Noise-free amplification of squeezed light via atomic coherence

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We derive general solutions for the two-mode operators after two laser modes interact with a beam of atoms coherently prepared in the lower levels in the Λ scheme. It is shown that under certain conditions, a "noise-free" energy transfer can take place from a strong coherent mode to a weak squeezed-field mode.

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I. INTRODUCTION

Recent interest in the quantum theories of linear optical amplifiers stems from their potential use in optical communication and the amplification of nonclassical light, e.g., squeezed states of the radiation field. It has been known that a linear amplifier not only amplifies the signal, but also adds noise. This added noise places limits on the performance of the amplifiers. For example, it has been shown that a phase-insensitive amplifier does not preserve squeezing if the gain is larger than 2 [1]. However, in a phase-sensitive amplifier, it should be possible to amplify the signal in such a way that no noise is added in the quadrature of interest and all the noise is fed into the conjugate quadrature [1,2]. Schemes based on a two-photon amplifier operating in a three-level atomic system in cascade configuration with upper and lower atomic states for such "noise-free amplification" have been proposed [3]. Other systems based on rigged reservoir [4] and parametric schemes [5] to achieve phase-sensitive amplification have also been studied and implemented. In these phase-sensitive amplifiers, even when no noise is added in the quadrature of interest, the amplification process itself degrades the amount of squeezing in the output. In this paper, we consider a two-mode linear amplifier operating on a three-level atomic system in Λ configuration, in which a "noise-free" energy transfer from a strong coherent field to a weak field in squeezed state can take place under certain conditions, thus leading to a noise-free amplification of squeezed light. Such a system has been studied previously in the context of noise-free energy transfer between two coherent modes in Ref. [6].

In general, the exchange of energy from one mode to another cannot be represented by unitary transformation. For example, the exchange between two field modes via interactions with an atomic beam will create entanglement between the field modes and the atoms. Such entanglement has been studied in the context of state reduction by continuous measurement [7,8]. In this paper we examine the evolution of the two-mode field density matrix, without selecting a particular atomic state. We are able to reach simple, general results which indicate not only an appreciable amount of energy transfer, but also a transfer of field fluctuations from one mode to the other. For one mode in a strong coherent state and the other in a weak squeezed state, a substantial energy transfer can take place from the coherent mode to the squeezed mode, and in the process, a significant amount of squeezing is transferred from the squeezed mode to the coherent mode. Under certain conditions the energy transfer can be made noise-free.

The paper is organized as follows. In Sec. II we derive the field master equation for atoms prepared in the two lower states of the Λ configuration. In Sec. III we discuss the incoherent superposition of lower states and show that there is no exchange between the two modes. In Sec. IV the evolution of quadrature expectations and their variances in both modes is derived for coherent superposition of lower states, and we analyze the approach to achieve noise-free energy transfer from one mode to the other. Section V contains a summary and discussion.

II. MODEL AND EQUATIONS

We consider the resonant interaction of two laser beams with an atomic beam of the Λ-type atoms as described in Ref. [8] (see Fig. 1). We assume that no more than one atom interacts with the laser beams at any given time. The atom is initially in the two lower levels $|b\rangle$ and $|b'\rangle$. Each of the two field modes couples one lower level (but not the other) to the upper level $|a\rangle$. The Hamiltonian in the interaction picture is

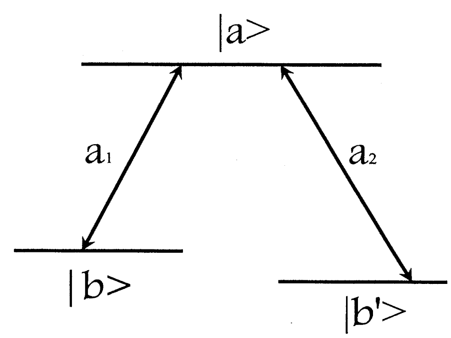

FIG. 1. Three-level atom in coherent superposition of two lower states. The two laser fields couple the lower levels $|b\rangle,|b'\rangle$ to the upper level $|a\rangle$.

