the complex ω-plane [33]. Furthermore, in view of (2.1) and (2.4), $R(\omega)$ has the property

$$R(\omega)\to 0 \quad \text{for } \omega\to\infty. \tag{4.21}$$

Thus, when the contribution of the first term in the large curly brackets is evaluated by conversion to polar coordinates as in (3.21), the integrand vanishes on the semicircle at infinity, and this term is zero. The canonical commutator thus has the expected form (2.18). When both coordinate positions lie in the vacuum, the expression (4.20) is replaced by

$$[\hat{A}(x,t),-\varepsilon_0\hat{E}(x',t)] = \frac{i\hbar}{2\pi cS}\int_{-\infty}^{\infty} d\omega \left\{ R(\omega)\exp\left(-\frac{i\omega(x+x')}{c}\right) + \exp\left(\frac{i\omega(x-x')}{c}\right)\right\}, \quad x<0,\ x'<0, \tag{4.22}$$

but similar arguments show that the first term in the large curly brackets makes no contribution and the desired result (2.18) for the canonical commutator is produced by the second term. When one position coordinate lies in the vacuum and the other in the dielectric, similar reasoning shows that the canonical commutator vanishes. Thus (2.18) holds for the entire quantized electromagnetic field associated with the semi-infinite dielectric and the other form (2.19) of canonical commutator follows from very similar derivations.

C. Vacuum field fluctuations

The contribution to the electromagnetic field fluctuations from the modes that propagate perpendicular to the interface is obtained with the use of the Green functions for the semi-infinite dielectric given in (4.3) and (4.5). The power spectrum needed for substitution in (3.33) is determined by the fluctuation-dissipation theorem in (3.34) as

$$\mathcal{S}(x,\omega)=2\hbar\omega^2\,\mathrm{Im}\mathcal{G}(x,x,\omega)=\begin{cases}\dfrac{\hbar\omega}{\varepsilon_0 cS}\mathrm{Re}\left\{-\dfrac{n(\omega)-1}{n(\omega)+1}\exp\left(-\dfrac{2i\omega x}{c}\right)+1\right\}, & x<0,\\[2ex] \dfrac{\hbar\omega}{\varepsilon_0 cS}\mathrm{Re}\left\{\dfrac{1}{n(\omega)}\left[\dfrac{n(\omega)-1}{n(\omega)+1}\exp\left(\dfrac{2i\omega n(\omega)x}{c}\right)+1\right]\right\}, & x>0.\end{cases} \tag{4.23}$$

The fluctuations are seen to be continuous through the interface at $x=0$, and the result (3.34) for an infinite homogeneous medium is reproduced far inside the dielectric where $\omega\kappa(\omega)x/c \gg 1$.

Figure 2 shows the spatial variation of the field fluctuations in the vicinity of the interface for typical values of the refractive index and extinction coefficient. The magnitude of the fluctuation is normalized to its free-space value. The oscillations to the left of the interface are a consequence of the partial standing-wave character of the excitation. The oscillations to the right of the interface decay rapidly to the value for the homogeneous dielectric given by (3.34). It should be emphasized that the fluctuations illustrated are only those associated with the modes that propagate perpendicular to the interface. The total field fluctuation, including modes that propagate in all directions, shows quite different behavior. Thus, for example, even for a lossless dielectric, the standing-wave form to the left of the interface is removed, and the total fluctuation decays quite rapidly to its free-space value [10].

