Photon statistics in spontaneous emission for the Dicke model in a lossless cavity and the generation of the Fock state

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Using a perturbation method, constructed in terms of SU(2)-group representations, we investigate the time evolution of the statistics of photons spontaneously emitted by a system of N identical two-level atoms immersed in a single-mode ideal cavity with s atoms initially excited. The phenomenon of collective collapses and revivals is discussed and the collapse and revival times, which are different for odd and even s, are determined. It is shown for s extremely small compared to N that at appropriately chosen time instants the model generates the s-photon-number state (Fock state).

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I. INTRODUCTION

It is our aim here to study the situation when s initially excited two-level atoms radiate spontaneously in the presence of $N-s$ unexcited atoms in a lossless single-mode cavity. The atoms are located within a distance much smaller than the wavelength of the radiation. However, the cooperative nature of spontaneous emission is assumed to result from indirect atom-atom coupling via the field-mode only. Such a system has been commonly referred to as the Dicke model [1]. Since the earliest numerical investigations [2] it has been known to present two limit cases when it may be treated as an almost "linear" system (equidistant eigenvalues spectrum) [2]. The first case occurs for $\mathcal{N} \gg N$, whereas the other for $\mathcal{N} \ll N$, where $\mathcal{N}$ is the excitation number, i.e., the number of photons plus the number of excited atoms. Then the motion of the model inside a subspace with a given excitation number is truly periodic. These may be called "strong-" and "weak-" field regimes, respectively. Obviously, the spontaneous emission of a partially inverted atomic system with $s \ll N$ belongs to the latter.

The problem in question is exactly solvable for $s \leqslant 8$. The first two cases, i.e., $s=1$ and $s=2$, are characterized by equidistant eigenvalues spectra (independently of N) and are described in terms of a two- and a three-level system, respectively. The time evolution of such systems is periodic. Cummings and Dorri [3] and Seke [4] have presented the solutions for spontaneous emission from a system of N two-level atoms in a cavity with only one atom initially inverted. Cummings and Dorri [3] considered nonsymmetrical initial excitation of the atomic system and showed that the effect of radiation trapping (cooperative inhibition of radiation [5,6]) holds for atoms in equivalent as well as inequivalent mode positions. If N becomes very large, the excited atom does not radiate at all. Nonsymmetrical excitation occurs if one knows which atom of the sample is excited. In such a case one atom has to be excited separately and injected into the cavity containing $N-1$ unexcited atoms. Seke [4] performed calculations for symmetrical initial excitation. Then all the atoms are supposed to experience the same field during the preparation of the initial state. As a result, the atomic state is a coherent superposition of the states with the jth atom excited and weighted with the same probability $1/N$. Such states are called Dicke states. In this case, the presence of $N-1$ unexcited atoms increases the short-time radiation rate N times, compared to single-atom emission. The effect of the symmetry properties of the initial state on the collective emission rate has been discussed by Crubellier *et al.* [7], while the effect of the spatial distribution of atoms of the sample on spontaneous emission has been studied by Bužek [8].

The solution for $s=2$ (N arbitrary) was found in Ref. [9]. The particular case $s=N=2$ was presented by Shumovsky *et al.* [10]. Beginning from $s=2$ for finite N, the effect of radiation trapping occurs for both symmetrical and nonsymmetrical excitations [9]. If N is large, the excited atoms do not radiate at all in the latter case [9]. A sketch of the solution for $s=3$ was proposed by Senitzky [6] and the complete solution for the time evolution of the atomic inversion was presented in Ref. [11]. In general, the eigenvalues spectrum is already unequidistant for $s=3$ and the eigenvalues are incommensurate quantities. Since the system oscillates with two incommensurate Rabi frequencies, the phenomenon of quantum beats leads to modulated oscillations. The case $s=3$ is the first in the hierarchy of those having unequidistant eigenvalues spectra. This unequidistance leads to the phenomenon of collective collapses and revivals of the oscillations of the system. However, if $s \geqslant 3$ is very small in comparison with N, these spectra may be approximately treated as equidistant [11–13]. In this situation the condition of weak-field regime is satisfied [2].

Many various topics for the model have been considered numerically. In particular, Seke and Rattay [14] studied spontaneous and induced emission in the presence of a Fock state field for different cavity damping. The influence of the Fock state field on the radiatively decaying atomic system was also discussed by Hasan *et al.* [15]. Barnett and Knight [16] studied collapses and revivals of an assemblage of two-level initially inverted atoms interacting with an initially coherent or chaotic quantum field in a high-Q cavity. Butler and Drummond [17] showed that the collectivity of the system enhances squeezing compared to the Jaynes-Cummings model (JCM). The same problem without the rotating-wave approximation was recently investigated by Seke [18]. In turn, Li *et al.* [19] discussed the problem of higher-order squeezing [20], while Drobný and Jex [21] discussed the statistical properties of field modes in trilinear optical processes. Quite recently this problem was considered by Fernee *et al.* [22].