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$$H_I = g a_1 |a\rangle\langle b| + g a_2 |a\rangle\langle b'| + \text{H.c.}, \tag{1}$$

where a_1 and a_2 are the annihilation operators for the two laser modes. Under this interaction Hamiltonian the evolution of the joint field-atom density matrix is, up to the second order of the interaction time τ,

$$\rho_{AF}(\tau) - \rho_{AF}(0) = -i\tau[H_I, \rho_{AF}] - \frac{\tau^2}{2}[H_I,[H_I,\rho_{AF}]], \tag{2}$$

where the initial joint density matrix $\rho_{AF}(0) = \rho_F(0)\rho_A(0)$,

$$\rho_A(0) = \rho_{bb}|b\rangle\langle b| + \rho_{b'b'}|b'\rangle\langle b'| + \rho_{bb'}|b'\rangle\langle b| + \rho_{b'b}|b\rangle\langle b'|. \tag{3}$$

Substituting Eqs. (1) and (3) into Eq. (2) and taking trace over the atomic states, we find the evolution of the field density matrix:

$$\begin{aligned}\rho_F(t) = \rho_F(0) - \frac{\tau^2|g|^2}{2}\{&(\rho_{bb}a_1^\dagger a_1 + \rho_{bb'}a_2^\dagger a_1 + \rho_{b'b}a_1^\dagger a_2 \\ &+ \rho_{b'b'}a_2^\dagger a_2)\rho_F(0) + \rho_F(0)(\rho_{bb}a_1^\dagger a_1 + \rho_{bb'}a_2^\dagger a_1 \\ &+ \rho_{b'b}a_1^\dagger a_2 + \rho_{b'b'}a_2^\dagger a_2) - 2[\rho_{bb}a_1\rho_F(0)a_1^\dagger \\ &+ \rho_{bb'}a_1\rho_F(0)a_2^\dagger + \rho_{b'b}a_2\rho_F(0)a_1^\dagger \\ &+ \rho_{b'b'}a_2\rho_F(0)a_2^\dagger]\}. \end{aligned} \tag{4}$$

If the injection rate of atoms is r, and $r\tau \ll 1$, then we obtain the time derivative of the field density matrix, i.e., the master equation from Eq. (4):

$$\begin{aligned}\dot{\rho}_F = -2R[&(\rho_{bb}a_1^\dagger a_1 + \rho_{bb'}a_2^\dagger a_1 + \rho_{b'b}a_1^\dagger a_2 + \rho_{b'b'}a_2^\dagger a_2)\rho_F \\ &+ \rho_F(\rho_{bb}a_1^\dagger a_1 + \rho_{bb'}a_2^\dagger a_1 + \rho_{b'b}a_1^\dagger a_2 + \rho_{b'b'}a_2^\dagger a_2) \\ &- 2(\rho_{bb}a_1\rho_F a_1^\dagger + \rho_{bb'}a_1\rho_F a_2^\dagger + \rho_{b'b}a_2\rho_F a_1^\dagger \\ &+ \rho_{b'b'}a_2\rho_F a_2^\dagger)]. \end{aligned} \tag{5}$$

Here $R = |g|^2\tau^2 r/4$.

III. INCOHERENT INJECTION

For atoms initially in incoherent superposition of $|b\rangle$ and $|b\rangle$, the off-diagonal elements $\rho_{bb'}$ and $\rho_{b'b}$ vanish. From the general formula Eq. (5) we get

$$\begin{aligned}\dot{\rho} = -2R(\rho_{bb} + \rho_{b'b'})[&(a_1^\dagger a_1\rho_F + \rho_F a_1^\dagger a_1 - 2a_1\rho_1\rho_F a_1^\dagger) \\ &+ (a_2^\dagger a_2\rho_F + \rho_F a_2^\dagger a_2 - 2a_2\rho_F a_2^\dagger)]. \end{aligned} \tag{6}$$

The master equation is additive. For each mode the atoms act as an absorber in a two-level model, therefore the two modes will decay independently at the same rate.