V. DIELECTRIC SLAB

The form of the dielectric function for a slab of thickness $2l$ is

$$\varepsilon(x,\omega)=\begin{cases}1 & \text{for } x<-l,\\ \varepsilon(\omega)=[n(\omega)]^2 & \text{for } -l<x<l,\\ 1 & \text{for } l<x.\end{cases} \tag{5.1}$$

The vector potential operator is once more determined by (2.13) and (2.14), with the particular integral part of the Fourier transform Green function given by (2.17). The complementary function parts of the Green function correspond to waves that travel away from the slab on both sides and travel in both directions in the slab interior. Their four coefficients are determined by imposition of the usual electric and magnetic boundary conditions at the two surfaces of the slab. The calculation is somewhat tedious but essentially straightforward, and the final expressions for the coordinate-space Green function, with x' assumed to lie between $-l$ and l, are

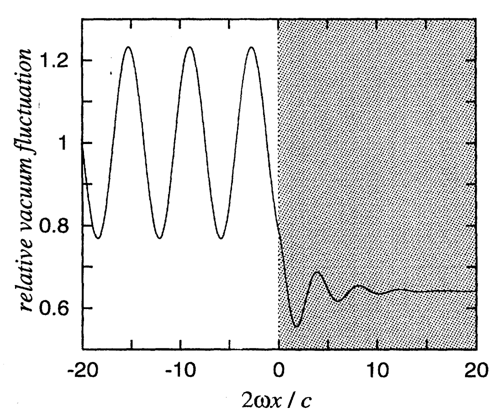

FIG. 2. Spatial variation of the spectrum $\mathcal{S}(x,\omega)$ of vacuum electric field fluctuations close to the surface of a semi-infinite dielectric, normalized to its free-space value, for complex refractive index $n(\omega)=1.5+i0.3$.

$$\mathcal{G}(x,x',\omega)=\frac{i}{2\varepsilon_0 c\omega S}\left\{V(\omega)\exp\left(\pm\frac{i\omega[n(\omega)x'-x]}{c}\right)+W(\omega)\exp\left(\mp\frac{i\omega[n(\omega)x'+x]}{c}\right)\right\},\quad x<-l,\quad x>l \tag{5.2}$$

where the upper and lower signs refer to the two ranges of x on the left and right of the slab, respectively,

$$V(\omega)=\frac{2[n(\omega)+1]}{D(\omega)}\exp\left(\frac{i\omega[n(\omega)-1]l}{c}\right), \tag{5.3}$$

$$W(\omega)=\frac{2[n(\omega)-1]}{D(\omega)}\exp\left(\frac{i\omega[3n(\omega)-1]l}{c}\right) \tag{5.4}$$

and

$$D(\omega)=[n(\omega)+1]^2-[n(\omega)-1]^2\exp[4i\omega n(\omega)l/c] \tag{5.5}$$

consistent with notation used previously [34] for the spatial modes of a lossless dielectric slab. The Green function for the interior of the slab is

$$\mathcal{G}(x,x',\omega)=\frac{i}{2\varepsilon_0 c\omega n(\omega)S}\left\{\frac{[n(\omega)]^2-1}{D(\omega)}\left[\exp\left(\frac{i\omega n(\omega)(2l+x+x')}{c}\right)+\exp\left(\frac{i\omega n(\omega)(2l-x-x')}{c}\right)\right]+\frac{[n(\omega)-1]^2}{D(\omega)}\left[\exp\left(\frac{i\omega n(\omega)(4l+x-x')}{c}\right)+\exp\left(\frac{i\omega n(\omega)(4l-x+x')}{c}\right)\right]+\exp\left(\frac{i\omega n(\omega)|x-x'|}{c}\right)\right\},\quad -l<x<l,\ -l<x'<l. \tag{5.6}$$

The structure of this Green function is typical of slab systems. Thus the first four terms correspond to communication between the points x and x' via the four distinct patterns of reflection in the slab surfaces, and the denominator $D(\omega)$ results from summation of multiple reflections of the same kinds. The final term corresponds to direct communication between the two points.

