Theory of phase locking of globally coupled laser arrays

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We have carried out an investigation of the globally coupled laser array with randomly spread eigenfrequencies. It is shown that as this spread increases, either the laser action vanishes or the coherent steady-state regime is replaced by a phase-unlocked regime. The regions of the stable phase locking have been analytically and numerically determined. It is found that the gain inertia leads to cooperative phase-locking effects when the maximum detuning is close to the relaxation frequency.

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I. INTRODUCTION

Many nonlinear systems [1,2] (coupled Josephson junctions, vortices in gas dynamics, neural networks, and evolution and economics models) belong to the class of "globally coupled" systems, i.e., with a feedback proportional to the mean value of the field over the entire system. One relatively simple example of this class is the "all-to-all" optically coupled laser array. This coupling can be experimentally provided with good approximation by placing a stop at the common focus of the array [3]. The investigation of such a configuration can clarify the complex dynamic behavior of the globally coupled arrays. Furthermore, this investigation is of practical importance since it gives the opportunity to attain high-power radiation with low angular divergence.

The attainment of this result requires the coherent emission of radiation from an N-laser array with the minimum phase spread at the output aperture. This mode can be achieved if the laser parameters, especially cavity eigenfrequencies, vary only slightly. At the present time only a static eigenfrequency spread has been taken into account. In particular for the case of the nearest-neighbor coupling it is shown in [4] that the breaking of the global order occurs through the creation of domains with phase differences between two adjacent ones close to π. The mean domain size is determined by the ratio between the optical coupling coefficient and the level of the eigenfrequency detunings. This correlation size restricts the divergence of the total output radiation by a value which does not depend on the size of the entire array. In [4,5] it is also shown that a small additional global coupling essentially expands the region of the phase-locking of all the lasers in the "nearest-neighbor" coupled array. In Ref. [5] the authors already noticed that the average field at the output aperture (which determines the system brightness) undergoes a phase transition. Precisely, the average field changes for an increase of detuning level as the magnetic moments in ferromagnetic media do for a temperature raise. The coherence (order parameter) violation can occur through creation of topological solitons initiated by eigenfrequency fluctuations.

The dynamics of the globally coupled array with negligible inertia of the active medium has been numerically studied in a recent work [6]. Reference [6] considers static variations of the eigenfrequencies with an almost Lorentzian distribution. The following regimes are numerically obtained: phase-locked, partially phase-locked (analogous to the domain regime in [4]), independent lasing of each laser, order-disorder oscillations. The authors of [6] found a sharp increase of the order-parameter itself, and this suggests an analogy with thermodynamic phase transitions. In that paper only regimes with large gain were studied, and lasing was possible at any level of detunings.

In this work we present an analytical study of the possible dynamic operations of the globally coupled laser array as well as a numerical simulation. Both analysis and simulation enable discrimination between transitions to phase-unlocked regimes, already introduced in [6] as phase locking–unlocking transition, and transitions to laser action damping. Explicit criteria for order violation are now determined for a wide range of gain values in the case of negligible gain medium inertia.

The active medium inertia can induce a complicated dynamic behavior as it is shown theoretically and experimentally in [7] for two coupled lasers. We show that in the case of globally coupled arrays, the active medium inertia causes an effect that we have called cooperative field phase locking (see Sec. III). This behavior is robust, since it persists also in the case of a small number of array elements and even for a long relaxation time of the active medium.

II. ANALYSIS OF PHASE-LOCKING REGIMES

Our approach to studying the field dynamics of optically globally coupled lasers is based on the following model. In the absence of coupling each laser operates in a single longitudinal and transverse mode, assumed to be the same for all the lasers, the gain is proportional to the population inversion on the two resonant levels (the two-level approach) and the only difference among the lasers in the array is in their cavity eigenfrequency. Generally speaking, the system of N optically coupled lasers has $3N$ degrees of freedom (N complex amplitudes E_k and N active medium gains g_k).

The system of nonlinear equations describing the dynamics of globally coupled lasers has the following form [3]:

$$\dot{E}_k=(g_k-1-M)E_k+i\Delta_k E_k+\frac{M}{N}\sum_{m=1}^{N} E_m, \quad (1)$$

$$\tau\dot{g}_k=g_0-g_k-|E_k|^2 g_k. \quad (2)$$

The following dimensionless variables are used here: $|E_k|^2=|\tilde{E}_k|^2 g_0/E_s^2 g_n$, $t=\tilde{t}g_n l/\tau_p$, $\Delta_k=\tilde{\Delta}_k\tau_p/g_n l$, $M=\tilde{M}/g_n l$, $g_0=\tilde{g}_0/g_n$, $\tau=\tilde{\tau}g_n l/\tau$, where N is the number of lasers in the system, $\tilde{E}_k$ is the complex amplitude of the field in the kth laser, E_s is the saturation field, $\tau_p=2L/c$ is the round-trip time of the single cavity, L is the cavity length, $g=\tilde{g}/g_n$, $\tilde{g}$ being the gain of the active medium and g_n the threshold gain, $\tilde{g}_0$ is the small-signal gain, l is the length of the active medium, $\tilde{\Delta}_k$ is the detuning of the cavity eigenfrequency from the mean frequency of the laser array, $\tilde{M}$ is the coefficient of the optical coupling (the same value for all lasers of the array), and $\tilde{\tau}$ is the active medium relaxation time.