IV. COHERENT INJECTION

Next we consider injected atoms in coherent superposition of $|b\rangle$ and $|b'\rangle$, $\rho_A(0) = |\psi_0\rangle\langle\psi_0|$, where

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|b\rangle + e^{-i\phi}|b'\rangle). \tag{7}$$

From Eq. (5) we find that the field master equation can be written in the form

$$\dot{\rho}_F = -2R(A_1^\dagger A_1\rho_F + \rho_F A_1^\dagger A_1 - 2A_1\rho_F A_1^\dagger). \tag{8}$$

Here we define the mode operators

$$A_{1,2} = \frac{a_1 \pm a_2 e^{-\phi}}{\sqrt{2}}, \tag{9a}$$

$$a_1 = \frac{A_1 + A_2}{\sqrt{2}}, \tag{9b}$$

$$a_2 = \frac{(A_1 - A_2)e^{-i\phi}}{\sqrt{2}}. \tag{9c}$$

The atom is only coupled to the fields through the mode operators A_1 and $A_1^\dagger$. If the initial field amplitudes for the two modes are α_1, α_2, then under the condition

$$\alpha_1 + \alpha_2 e^{i\phi} = 0 \tag{10}$$

the two modes are effectively decoupled from the atom, and the amplitudes will not change. In the following we assume $\phi = 0$. For any arbitrary operator of normally ordered form,

$$Q = (A_1^\dagger)^p (A_2^\dagger)^r A_1^q A_2^s, \tag{11}$$

we find that

$$\frac{d}{dt}\langle Q\rangle = -2R(p+q)\langle Q\rangle, \tag{12a}$$

$$\langle Q(t)\rangle = e^{-2R(p+q)t}\langle Q(0)\rangle. \tag{12b}$$

Similarly, we can write the time dependence of any normally ordered operators of the original modes as

$$\begin{aligned}\langle (a_1^\dagger)^p (a_2^\dagger)^r a_1^q a_2^s\rangle(t) = \Bigg\langle &\left[\frac{a_1^\dagger(1+e^{-2Rt}) - a_2^\dagger(1-e^{-2Rt})e^{-i\phi}}{2}\right]^p \left[\frac{a_2^\dagger(1+e^{-2rt}) - a_1^\dagger(1-e^{-2Rt})e^{i\phi}}{2}\right]^r \\ &\times \left[\frac{a_1(1+e^{-2Rt}) - a_2(1-e^{-2Rt})e^{i\phi}}{2}\right]^q \left[\frac{a_2(1+e^{-2Rt}) - a_1(1-e^{-2Rt})e^{-i\phi}}{2}\right]^s \Bigg\rangle. \end{aligned} \tag{13}$$

The quadrature operators are defined as follows:

$$x_1=\frac{a_1+a_1^\dagger}{2},\quad x_2=\frac{a_2e^{i\phi}+a_2^\dagger e^{-i\phi}}{2};$$

$$y_1=\frac{a_1-a_2^\dagger}{2i},\quad y_2=\frac{a_2e^{i\phi}-a_2^\dagger e^{-i\phi}}{2i};$$

$$X_1=\frac{A_1+A_1^\dagger}{2},\quad X_2=\frac{A_2+A_2^\dagger}{2};$$

$$Y_1=\frac{A_1-A_1^\dagger}{2i},\quad Y_2=\frac{A_2-A_2^\dagger}{2i}. \tag{14}$$