The Green functions (5.2) and (5.6) determine the electromagnetic field quantization, but the calculation of vacuum field fluctuations also requires the corresponding expressions for positions outside the slab, given by

$$\mathcal{G}(x,x',\omega)=\frac{i}{2\varepsilon_0 c\omega S}\left\{\frac{[n(\omega)]^2-1}{D(\omega)}\exp\left(\frac{-i\omega[2l\pm(x+x')]}{c}\right)\left[\exp\left(\frac{4i\omega n(\omega)l}{c}\right)-1\right]+\exp\left(\frac{i\omega|x-x'|}{c}\right)\right\},\quad x,x'<-l,\ x,x'>l, \tag{5.7}$$

where the upper and lower signs refer to the left and right of the slab, respectively. The Green function for positions that lie on opposite sides of the slab is not needed here. The slab Green functions (5.2), (5.6), and (5.7) reduce to those for the semi-infinite dielectric given in (4.3) and (4.5) when the coordinate origin is shifted to the left-hand surface of the slab and the limit $l\rightarrow\infty$ is taken. Related forms of the Green function, in the time domain and in Laplace transform space, have been derived for the absorbing dielectric slab [23].

A particular integral solution for the vector potential operator is now obtained by substitution of the Green function given by (5.2) and (5.6) into the integrand of (2.14). The complete solution also contains complementary function parts that correspond to free fields incident on the slab surfaces from the regions of vacuum to its left and right. The notation for the operators associated with the rightwards and leftwards parts of the fields in the three spatial regions is illustrated in Fig. 1(c). The operators for the incoming fields on the left and right of the slab have the free-space commutators

$$[\hat{a}_R(\omega),\hat{a}_R^\dagger(\omega')]=[\hat{\ell}_L(\omega),\hat{\ell}_L^\dagger(\omega')]=\delta(\omega-\omega'), \tag{5.8}$$

similar to (4.7), and the operators for the two kinds of incoming wave commute,

$$[\hat{a}_R(\omega),\hat{\ell}_L^\dagger(\omega')]=0, \tag{5.9}$$

analogous to (4.16).

Consider first the complete fields exterior to the slab. The field on the left can be written in a form similar to (4.6),

$$\hat{A}^+(x,t)=\int_0^\infty d\omega\left(\frac{\hbar}{4\pi\varepsilon_0 c\omega S}\right)^{1/2}[\hat{a}_R(\omega)e^{i\omega x/c}+\hat{a}_L(\omega)e^{-i\omega x/c}]e^{-i\omega t},\quad x<-l, \tag{5.10}$$

where the operator for the leftwards-propagating outgoing field is given by

$$\hat{a}_L(\omega)=R_s(\omega)\hat{a}_R(\omega)+T_S(\omega)\hat{b}_L(\omega)+i\left(\frac{2\omega\eta(\omega)\kappa(\omega)}{c}\right)^{1/2}\times\int_{-l}^{l}dx'\left\{V(\omega)\exp\left(\frac{i\omega n(\omega)x'}{c}\right)+W(\omega)\exp\left(-\frac{i\omega n(\omega)x'}{c}\right)\right\}\hat{f}(x',\omega), \tag{5.11}$$

and (3.9) and (3.18) have been used. The amplitude transmission and reflection coefficients for the slab are given by [34]

$$T_S(\omega)=\frac{4n(\omega)}{D(\omega)}\exp\left(\frac{2i\omega[n(\omega)-1]l}{c}\right) \tag{5.12}$$

and

$$R_s(\omega)=-\frac{[n(\omega)]^2-1}{D(\omega)}\exp\left(-\frac{2i\omega l}{c}\right)\times\left\{1-\exp\left(\frac{4i\omega n(\omega)l}{c}\right)\right\}. \tag{5.13}$$