The first term on the right-hand side of (1) governs gain and loss of the field per cavity trip (including coupling out of radiation to other lasers), the second term governs variation of the field phase (we assume $|\Delta_k\tau_p|\ll 1$), and the third term represents the injection of the mean field to the laser.

In Eq. (2), the first term on the right-hand side is the pump rate, the second term is the gain relaxation, and the last term governs the radiation induced decrease of the resonance level population.

In the case of the fast relaxation of the active medium, Eq. (2) gives

$$g(I_k)=\frac{g_0}{1+|E_k|^2}. \quad (3)$$

The time dependence is reduced to the same set of equations of Ref. [6] for low intensities. In the numerical simulations we assumed cavity eigenfrequency detunings to be uniformly distributed in the interval $[-\Delta_0/2, \Delta_0/2]$ (that differs from Lorentzian distribution used in [6]. Here Δ_0 is the distribution width. In the numerical simulation of (1) and (2), detunings Δ_k were determined by random number generator.

The coherence degree of the array fields can be characterized by the axial far-field radiation brightness which is proportional to the squared sum of individual near fields. At the low level of eigenfrequency detunings the phase of each laser field has a steady-state value which is different from the average field phase. The phase difference increases with the detuning level. The different kinds of coherence violation and consequent decrease of the system brightness for increasing detuning take place under different relations between the optical coupling strength M and the active medium gain g_0.

The time-average radiation brightness for a fixed value $M=0.1$ and different values of g_0 are shown in Fig. 1 as a function of Δ_0. The numerical simulations have been carried out for numbers of lasers $N=100$ and $N=1000$ showing that for $N>100$, the results do not significantly depend on different selections of random detunings.

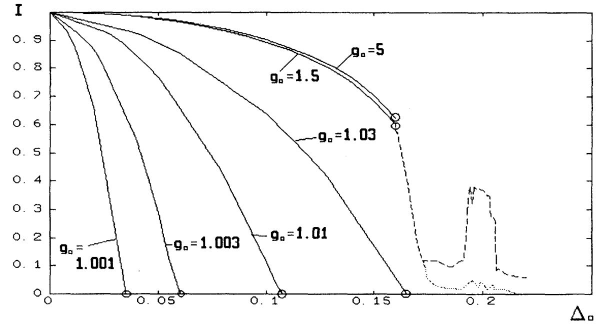

FIG. 1. The average field brightness as function of Δ_0 for $N=100$ at $M=0.1$ and different values of g_0: solid lines, coherent steady-state regimes; dashed line, dynamic regimes at $\tau=50$; dotted line, dynamic regimes in the case of the inertialess active medium. The vertical axis is normalized to the brightness of the in-phase locked array.

At small values of the small-signal gain g_0 the laser array operation remains coherent with the increase of the detuning distribution width (curves $g_0=1.001-1.03$ in Fig. 1 but the axial brightness decreases to zero. As it is seen from Eq. (1), when $g_0-1<M$, lasing of each single laser is impossible in the absence of the injection of the average field. Increasing Δ_0 the average field decreases and at some threshold value of Δ_0 all the lasers quit lasing.

A qualitatively different violation of the coherence occurs under large g_0 conditions. Indeed in the curves $g_0=1.5$ and $g_0=5$ of Fig. 1 a transition takes place from steady-state coherent emission to phase unlocking at some critical value of Δ_0. Phase unlocking leads to a significant decrease of the total array brightness. At the high pump rates the field amplitudes vary only slightly and the brightness decrease is caused by averaging of the randomly distributed and dynamically varying phases. This brightness dependence on the eigenfrequency spread is analogous to that numerically obtained in [6] in the case of a Lorentzian distribution.

To determine how the critical values of Δ_0 depend upon the array parameters, let us consider the conditions for coherent emission in more detail, both for low and high gains and for different eigenfrequency distributions.