Bonifacio and Preparata [23] and Kumar and Mehta [24] have presented an approximate solution for the time evolution of the system in question in terms of elliptic functions. Recently, we have proposed another analytical approach to the problem [11–13,25], consisting of the construction of a perturbation theory in terms of SU(2)-group representations.

II. PERTURBATION APPROACH

Applying the perturbation method mentioned above, we study a system of N identical two-level atoms interacting with a single-mode quantized radiation field in an ideal cavity from the point of view of photon-number statistics. Initially, s of these atoms are excited. The linear dimensions of the atomic sample are assumed to be much smaller than the wavelength of the radiation and all the atoms are treated as being in equivalent-mode positions. However, the wave functions of the atoms are assumed not to overlap. The cooperative nature of spontaneous emission from the system is then due to indirect atom-atom coupling via the field mode.

The electric-dipole Hamiltonian in the rotating-wave approximation for the system under consideration reads ($\hbar=1$)

$$H=H_{\text{free}}+H_{\text{int}},$$

$$H_{\text{free}}=\omega a^\dagger a+\omega_a\sum_{j=1}^{N} S_3^{(j)},$$

$$H_{\text{int}}=g\sum_{j=1}^{N}[a^\dagger S_-^{(j)}+aS_+^{(j)}], \tag{1}$$

where the photon creation $a^\dagger$ and annihilation a operators satisfy the usual boson commutation rule $[a,a^\dagger]=1$ and $S_-^{(j)}$, $S_+^{(j)}$, and $S_3^{(j)}$ are pseudospin lowering, raising, and inversion operators of the jth atom, respectively, whereas ω denotes the frequency of the field mode while ω_a is the atomic transition frequency. In what follows, we assume exact resonance $\omega=\omega_a$. In the small sample approximation the coupling coefficient g is the same for all the atoms. The above Hamiltonian is a particular case of the trilinear Hamiltonian [2] describing inelastic Brillouin and Raman scattering and parametric conversion processes.

The excitation number operator $\mathcal{N}$

$$\mathcal{N}=a^\dagger a+\sum_{j=1}^{N} S_3^j+\frac{N}{2}$$

is an integral of motion. This means that the system, initially in a state with fixed eigenvalue of the operator $\mathcal{N}$, will never leave the subspace corresponding to this excitation number in the course of time evolution. The representation of the group SU(2) acts in each subspace of this kind. The spectrum of the eigenvalues of its generators is well known.

The basis vectors of the model read

$$|s,m\rangle=|s-m\rangle_a\otimes|m\rangle_f, \tag{2}$$

where $|m\rangle_f$ denotes the Fock state of the field while $|s-m\rangle_a$ is the normalized symmetric Dicke state of the atomic subsystem with $s-m$ atoms excited. The dimension of the subspace spanned by these vectors is $s+1$. In particular, the time evolution of the system in this subspace is related with the initial condition $|s,0\rangle$ when s atoms are initially excited and no photon is present (spontaneous emission).

Our perturbation approach [11–13,25] was constructed with the help of a small parameter ϵ

$$\epsilon=(N-s/2+1/2)^{-1}.$$

In principle, not only should ϵ be small but s/N as well. However, as it is also the case in other problems solved by perturbation methods, our approximate formulas work satisfactorily well even for not small values of ϵ and s/N. The free part of the Hamiltonian (1) leads to an unimportant phase factor in the transition amplitudes. Therefore, only the interaction Hamiltonian H_{int} was expanded in a power series in ϵ and its zeroth-order term, proportional to the generator S_x of the group SU(2), was treated as the unperturbed term.

Let us denote by $\Lambda_{s,p}$ the eigenvalues of the system and by A_{mp} the matrix elements of the transition from the basis $|s,m\rangle$ to the eigenvector basis of the interaction Hamiltonian H_{int}. By standard perturbation methods, within an accuracy of ϵ^2, we found the following form of the eigenvalues [11–13]:

$$\Lambda_{s,p}^{(2)}=\Lambda_{s,p}^{(0)}\left\{1-\frac{\epsilon^2}{16}\left[5p(s-p)-\frac{(s-1)(s-2)}{2}\right]\right\},\qquad 0\leq p\leq s, \tag{3}$$

where $\Lambda_{s,p}^{(0)}$ are the zeroth-order eigenvalues

$$\Lambda_{s,p}^{(0)}=(s-2p)\Omega/2,\quad 0\leq p\leq s, \tag{4}$$

and the frequency Ω reads

$$\Omega=2\,g/\sqrt{\epsilon}=2g\sqrt{N-s/2+1/2}. \tag{5}$$

In turn, within an accuracy of ϵ we obtained

$$A_{mp}^{(1)}=\alpha_{mp}^{s}+\frac{\epsilon}{8}\{(s-2p+1)[p(s-p+1)]^{1/2}\alpha_{mp-1}^{s}-(s-2p-1)[(p+1)(s-p)]^{1/2}\alpha_{mp+1}^{s}\}, \tag{6}$$