These coefficients reduce to the corresponding expressions (4.4) for the surface of a semi-infinite dielectric when $l\to\infty$ and account is taken of the different positions chosen for the coordinate origins in the two cases. The field on the right of the slab is similarly given by

$$\hat{A}^+(x,t)=\int_0^\infty d\omega\left(\frac{\hbar}{4\pi\varepsilon_0 c\omega S}\right)^{1/2}[\hat{b}_R(\omega)e^{i\omega x/c}+\hat{b}_L(\omega)e^{-i\omega x/c}]e^{-i\omega t},\quad x>l, \tag{5.14}$$

where the operator for the rightwards-propagating outgoing field is given by

$$\hat{b}_R(\omega)=R_S(\omega)\hat{b}_L(\omega)+T_S(\omega)\hat{a}_R(\omega)+i\left(\frac{2\omega\eta(\omega)\kappa(\omega)}{c}\right)^{1/2}\times\int_{-l}^{l}dx'\left\{V(\omega)\exp\left(-\frac{i\omega n(\omega)x'}{c}\right)+W(\omega)\exp\left(\frac{i\omega n(\omega)x'}{c}\right)\right\}\hat{f}(x',\omega). \tag{5.15}$$

It is again straightforward, but algebraically lengthy, to show with the use of the forms of the coefficients (5.3), (5.4), (5.12), and (5.13) that the outgoing field operators have the simple free-space commutators

$$[\hat{a}_L(\omega),\hat{a}_L^\dagger(\omega')]=[\hat{b}_R(\omega),\hat{b}_R^\dagger(\omega')]=\delta(\omega-\omega'). \tag{5.16}$$

Expressions for the electric and magnetic field operators on the left and right of the slab are readily obtained from (5.10) and (5.14), respectively, with the use of (2.12). The expressions for the vector potential operator on either side of the slab obtained from (5.10), (5.11), (5.14), and (5.15) agree with those used in calculations of the Casimir force between a pair of absorbing dielectric plates [25] when the plates are coalesced and account is taken of different units and normalization conventions.

Consider now the complete field inside the dielectric slab, which again has the same form as (3.23) but with new definitions of the left and right destruction operators. Thus with use of the interior Green function (5.6) and addition of the complementary function parts obtained from free fields incident on the slab from the vacua on its left and right, the complete expressions for the operators inside the dielectric are

$$\hat{c}_R(x,\omega)=\left\{\frac{n(\omega)}{[\eta(\omega)]^{1/2}}[V(\omega)\hat{a}_R(\omega)+W(\omega)\hat{b}_L(\omega)]+i\left(\frac{2\omega\kappa(\omega)}{c}\right)^{1/2}\left[\int_{-l}^{l}dx'\left[\frac{[n(\omega)]^2-1}{D(\omega)}\exp\left(\frac{i\omega n(\omega)(2l+x')}{c}\right)+\frac{[n(\omega)-1]^2}{D(\omega)}\exp\left(\frac{i\omega n(\omega)(4l-x')}{c}\right)\right]\hat{f}(x',\omega)+\int_{-l}^{x}dx'\exp\left(-\frac{i\omega n(\omega)x'}{c}\right)\hat{f}(x',\omega)\right]\right\}\exp\left(\frac{i\omega n(\omega)x}{c}\right) \tag{5.17}$$

and

$$\hat{c}_L(x,\omega)=\left\{\frac{n(\omega)}{[\eta(\omega)]^{1/2}}[W(\omega)\hat{a}_R(\omega)+V(\omega)\hat{b}_L(\omega)]+i\left(\frac{2\omega\kappa(\omega)}{c}\right)^{1/2}\left[\int_{-l}^{l}dx'\left[\frac{[n(\omega)]^2-1}{D(\omega)}\exp\left(\frac{i\omega n(\omega)(2l-x')}{c}\right)+\frac{[n(\omega)-1]^2}{D(\omega)}\exp\left(\frac{i\omega n(\omega)(4l+x')}{c}\right)\right]\hat{f}(x',\omega)+\int_{x}^{l}dx'\exp\left(\frac{i\omega n(\omega)x'}{c}\right)\hat{f}(x',\omega)\right]\right\}\exp\left(-\frac{i\omega n(\omega)x}{c}\right). \tag{5.18}$$