Using the result in Eq. (12b), and noticing that the quadrature operators must satisfy proper commutation relations, we can write

$$X_1(t)=X_1(0)e^{-2Rt}+\sqrt{1-e^{-4Rt}}F_X, \tag{15a}$$

$$Y_1(t)=Y_1(0)e^{-2Rt}+\sqrt{1-e^{-4Rt}}F_Y, \tag{15b}$$

$$X_2(t)=X_2(0), Y_2(t)=Y_2(0). \tag{15c}$$

The random noise operators F_X, F_Y satisfy $\langle F_X\rangle=\langle F_Y\rangle=\langle F_XF_Y\rangle=0$, and $\langle F_X^2\rangle=\langle F_Y^2\rangle=\frac{1}{4}$. Consequently, the interaction with the atomic beam introduces noises to the laser beams:

$$x_{1,2}(t)=x_{1,2}(0)\frac{1+e^{-2Rt}}{2}-x_{2,1}(0)\frac{1-e^{-2Rt}}{2}+\left(\frac{1-e^{-4Rt}}{2}\right)^{1/2}F_X, \tag{16a}$$

$$y_{1,2}(t)=y_{1,2}(0)\frac{1+e^{-2Rt}}{2}-y_{2,1}(0)\frac{1-e^{-2Rt}}{2}+\left(\frac{1-e^{-4Rt}}{2}\right)^{1/2}F_Y. \tag{16b}$$

The variances of these quadrature operators become mixed; for example,

$$\begin{aligned}\langle(\Delta x_{1,2}(t))^2\rangle=&\tfrac{1}{4}(1+e^{-2Rt})^2\langle(\Delta x_{1,2}(0))^2\rangle\\&+\tfrac{1}{4}(1-e^{-2Rt})^2\langle(\Delta x_{2,1}(0))^2\rangle-\tfrac{1}{2}(1-e^{-4Rt})\\&\times\langle\Delta x_1(0)\Delta x_2(0)\rangle+\tfrac{1}{8}(1-e^{-4Rt}),\end{aligned} \tag{17a}$$

$$\begin{aligned}\langle\Delta x_1(t)\Delta x_2(t)\rangle=&-\tfrac{1}{4}(1-e^{-4Rt})[\langle(\Delta x_1(0))^2\rangle\\&+\langle(\Delta x_2(0))^2\rangle]+\tfrac{1}{2}(1+e^{-4Rt})\\&\times\langle\Delta x_1(0)\Delta x_2(0)\rangle+\tfrac{1}{8}(1-e^{-4Rt}).\end{aligned} \tag{17b}$$

We emphasize here that these results are applicable to arbitrary initial two-mode states, that the two modes are not necessarily decoupled, not do they need to be in a pure state. When the two inputs are both coherent states as discussed in Ref. [8], the outputs remain coherent states. Thus neither $\langle(\Delta x_1)^2\rangle$, $\langle(\Delta x_2)^2\rangle$, nor $\langle\Delta x_1\Delta x_2\rangle$ will be changed, and the interaction with the atoms neither increases nor decreases the noise. However, when one input is a coherent state and the other is a squeezed vacuum, we see the noise in one quadrature of the former can be reduced. With the coherent-state input $\langle(\Delta x_2)^2\rangle=\frac{1}{4}$, we get from Eq. (17a)

$$\langle(\Delta x_1(t))^2\rangle=\tfrac{1}{4}+\tfrac{1}{4}(1+e^{-2Rt})^2[\langle(\Delta x_1(0))^2\rangle-\tfrac{1}{4}], \tag{18a}$$

$$\langle(\Delta x_2(t))^2\rangle=\tfrac{1}{4}+\tfrac{1}{4}(1-e^{-2Rt})^2[\langle(\Delta x_1(0))^2\rangle-\tfrac{1}{4}]. \tag{18b}$$

Hence the quadrature x_2 will be squeezed if the quadrature x_1 is squeezed initially. In the mean time, x_1 will become less squeezed. In the limit $Rt\to\infty$ both modes are squeezed to the same amount, which has the maximum value of 25%:

$$\langle(\Delta x_1)^2\rangle_{t\to\infty}=\langle(\Delta x_2)^2\rangle_{t\to\infty}=\tfrac{1}{4}[\langle(\Delta x_1(0))^2\rangle+\tfrac{3}{4}]. \tag{19}$$