where α_{mp}^{s} are the matrix elements of the transition from the basis (2) to the basis of the eigenvectors of the zeroth-order Hamiltonian $H_{\text{int}}^{(0)}$ [the components of the zeroth-order eigenvectors; see Eq. (8) of Ref. [25]]. Due to the form of the expansion parameter, the first-order (in general, odd-order) corrections to the eigenvalues vanish. As long as the initial state is an eigenstate of the excitation number operator, the only value of s that contributes to the initial state and the spectrum of the zeroth-order eigenvalues is equidistant irrespective of the ratio s/N. In particular, for $s=1$ and $s=2$ all higher-order corrections to the eigenvalues are equal to zero, both these spectra are equidistant independently of N, and the evolution of such systems is periodic. In turn, for $s \geqslant 3$ the eigenvalues (3) contain a term nonlinear in p that destroys the equidistance of the spectrum. However, as seen from the formula (3), if $s \geqslant 3$ is extremely small in comparison with N, these spectra may be approximately treated as equidistant (weak-field regime).

III. TIME EVOLUTION

The time evolution of the expectation value of any operator $\mathcal{O}$ bearing on the system is obtained with the standard formula

$$\langle \mathcal{O} \rangle = \langle s,0 | e^{iH_{\rm int}t} \mathcal{O} e^{-iH_{\rm int}t} | s,0 \rangle .$$

For the expectation value of the kth power of the photon number we have

$$\begin{aligned}\langle n^k(t) \rangle &= \sum_{p,q=0}^{s} \sum_{m=0}^{s} m^k A_{0p} A_{0q} A_{mp} A_{mq} e^{i(\Lambda_{s,q} - \Lambda_{s,p})t} \\ &= \sum_{p=0}^{s} A_{0p}^2 \sum_{m=0}^{s} m^k A_{mp}^2 \\ &\quad + \sum_{p=0}^{s} \sum_{q \neq p}^{s} A_{0p} A_{0q} \cos[(\Lambda_{s,q} - \Lambda_{s,p})t] \\ &\quad \times \sum_{m=0}^{s} m^k A_{mp} A_{mq} . \end{aligned} \tag{7}$$

From Eqs. (3) and (6), in the second-order approximation for the eigenvalues and in the first-order approximation for the eigenvectors (subscript 21) we arrive at

$$\begin{aligned}\langle n(t) \rangle_{21} &= \frac{s}{2} - \frac{\epsilon s(s-1)}{16} \\ &\quad - \sum_{p=1}^{s} C_p^s \left\{ \left(1 - \frac{\epsilon}{8}[(s-2p)^2 - 2p + 1] \right) \right. \\ &\quad \times \cos[(\Lambda_{s,p-1}^{(2)} - \Lambda_{s,p}^{(2)})t] - \frac{\epsilon(p-1)}{4} \\ &\quad \left. \times \cos[(\Lambda_{s,p-2}^{(2)} - \Lambda_{s,p}^{(2)})t] \right\} , \end{aligned} \tag{8}$$

where

$$C_p^s = \frac{p}{2^s} \binom{s}{p} = \frac{s}{2^s} \binom{s-1}{p-1} \tag{9}$$

is simply equal to the binomial distribution function multiplied by $s/2$. In the zeroth-order approximation for the eigenvectors only the frequencies related to transitions between two neighboring levels of the Hamiltonian (1) contribute to the time evolution of the mean photon number $\Lambda_{s,p-1}^{(2)} - \Lambda_{s,p}^{(2)}$. This Dicke frequency involved in the main term of Eq. (8) reads

$$\begin{aligned}\Omega^{(p)} &= \Lambda_{s,p-1}^{(2)} - \Lambda_{s,p}^{(2)} \\ &= \Omega \left\{ 1 + \frac{3\epsilon^2}{16}[5(p-1)(p-s) + (s-1)(s-2)] \right\} , \end{aligned} \tag{10}$$

where Ω is given by Eq. (5). Due to the summation over p in Eq. (8) we deal with a spread of the Dicke frequency caused by the collectivity of the system and p labels the frequencies of the spread. In the first-order approximation for the eigenvectors new transition frequencies $\Lambda_{s,p-2}^{(2)} - \Lambda_{s,p}^{(2)}$ intervene. Equation (8) contains the term that does not vanish under the operation of time averaging: $s/2 - \epsilon s(s-1)/16$. Owing to the term connected with the first-order approximation for the eigenvectors, its value is smaller than $s/2$ (except for $s=1$) which means that some portion of the energy is trapped in the atomic subsystem. Hence, on the average, the probability of finding the system with a smaller number of photons is slightly greater than with a larger number of photons, although the approximate solution allows the mean photon number to reach the value s.