These expressions reduce to the forms (3.13) and (3.14) for the infinite homogeneous dielectric and to the forms (4.10) and (3.14) for the semi-infinite dielectric when appropriate limits are taken. It is straightforwardly verified that (3.24) and (3.25) hold for the redefined operators $\hat{c}_R(x,\omega)$ and $\hat{c}_L(x,\omega)$, respectively, and the forms of electric and magnetic field operator given in (3.26) and (3.27) remain valid inside the dielectric slab. More complicated forms of the quantized field operators have been derived [24] for an atomic model of a dielectric slab in which allowances are made for initial excitations of the atoms and elevated material temperatures.

If the Green functions are not required, the field operators can again be calculated more directly by straightforward application of the usual boundary conditions. Thus with the forms of the electric and magnetic field operators described above, the boundary conditions at $x=-l$ give

$$\hat{a}_R(\omega)e^{-i\omega l/c}+\hat{a}_L(\omega)e^{i\omega l/c}=\frac{[\eta(\omega)]^{1/2}}{n(\omega)}\times[\hat{c}_R(-l,\omega)+\hat{c}_L(-l,\omega)] \tag{5.19}$$

and

$$\hat{a}_R(\omega)e^{-i\omega l/c}-\hat{a}_L(\omega)e^{i\omega l/c}=[\eta(\omega)]^{1/2}\times[\hat{c}_R(-l,\omega)-\hat{c}_L(-l,\omega)], \tag{5.20}$$

while the boundary conditions at $x=l$ give

$$\hat{b}_R(\omega)e^{i\omega l/c}+\hat{b}_L(\omega)e^{-i\omega l/c}=\frac{[\eta(\omega)]^{1/2}}{n(\omega)}[\hat{c}_R(l,\omega)+\hat{c}_L(l,\omega)] \tag{5.21}$$

and

$$\hat{b}_R(\omega)e^{i\omega l/c}-\hat{b}_L(\omega)e^{-i\omega l/c}=[\eta(\omega)]^{1/2}[\hat{c}_R(l,\omega)-\hat{c}_L(l,\omega)]. \tag{5.22}$$

The operators $\hat{c}_R(x,\omega)$ and $\hat{c}_L(x,\omega)$ evaluated at $x=-l$ are related to the same operators evaluated at $x=l$ by (3.28) and (3.29), respectively. The above four equations are therefore sufficient to determine $\hat{a}_L(\omega)$, $\hat{b}_R(\omega)$, $\hat{c}_R(x,\omega)$, and $\hat{c}_L(x,\omega)$ in terms of the independent input operators $\hat{a}_R(\omega)$ and $\hat{b}_L(\omega)$, and when this is done, the expressions (5.11), (5.15), (5.17), and (5.18) are rederived.

The commutators of the six varieties of creation and destruction operator shown in Fig. 1(c) are readily calculated from the above definitions. Some of these have quite simple forms, as for example the results given in (5.8), (5.9), and (5.15). The commutators of incoming with outgoing operators obtained from (5.11) and (5.15) are

$$[\hat{a}_L(\omega),\hat{a}_R^\dagger(\omega')]=[\hat{b}_R(\omega),\hat{b}_L^\dagger(\omega')]=\delta(\omega-\omega')R_S(\omega) \tag{5.23}$$

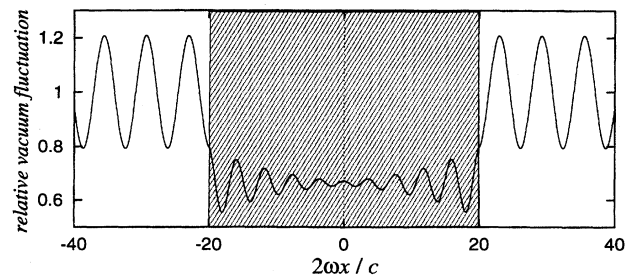

FIG. 3. Spatial variation of the spectrum $\mathscr{S}(x,\omega)$ of vacuum electric field fluctuations in the vicinity of a dielectric slab, normalized to its free-space value. The slab thickness is $2l=20c/\omega$ and the complex refractive index is $n(\omega)=1.5+i0.1$.