In the mean time, the amplitude of the coherent-state mode is reduced, while the squeezed vacuum gains a nonzero amplitude according to Eq. (16a):

$$\langle x_1(t)\rangle=\frac{-1}{2}(1-e^{-2Rt})\langle x_2(0)\rangle, \tag{20a}$$

$$\langle x_2(t)\rangle=\tfrac{1}{2}(1+e^{-2Rt})\langle x_2(0)\rangle. \tag{20b}$$

So in the limiting case we can preserve 50% of the input coherent-state amplitude, and apply up to 25% squeezing to it.

For the prospect of obtaining noise-free amplification, we noticed that one can start with a squeezed input of small amplitude for mode 1, and an unsqueezed coherent state of large amplitude for mode 2; as in the above discussion, the quadrature variance does not depend on the amplitudes; it is possible to achieve a larger amplitude for the squeezed mode while preserving its squeezed nature.

Assuming $Rt\gg 1$, which can be realized by adjusting τ and r, we can write Eq. (18a) as

$$\begin{aligned}\langle(\Delta x_1(t))^2\rangle&=\langle(\Delta x_1(0))^2\rangle+2Rt[\tfrac{1}{4}-\langle(\Delta x_1(0))^2\rangle]\\&\approx\langle(\Delta x_1(0))^2\rangle.\end{aligned} \tag{21}$$

This means that the squeezed mode can preserve its squeezed nature, or more precisely, the increase of noise is negligible. From Eq. (20a) we have

$$\langle x_1(t)\rangle=Rt\langle x_2(0)\rangle. \tag{22}$$

For large initial coherent state of mode 2, we can have $Rt\langle x_2(0)\rangle=\text{const}\gg 1$. That is to say, the amplitude of mode 1 (initially in squeeze state with very small amplitude) can be amplified to an arbitrary value while retaining its original low noise.

V. DISCUSSION

The method described in this paper uses atomic coherence to transfer energy from one mode to another while eliminating the noise transferred in the mean time. To illustrate this point we used coherent-state and squeezed-state inputs in the above discussion; however, noise-free transfer is not restricted to such inputs. We notice that $\langle(\Delta x_1(t))^2\rangle \approx \langle(\Delta x_1(0))^2\rangle$ for $Rt \ll 1$, with arbitrary inputs from Eq. (17a). Further, it must be pointed out that the output in either mode is not a squeezed state. From Eq. (17b) one finds

$$\langle\Delta x_1(t)\Delta x_2(t)\rangle = -\tfrac{1}{4}(1-e^{-4Rt})[\langle(\Delta x_1(0))^2\rangle - \tfrac{1}{4}]. \tag{23}$$

x_1 and x_2 are either correlated [if $\langle(\Delta x_1(0))^2\rangle - \frac{1}{4} > 0$] or anticorrelated [if $\langle(\Delta x_1(0))^2\rangle - \frac{1}{4} < 0$]. The correlation coefficient

$$C(x_1, x_2)_{t\to\infty} = \frac{\langle\Delta x_1 \Delta x_2\rangle}{\sqrt{\langle(\Delta x_1)^2\langle(\Delta x_2)^2\rangle}}(t\to\infty) = \frac{-[\Delta^2 x_1(0) - \frac{1}{4}]}{\Delta^2 x_1(0) + \frac{1}{4}}. \tag{24}$$

We also point out that the change of the two-mode state is the result of interaction with the atoms, which not only changes the field modes but the atoms as well. In deriving the field master equation we have assumed that the atoms are not collected and observed. Thus it differs from the schemes that use the detection of atoms to realize field state changes. Our results given here can be interpreted as the weighted averages over all possible number of atoms in the excited state. In each such case it is also possible to transfer squeezing from one mode to another as well as amplitude. However, no general formalism exists for these projected field states corresponding to a different number of excited atoms; they have to be studied case by case. These studies will be presented separately.

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