The expectation value of the squared photon-number operator in the approximation 21 reads

$$\begin{aligned}\langle n^2(t) \rangle_{21} &= \frac{s(3s+1)}{8} - \frac{\epsilon s(s-1)^2}{16} - \sum_{p=1}^{s} C_p^s \left\{ \left[s - \frac{\epsilon}{8}(5s^3 + s^2 - 6s - 11s^2 p - sp^2 + 12sp \right. \right. \\ &\quad \left. + 6p^3 - 19p^2 + 21p - 8) \right] \cos[(\Lambda_{s,p-1}^{(2)} - \Lambda_{s,p}^{(2)})t] - \frac{p-1}{2}\left[1 - \frac{\epsilon}{2}(2s^2 + 3s - 5sp + 2p^2 - 3p + 3) \right] \\ &\quad \left. \times \cos[(\Lambda_{s,p-2}^{(2)} - \Lambda_{s,p}^{(2)})t] - \frac{\epsilon}{8}(s-2p+1)(p-1)(p-2)\cos[(\Lambda_{s,p-3}^{(2)} - \Lambda_{s,p}^{(2)})t] \right\} . \end{aligned} \tag{11}$$

If the number of excited atoms satisfies the condition $s \ll N$, the pure zeroth-order approximation is sufficient to describe the process, which is then periodic (subscript 00):

$$\langle n(t)\rangle_{00} = \frac{s}{2}(1-\cos\Omega t), \tag{12}$$

$$\langle n^2(t)\rangle_{00} = \frac{s}{8}[1+3s-4s\cos\Omega t+(s-1)\cos 2\,\Omega t],$$

where the frequency Ω is given by Eq. (5). For $s=N=1$ the results for the JCM are recovered.

IV. SECOND-ORDER REVIVALS

In this section we estimate the revival and collapse times of the mean photon number (8). To start with, however, we wish to present some general remarks about revivals, not only concerned with the Dicke model. Let the index p label different frequencies Ω_p of a spread, distributed with some function peaked in the point $p=p_m$. As usual, we treat the frequency as a continuous quantity and expand the dispersion curve Ω_p around the point p_m

$$\Omega^{(p)} = \Omega_0^{(p_m)} + \Omega_1^{(p_m)}(p-p_m) + \Omega_2^{(p_m)}(p-p_m)^2 + \cdots,$$

$$\Omega_r^{(p_m)} = \frac{1}{r!}\frac{d^r\Omega^{(p)}}{dp^r}\bigg|_{p=p_m}. \tag{13}$$

The first term represents the rapid oscillations of this fictitious system while the remaining terms represent their envelope. If $|\Omega_1^{(p_m)}| \gg |\Omega_k^{(p_m)}|$, $k=2,3,\ldots$, the linear term is the source of collapses and revivals of the oscillations. The revival time is estimated by the standard argument [26] that at least the oscillations with the nearest-neighbor frequencies, contributing to the evolution with the greatest weights, are in phase. In the linear approximation, irrespective of whether p_m is an integer or half-integer, the following formula is universal if both odd and even p's contribute to the evolution (e.g., the coherent JCM):

$$|\Omega^{(p_m+1)} - \Omega^{(p_m)}|T_R \approx |\Omega_1^{(p_m)}|T_R = 2\,\pi. \tag{14}$$

One can check that the terms oscillating with the frequencies $\Omega^{(p+l)}$ and $\Omega^{(p)}$ (l is an arbitrary integer) will also be phased at T_R provided all higher-order nonlinear terms in the expansion (13) are negligible. In this pure linear (harmonic) approximation the revivals of the system are perfectly periodic [27]. If higher-order derivatives in Eq. (13) are significant, they additionally spread revivals arising from the linear expansion and, in particular, lead to their incompleteness as in the resonant coherent JCM [26] or, at another photon-number distribution function, additionally contribute to the emergence of ringing revivals as in the JCM coupled to an initially strongly squeezed coherent field [28].

The behavior of an observable is quite different if the first derivative of the frequency vanishes: $\Omega_1^{(p_m)}=0$. In general, one can consider revivals of the kth order if all frequencies $\Omega_r^{(p_m)}$, $r=1,2,\ldots k-1$ are zero and the first nonvanishing derivative is $\Omega_k^{(p_m)}$. Then the shape of the revivals obviously

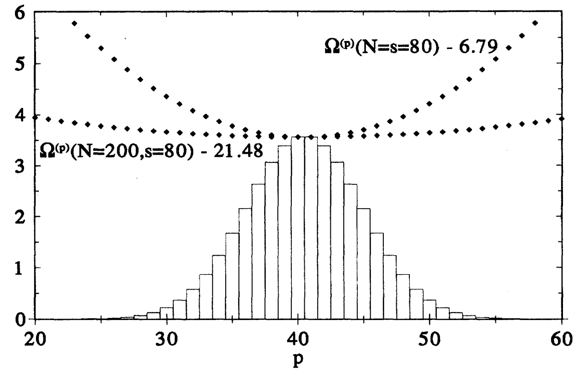

FIG. 1. Distribution C_p^s (12) (bar graph) and frequencies $\Omega^{(p)}$ (10) for $N=80$ and 200, $s=80$, and $g=1$.

undergoes a modification. Moreover, the formula (14) is not valid anymore. Additionally, the revival time has to be estimated separately for integer and half-integer numbers of p_m. The latter point is especially important as it leads to a qualitatively different outlook on the phenomenon of revivals. In particular, if the frequency $\Omega_2^{(p_m)}$ is the first nonzero one, we are faced with second-order revivals as in the JCM in the presence of a nonlinear Kerr medium and the phenomena caused by such anharmonicities have been termed "superstructures" [29]. In fact, in our present investigations we deal with collapses and revivals of this kind.