and

$$[\hat{a}_L(\omega),\hat{b}_L^\dagger(\omega')]=[\hat{b}_R(\omega),\hat{a}_R^\dagger(\omega')]=\delta(\omega-\omega')T_S(\omega). \tag{5.24}$$

The outgoing wave operators on opposite sides of the slab commute,

$$[\hat{a}_L(\omega),\hat{b}_R^\dagger(\omega')]=0, \tag{5.25}$$

where this result follows with the use of (5.3), (5.4), (5.12), and (5.13). The commutators of the operators $\hat{c}_R(x,\omega)$ and $\hat{c}_L(x,\omega)$ given by (5.17) and (5.18) are more complicated and their explicit forms are given in the Appendix. The degrees of freedom for the dielectric slab include the incoming fields associated with $\hat{a}_R(\omega)$ and $\hat{b}_L(\omega)$ in addition to the continuum of noise currents, and the Hilbert space is augmented by the excited states of these fields.

The canonical commutation relations (2.18) and (2.19) are satisfied by the vector potential operators derived above for the three spatial regions associated with the slab geometry. The calculations needed to verify these relations are straightforward but very protracted, and brief details are given in the Appendix.

The power spectrum of the vacuum field fluctuations is again determined by the fluctuation-dissipation theorem (3.34), where the required Green functions are given by (5.6) and (5.7). Figure 3 shows the spatial variation of the fluctuation spectrum, whose magnitude is normalized to its free-space value. The figure resembles a reflected version of the corresponding Fig. 2 for the semi-infinite dielectric, and the spectrum for the dielectric slab shows similar features, with undamped oscillatory behavior outside the dielectric and oscillations inside the slab that decay with distance from its surfaces. The behavior in the central region of the slab depends upon the way in which the damped oscillations from its opposite sides interfere; for the parameters chosen in Fig. 3, about 9.5 periods of the oscillation fit into the thickness of the slab, and a certain amount of constructive interference occurs. As in Fig. 2, it should be emphasized that the fluctuations illustrated are only those associated with the modes that propagate perpendicular to the surfaces, and again even the lossless slab shows a complete three-dimensional fluctuation spectrum whose oscillations decay with distance from the surfaces, both within the slab and in the free-space regions on either side [11].

VI. CONCLUSIONS

We have presented expressions for the electromagnetic field operators in absorbing media whose spatial distributions take the forms of an infinite homogeneous dielectric, a semi-infinite dielectric, and a dielectric slab. The three distributions are illustrated in Fig. 1, together with the notations for the destruction operators that provide the most convenient expressions for the quantized fields. The vector potential operator inside the dielectric medium can always be written in the form of (3.23), but the operators $\hat{c}_R(x,\omega)$ and $\hat{c}_L(x,\omega)$ have the different expressions (3.13) and (3.14) for the infinite homogeneous dielectric, (4.10) and (3.14) for the semi-infinite dielectric, and (5.17) and (5.18) for the dielectric slab. These expressions include contributions from the noise operators $\hat{f}(x,\omega)$ associated with the dielectric loss mechanisms, also from a rightwards incoming free field with destruction operator $\hat{a}_R(\omega)$ for the semi-infinite dielectric, and from rightwards and leftwards incoming fields with destruction operators $\hat{a}_R(\omega)$ and $\hat{b}_L(\omega)$, respectively, for the dielectric slab. The vector potential operator in the vacuum to the left of the dielectric can always be written in the form of (4.6), where the operator $\hat{a}_R(\omega)$ is associated with an incoming free field and the operator $\hat{a}_L(\omega)$ is given by the expression (4.8) for the semi-infinite dielectric and by (5.11) for the dielectric slab. Finally, the vector potential operator to the right of the dielectric slab is given by (5.14), where the operator $\hat{b}_R(\omega)$ is given by the expression (5.15) and $\hat{b}_L(\omega)$ is associated with an incoming free field. There are no incoming free fields for the infinite homogeneous dielectric since any electromagnetic wave launched from infinite distance is totally absorbed.