Before proceeding to a more detailed discussion let us note what follows. The distribution C_p^s has its maximum at $p_m=(s+1)/2$ for odd s while for even s it reaches two identical maximal values at $p_m=s/2$ and $p_m=(s+2)/2$. The Dicke frequency (10) has a form parabolic at p and reaches its minimum (minima) at the same point (points) at which the distribution has its maximum (maxima). Their plots for $s=80$ and $N=200$ and $N=80$ versus p are presented in Fig. 1. For clarity of the graph the values of the frequencies have appropriately been diminished by the values presented on the graph. From Eqs. (8) and (9) it is clear that the time evolution of the model is mainly governed by the eigenvectors with the smallest absolute values of the eigenfrequencies [16,30,31]. Studies of the coherent JCM have shown that its most interesting dynamics occurs when the nonlinearity of the Rabi frequency is slight. In the resonant JCM this takes place for great numbers of initially coherent photons [26,32]. Here the source of the spread is different. However, similarly, one can expect a regular dynamical behavior of the system if the Dicke frequency is only slightly nonlinear in the vicinity of the maximum of the binomial distribution (Fig. 1, N=200, s=80). For a given s its nonlinearity diminishes as N grows. For s extremely small in comparison with N, the dispersion curve of the frequency (10) reduces to a straight line and the time evolution of the system is periodic (approximation 00). Distinct collapses and revivals ("the superstructures") become more evident with growing s (but still less than N; Figs. 2 and 3). If s is comparable with N, the nonlinearity of the Dicke frequency (10) becomes significant and more noticeable; the frequency then varies rapidly in the vicinity of the maximum of the binomial distribution (Fig. 1, $s=N=80$) and collapses and revivals of

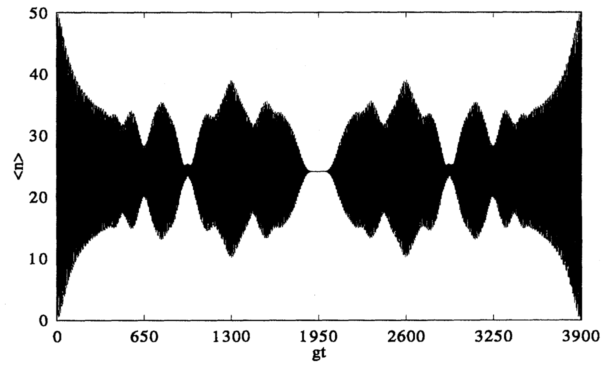

FIG. 2. Superstructures for $N=200$ and $s=50$.

the oscillations are not as clear as in the case $s<N$. Computer simulations show that the evolution is rather characterized by modulated oscillations [21]. By definition, our method is unable to describe correctly the evolution of the model for $s\approx N$. Contrary to the computer simulations it still gives distinct collapses and revivals. Moreover, for $N\geqslant 10$ if s becomes very close to N, the formula (8) gives for some time instants unphysical, slightly negative values of the mean photon number (e.g., if $N=20$ this occurs for $17\leqslant s\leqslant 20$).

Now we shall estimate the revival time. To achieve this goal we may neglect in Eq. (8) the contributions proportional to approximately ϵ and take into account only the terms $\cos\Omega^{(p)}t$. Due to the form of the frequency function (10), or as evident from Fig. 1, the pairs of the cosines functions with the frequencies $\Omega^{(p)}$ and $\Omega^{(s+1-p)}$ always oscillate in phase and each constituent of the couple contributes to the evolution with the same weight. The Dicke frequency, considered as a continuous function, has its minimum at $p_m=(s+1)/2$, which simply means vanishing of its first-order derivative in this point. Hence the appearance of revivals has to be attributed to the second-order derivative of the frequency. In fact, all higher-order derivatives of the Dicke frequency (10) are equal to zero in the approximation considered and we deal here with pure second-order revivals

$$\Omega^{(p)}=\Omega_0^{(p_m)}+\Omega_2(p-p_m)^2,$$

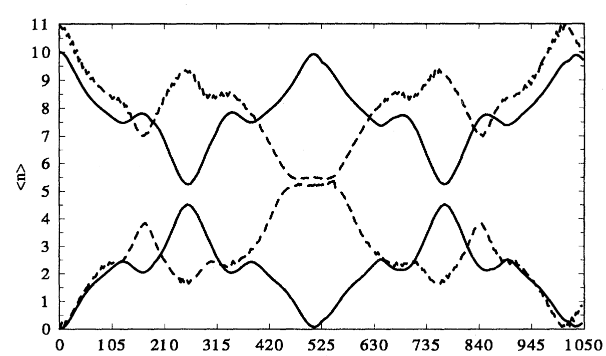

FIG. 3. Envelopes of the mean photon number for $N=50$: $s=11$, dashed line; $s=10$, solid line.

$$\Omega_2=15g\epsilon^{3/2}/8=15g^4/\Omega^3. \tag{15}$$

Since the second-order derivative is p independent, for the sake of simplicity the superscript at the frequency Ω_2 has been omitted. Revival occurs if at least the nearest-neighbor terms of the spread, contributing to the evolution with the greatest weights, oscillate in phase. For odd s, p_m is an integer number. As mentioned, the contributions with $p=p_m\pm 1$ are always in phase. Therefore we are interested in those with $\Omega^{(p_m+1)}$ and $\Omega^{(p_m)}$. They are in phase if

$$[\Omega^{(p_m+1)}-\Omega^{(p_m)}]T_R^{\text{odd } s}=\Omega_2 T_R^{\text{odd } s}=2\pi. \tag{16}$$

If the above condition is satisfied then, in fact, all terms engaged in the evolution acquire a common phase at this time

$$\Omega^{(p+l)}-\Omega^{(p)}=\Omega_2\mathcal{T},\quad \mathcal{T}=2l(p-p_m)+l^2$$

where $\mathcal{T}$ denotes an integer, and completeness of the revivals is expected, attributable to the vanishing of all higher-order derivatives. From (16) the revival time reads

$$T_R^{\text{odd } s}=\frac{16\pi}{15g}(N-s/2+1/2)^{3/2}. \tag{17}$$

For even s, if we treat the frequency as a continuous function, it takes a single extremum in the point $p_m=(s+1)/2$, as for odd s, but now p_m is a half-integer. The two most heavily weighted terms, corresponding to $p=p_m\pm 1/2$, are in phase during the whole evolution and in order to estimate the revival time, one has to discuss the contributions related to $p=p_m+1/2$ and $p=p_m+3/2$,

$$[\Omega^{(p_m+\frac{3}{2})}-\Omega^{(p_m+\frac{1}{2})}]T_R^{\text{even } s}=2\,\Omega_2 T_R^{\text{even } s}=2\pi,$$

and finally we find

$$T_R^{\text{even } s}=\frac{8\pi}{15g}(N-s/2+1/2)^{3/2}. \tag{18}$$

It is easily verified that also in this case all terms $p=p_m+(2l+1)/2$ are in phase at this time instant. An intriguing feature of the model, overlooked by us earlier, is apparent: for a given N and for the two nearest-neighbor s, the revival time is almost twice as short for even s. In particular, for $N=50$ and $s=11$ the revival time amounts to $T_R^{\text{odd } s}\simeq 1011.5/g$, whereas for $N=50$ and $s=10$ it is equal to $T_R^{\text{even } s}\simeq 514/g$, in excellent agreement with the graph (Fig. 3).

As seen from Fig. 3 in both cases of odd and even s, the collapse times are equal to one half of the corresponding revival times

$$T_C^{\text{odd } s}=T_R^{\text{odd } s}/2,\quad T_C^{\text{even } s}=T_R^{\text{even } s}/2. \tag{19}$$

To make this result clear and to explain for small even s the significant amplitudes of "residual" oscillations in the quiescent period, we have to find which terms may acquire opposite phases. As previously, we consider odd s first. With regard to the symmetry of the problem, we take the differences of the frequencies only in one wing of the binomial distribu-

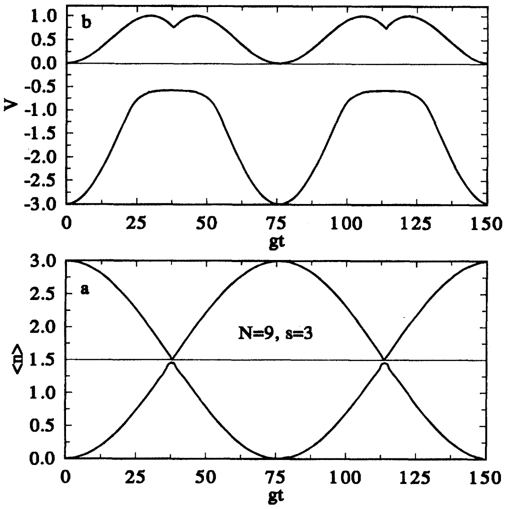

FIG. 4. Envelopes of the (a) mean photon number and (b) normally ordered variance for $N=9$ and $s=3$.

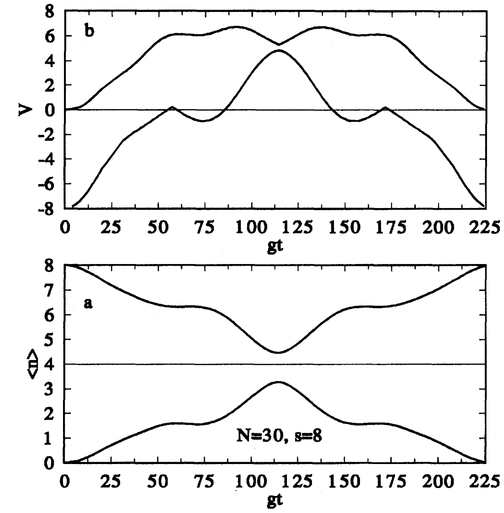

FIG. 5. Envelopes of the (a) mean photon number and (b) normally ordered variance for $N=30$ and $s=8$.