The detailed expressions for the electromagnetic field operators in terms of incoming fields and noise operators thus depend on the local environment, and the commutators of the destruction operators, listed above, with their conjugate creation operators are strongly influenced by the dielectric boundaries. The operator commutators inside the dielectric always simplify to the equal-space forms (3.12) at positions separated from any boundary by distances large compared to the characteristic decay length associated with the dielectric absorption. Particular attention has been paid to the commutator of the vector potential operator with its canonically conjugate momentum, given by (2.18) and (2.19) for two choices of transverse gauge. While it follows from general principles that the quantized field expressions must satisfy these commutation relations, they are extremely useful in providing verifications of the accuracies of the overall structures and detailed algebraic coefficients that occur in the field operators [9]. All of the quantized field expressions derived here conform to the canonical commutation relations.

The scope of the present paper is limited to the derivations of formal expressions for the field operators in the three spatial arrangements of dielectric. However, the expressions are easy to use in practical calculations, and they can be applied immediately to such problems as the effects of propagation through absorbing dielectrics on light that initially displays nonclassical features, or to the mechanical effects of light on absorbing media. The quantization procedure is also readily extended to other configurations of dielectric media, for example different dielectrics in contact or separated by free space. Although the Langevin force operator is not derived here from any detailed model of the dielectric loss mechanisms, in contrast to the calculations reported in [19], all physical predictions of the theory are determined in the usual way by expectation values of the operators. With the usual assumptions of a noise reservoir effectively in its zero temperature vacuum state for experiments in the visible frequency region, the required expection values can be found with use of the commutator (3.10) and the basic results

$$\langle 0|\hat{f}(x,\omega)|0\rangle=\langle 0|\hat{f}^{\dagger}(x,\omega)|0\rangle=0 \tag{6.1}$$

and

$$\langle 0|\hat{f}^{\dagger}(x',\omega')\hat{f}(x,\omega)|0\rangle=0. \tag{6.2}$$

These simple conditions are sufficient for the extraction of useful results from the theory.

The quantized field expressions are restricted to propagation in one dimension perpendicular to the dielectric surfaces, and these are adequate for a wide range of optical systems, where plane parallel light beams are confined in fibres or arranged to fall perpendicularly on material bodies. However, a further range of observable effects, notably spontaneous emission by atoms close to dielectric surfaces or vacuum field Casimir forces on dielectric surfaces, require a quantized field theory that incorporates propagation in all spatial directions. The extension of the one-dimensional theory presented here to three dimensions is in progress, together with the applications of the one-dimensional theory outlined above, and the results will be reported subsequently.

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APPENDIX

In this Appendix, we give further results for the operator and field commutation relations associated with the dielectric slab. Consider first the operators in the interior of the slab defined in (5.17) and (5.18). Their commutators are obtained with the use of (3.10), (5.8), and (5.9) in the forms

$$[\hat{c}_R(x,\omega),\hat{c}_R^\dagger(x',\omega')]=\delta(\omega-\omega')\left\{\exp\left(\frac{i\omega\eta(\omega)(x-x')-\omega\kappa(\omega)|x-x'|}{c}\right)+\left[M(\omega)\exp\left(\frac{i\omega n(\omega)(x-x')}{c}\right)\right.\right.$$
$$\left.+M^*(\omega)\exp\left(\frac{i\omega n^*(\omega)(x-x')}{c}\right)\right]-\frac{i\kappa(\omega)}{\eta(\omega)}\left[N(\omega)\exp\left(\frac{i\omega n(\omega)(x+x')}{c}\right)\right.$$
$$\left.\left.-N^*(\omega)\exp\left(-\frac{i\omega n^*(\omega)(x+x')}{c}\right)\right]\right\}, \quad (A1)$$

$$[\hat{c}_L(x,\omega),\hat{c}_L^\dagger(x',\omega')]=[\hat{c}_R(-x,\omega),\hat{c}_R^\dagger(-x',\omega')] \quad (A2)$$

and

$$[\hat{c}_R(x,\omega),\hat{c}_L^\dagger(x',\omega')]=[\hat{c}_L(x,\omega),\hat{c}_R^\dagger(x',\omega')]^*=\delta(\omega-\omega')\left\{-\frac{i\kappa(\omega)}{\eta(\omega)}\theta(x-x')\left[\exp\left(\frac{i\omega n(\omega)(x-x')}{c}\right)\right.\right.$$
$$\left.-\exp\left(-\frac{i\omega n^*(\omega)(x-x')}{c}\right)\right]+\left[N(\omega)\exp\left(\frac{i\omega n(\omega)(x+x')}{c}\right)+N^*(\omega)\exp\left(\frac{i\omega n^*(\omega)(x+x')}{c}\right)\right]$$
$$\left.-\frac{i\kappa(\omega)}{\eta(\omega)}\left[M(\omega)\exp\left(\frac{i\omega n(\omega)(x-x')}{c}\right)-M^*(\omega)\exp\left(-\frac{i\omega n^*(\omega)(x-x')}{c}\right)\right]\right\}, \quad (A3)$$

where

$$M(\omega)=\frac{(n(\omega)-1)^2}{D(\omega)}\exp\left(\frac{4i\omega n(\omega)l}{c}\right), \quad (A4)$$

$$N(\omega)=\frac{[n(\omega)]^2-1}{D(\omega)}\exp\left(\frac{2i\omega n(\omega)l}{c}\right) \quad (A5)$$

and $D(\omega)$ is given by (5.5).

The vector potential and electric field operators inside the dielectric slab are expressed in terms of $\hat{c}_R(x,\omega)$ and $\hat{c}_L(x,\omega)$ by (3.23) and (3.26), respectively. The value of the equal-time canonical commutator (2.18) in the slab is thus determined by the commutation relations listed above, and it simplifies, after some algebra, to

$$[\hat{A}(x,t),-\varepsilon_0\hat{E}(x',t)]=\frac{i\hbar}{2\pi cS}\int_{-\infty}^{\infty}d\omega\left\{\frac{1}{n(\omega)}\exp\left(\frac{i\omega n(\omega)|x-x'|}{c}\right)+\frac{M(\omega)}{n(\omega)}\left[\exp\left(\frac{i\omega n(\omega)(x-x')}{c}\right)\right.\right.$$
$$\left.\left.+\exp\left(-\frac{i\omega n(\omega)(x-x')}{c}\right)\right]+\frac{N(\omega)}{n(\omega)}\left[\exp\left(\frac{i\omega n(\omega)(x+x')}{c}\right)+\exp\left(-\frac{i\omega n(\omega)(x+x')}{c}\right)\right]\right\}. \quad (A6)$$

The functions $M(\omega)$ and $N(\omega)$ have no poles in the upper half of the complex ω plane, and they both tend to zero as the frequency ω tends to infinity. Thus, similar to the evaluation of (4.20), only the first term in the integrand of (A6) makes a nonzero contribution. This remaining integral is identical to that in (3.19), and the canonical commutator therefore has the required value given by (2.18).

The canonical commutator is evaluated for positions outside the slab with the use of the expressions for the vector potential operator obtained from (5.10) and (5.14). The commutator takes the same form as that for the exterior of the semi-infinite dielectric given by (4.22), except that the reflection coefficient $R(\omega)$ is replaced by the slab coefficient $R_S(\omega)$, and similar arguments lead to the required value given by (2.18).

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