tion. An attenuation of the oscillation amplitudes related to $\cos\Omega^{(p)}t$ would occur if at least the contributions

$$[\Omega^{(p_m-1)}-\Omega^{(p_m)}]T_C^{\text{odd } s}=\Omega_2 T_C^{\text{odd } s}=\pi$$

were out of phase. Having determined the collapse time $T_C^{\text{odd } s}$ by the above condition, let us note that, in fact, all nearest-neighbor contributions then acquire opposite phases

$$[\Omega^{(p_m-k)}-\Omega^{[p_m-(k-1)]}]T_C^{\text{odd } s}=(2k-1)\pi,$$

$$k=1,2,\ldots,(s-1)/2.$$

In other words, all terms with odd p's are in phase at this time and remain out of phase with those corresponding to even p's. From the properties of the binomial distribution one can check that the total statistical weights of these two groups terms are identical. In consequence, the resulting oscillation amplitude vanishes, which is equivalent to

$$\sum_{p=1}^{s}(-1)^{p-1}C_p^s\equiv 0.$$

Hence some residual oscillations of the mean photon number for small odd s in the quiescent period (Fig. 4) may solely be attributed to the terms connected with the first-order approximation for the eigenvectors.

The discussion of the collapse time for even s follows the same lines as above. There are also two groups of the terms oscillating with opposite phases at $T_C^{\text{even } s}$. In this case, however, the two greatest equally weighted contributions with $p=s/2$ and $p=(s+2)/2$ always remain in phase. Because of the correlation of the greatest contributions, the summarized weights of the two cosines with opposite signs are now different and this difference reads

$$\Delta=2\sum_{p=1}^{s/2}(-1)^{\frac{s}{2}-p}C_p^s. \tag{20}$$

With respect to the general relation

$$\sum_{r=0}^{m}(-1)^{m-r}\binom{n}{r}=\binom{n-1}{m},$$

from (20), after simple algebra, one gets

$$\Delta=\frac{s}{2^{s-1}}\binom{s-2}{\frac{s}{2}-1}. \tag{21}$$

So the oscillations of the mean photon number, connected with the zeroth-order approximation for the eigenvectors, do not collapse totally now. The magnitude of Δ increases with growing s (for large s it is approximately equal to $\sqrt{s/2\pi}$), but the relative quantity Δ/s decreases. Hence the amplitudes of residual oscillations during the quiescent period, so significant on the graphs for $s=4$ and $s=6$ (see Figs. 4 and 5 presented in Ref. [13]), monotonically diminish with growing s (Figs. 5 and 3) and, for instance, for $s=50$ are already unnoticeable on the scale of the graph (Fig. 2).

V. PHOTON-NUMBER STATISTICS

The second-order factorial moment of the photon-number operator, i.e., the second-order normally ordered field correlation function $G^{(2)}$, is, by definition,

$$G^{(2)}(t)=\langle a^\dagger a^\dagger aa\rangle=\langle n^2(t)\rangle-\langle n(t)\rangle, \tag{22}$$

while the normalized second-order coherence degree $g^{(2)}$ is

$$g^{(2)}(t)=\frac{G^{(2)}(t)}{\langle n(t)\rangle^2}=1+\frac{\langle[\Delta n(t)]^2\rangle-\langle n(t)\rangle}{\langle n(t)\rangle^2}, \tag{23}$$

where $\langle[\Delta n(t)]^2\rangle=\langle n^2(t)\rangle-\langle n(t)\rangle^2$ is the photon-number variance. A very convenient quantity, often used, is the normally ordered photon-number variance V

$$V(t)=G^{(2)}(t)-\langle n(t)\rangle^2=\langle[\Delta n(t)]^2\rangle-\langle n(t)\rangle. \quad (24)$$

The values $g^{(2)}<1$ or $\langle(\Delta n)^2\rangle<\langle n\rangle$ or simply $V<0$ indicate sub-Poissonian fields. Such fields have no classical counterparts (for a review see Peřina [33]).

At least two photons are necessary to have the second-order correlation function (22) be nonzero. Hence it is obvious that for $s=1$ this function is just equal to zero because of the presence of one photon only at every atomic jump. In turn, for $s=2$ the number of photons in the cavity can be at most equal to 2 and the normally ordered variance does not exceed 0. The field remains sub-Poissonian during the whole time evolution. However, for $s=3$ the photon-number statistics already oscillates, in general, between sub-Poissonian and super-Poissonian. The graphs in Fig. 4 represent the envelopes of the oscillations of the mean photon number and normally ordered variance (8), (11), and (24) for $s=3$ and $N=9$. The mean photon number collapses and only residual oscillations during the short quiescent period, related to the first-order approximation for the eigenvectors, are present. The oscillations of the variance do not reveal collapse but only a decrease of their amplitude. When the oscillations of the mean photon number completely revive the photon number statistics tends to be sub-Poissonian only (time intervals around $gt\approx 75$ and 150). The revival time of the oscillations of the mean photon number, calculated from the formula (17), is equal to $T_R\simeq 76/g$, in full agreement with the graph. Subsequent revivals will occur at $t=kT_R$ ($k=2,3,\ldots$). The oscillations of both quantities are of irregular nature during collapses of $\langle n\rangle$ because of only the two-frequency beat mechanism in this case. For the clarity of both graphs the local maxima and minima in these time regions have been cut out. In Fig. 5 the envelopes of the oscillations of the same quantities are plotted for $s=8$ and $N=30$. The revival time of the mean photon number given by the formula (18) amounts to $T_R^{\text{even } s}\simeq 228.5/g$. In this case a greater number of the frequencies (3) intervene. Therefore "residual" oscillations, the amplitudes of which are now significant (even s), have a regular character. Approximately, in the time interval $85\leqslant gt\leqslant 135$ the photon number statistics remains super-Poissonian solely. The shift of the quasistationary value of $\langle n\rangle$ below $s/2=4$ is well noticeable.

In the zeroth-order approximation for both the eigenvectors and eigenfrequencies from (12) one finds a rather interesting result, namely, the time behavior of $G_{00}^{(2)}(t)$, $\langle n(t)\rangle_{00}^2$, and $V_{00}(t)$ is identical. Only the oscillations amplitudes are different. The second-order coherence degree (23) becomes time independent in this approximation $g_{00}^{(2)}=1-1/s$ while the photon number variance has the form:

$$\langle[\Delta n(t)]^2\rangle_{00}=\frac{s}{8}(1-\cos 2\,\Omega t). \quad (25)$$

The emitted field is sub-Poissonian for all times. In other words, the equidistance of the eigenvalues spectra and vanishing of the first-order corrections to the eigenvectors are the necessary conditions for having nonclassical field statistics in the spontaneous emission of the Dicke model during the whole evolution of the system. The atomic system then behaves like an oscillator. This simple result is, in fact, nontrivial. Namely, the spontaneous emission from a partially inverted Dicke model is a potential source of generation of the s-photon Fock state if $s\ll N$. In fact, for time instants such that $\Omega t=(2k+1)\pi$ ($k=0,1,\ldots$) the mean photon number (12) amounts to s, whereas $\langle(\Delta n)^2\rangle_{00}$ is equal to 0. The latter is just the immanent feature of the photon-number state, in this case the s-photon Fock state.

Photon-number states are commonly used in the description of quantum fields. On the other hand, their practical realization is by no means a trivial task. Hong and Mandel [34] have experimentally realized the one-photon state in spontaneous parametric down-conversion. A highly excited Fock state may be created in a kicked cavity into which two-level atoms are injected [35]. Brune *et al.* [36] have proposed a method for the preparation of such a state based on quantum nondemolition measurement. Quite recently, Leoński and Tanaś [37] have pointed to the possibility of obtaining the one-photon state in a cavity periodically kicked with classical pulses and containing a nonlinear Kerr medium, while Liebman and Milburn [38] have proposed creating number states in the micromaser using feedback. The spontaneous emission of the Dicke model in a cavity, if only a small fraction of the total number of atoms is initially excited, also offers production of the s-photon number state.

VI. CONCLUSIONS

We have analytically investigated the time evolution of the mean photon number and the photon-number statistics for the process of spontaneous emission from a collection of symmetrically excited two-level atoms in the presence of a large number of the same unexcited atoms inside a high-Q cavity. The photon-number evolution exhibits collapses and revivals that are not related to the initial photon distribution but arise from the cooperativity of the system. Therefore, they differ strongly from the familiar collapses and revivals of the coherent JCM by both their origin and shape. The revival shape is reminiscent of "superstructures" or "strange revivals" [29]. The collective revivals are very sensitive to the parity of the initial number of atomic excitations and the intriguing thing is that the collapse and revival times for the odd initial number of exited atoms s are approximately twice those for the nearest-neighbor even s. The collapse and revival times are not mutually independent parameters (as they are, e.g., in the coherent JCM, where the collapse time is a function of the photon-number distribution width). In our case, the width of the binomial distribution does not affect the collapse time. The collapse (revival) time becomes shorter with growing s. For a given N and even s the greatest collapse (revival) time obviously occurs for $s=4$. There is no odd number s for which at the same N the collapse (revival) time would be at least equal to that time; as for any odd s it is simply always greater than that corresponding to $s=4$.

One way of preparing the initial state of the atomic system considered here is as follows. First, let the system of the unexcited atoms be effectively inverted by a π pulse. This fully inverted state is symmetric with respect to the permutation of the atoms. The system starts to radiate before it enters the cavity. The evolution goes through symmetric states only. By controlling the time elapse between the inverting irradiation and entering the cavity by the atoms we can get the symmetric Dicke state of the system with a certain, greater or smaller, number of excited atoms. The above controling we also understand as a measurement of the number of photons emitted by the system which permits determination of the number of atoms s that remain excited. In the main measurements of the collapse (revival) times, significantly different values are archieved for similarly prepared systems, depending on the parity of s.

The photon-number statistics can be essentially sub-Poissonian for a wide time range. Moreover, the field state becomes almost a number state (with s photons) at some time instants. Hence the Dicke model reveals the possibility of generation of the number state of the electromagnetic field.

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