(a) $g_0-1<M$. Close to threshold, the increase of the detuning distribution width determines the decrease of each laser field amplitude down to complete quenching. The criterion of the coherent lasing when $g_0-1<M$ can be easily obtained since the filed intensity goes to zero and the system (1) becomes linear ($g_k\equiv g_0$). The condition for lasing is determined by the roots of the characteristic equation

$$\begin{vmatrix} D_1-\lambda & M/N & \cdots & M/N \\ M/N & D_2-\lambda & \cdots & M/N \\ \vdots & \vdots & \ddots & \vdots \\ M/N & M/N & \cdots & D_N-\lambda \end{vmatrix}=0, \quad (4)$$

where

$$D_k=g_0-1-M+i\Delta_k+M/N. \quad (5)$$

It is interesting to note that the proof of the existence of a gap in the electron energy level spectrum in the Bardeen-

Cooper-Shiffer (BCS) theory of superconductivity is reduced to a problem similar to (4) (see, for instance [8,9],). Let us call $f(\Delta)$ the detuning distribution function normalized to the number of lasers [$\int f(\Delta)d\Delta=N$ with $N\gg 1$]. If $\mathrm{Re}(M)>0$, then the eigenvalue with the maximum real part $\lambda_{\max}$ (the analog of the ground energy level in the BCS theory) is determined by the integral equation

$$\int \frac{f(\Delta)d\Delta}{\lambda_{\max}-i\Delta-(g_0-1-M+M/N)}=\frac{N}{M}. \tag{6}$$

If the random detunings are uniformly distributed in the interval $[-\Delta_0/2,\ \Delta_0/2]$ then

$$\lambda_{\max}=g_0-1-M+\frac{\Delta_0}{2}\left[\tan\frac{\Delta_0}{2M}\right]^{-1}. \tag{7}$$

Equation (7) holds in the general case of a complex coupling factor M. It is easily shown that the lasing condition reduces to $\mathrm{Re}(\lambda_{\max})>0$. Using (7) we obtain a coherent emission condition:

$$g_0-1>\mathrm{Re}\left\{M-\frac{\Delta_0}{2}\left[\tan\frac{\Delta_0}{2M}\right]^{-1}\right\}. \tag{8}$$

For $\mathrm{Re}(M)>\ \mathrm{Im}(M)$ the order-parameter dependence on detuning results in

$$C=1-\frac{\Delta^2}{12(g_0-1)\mathrm{Re}(M)}. \tag{9}$$

For the Lorentzian detuning distribution,

$$f(\Delta)=\frac{\gamma}{\pi}\frac{N}{\gamma^2+\Delta^2}, \tag{10}$$

Eq. (6) leads to a criterion for coherent array lasing which is independent from M:

$$g_0-1>\gamma, \tag{11}$$

and yields an order parameter C given by

$$C=1-\frac{\gamma}{(g_0-1)}. \tag{12}$$

For the Gaussian eigenfrequency distribution

$$f(\Delta)=\frac{N}{\gamma\sqrt{\pi}}\exp\left(\frac{-\Delta^2}{\gamma^2}\right), \tag{13}$$

the coherent emission condition is

$$g_0-1>\frac{\gamma^2}{2M}, \tag{14}$$

and for $\mathrm{Re}(M)>\ \mathrm{Im}(M)$ the order parameter is

$$C=1-\frac{\gamma^2}{2M(g_0-1)}. \tag{15}$$

(b) $g_0-1>M$ (M real). In this case the lasing does not die out with increasing detunings, because the gain in each laser is sufficiently large to let it operate without the injection of a coupling signal. Beyond a critical value of Δ_0 the transition occurs from a steady-state coherent regime to an unsteady regime with a sharp brightness decrease caused by the interference of the laser fields with randomly distributed phases.

This critical value of Δ_0 can be obtained by perturbation theory in the limit $g_0-1\gg M$ (field amplitude values vary only slightly). Let us use the steady-state laser fields in the form

$$E_k=A_k\exp i\Psi_k. \tag{16}$$

If the coupling coefficient is real the equations for amplitudes A_k and phase Ψ_k have the form

$$\dot{A}(\Delta_k)=[g(\Delta_k)-1-M]A(\Delta_k)+MCA(0)\cos\Psi(\Delta_k), \tag{17}$$

$$\dot{\Psi}(\Delta_k)=\Delta_k-MC\frac{A(0)}{A(\Delta_k)}\sin\Psi(\Delta_k), \tag{18}$$

where C is a common constant determined by

$$CNA(0)=\sum_k E(\Delta_k). \tag{19}$$

At steady state we have

$$\sin\Psi(\Delta_k)=\frac{\Delta_k A(\Delta_k)}{MCA(0)}, \tag{20}$$

$$\cos\Psi(\Delta_k)=\frac{[1+M-g(\Delta_k)]A(\Delta_k)}{MCA(0)}. \tag{21}$$

Thus, the phase of each laser field with respect to the average field takes a nonzero value depending on the laser detuning. The locking destabilizes as the maximum phase difference exceeds $\pi/2$. This could be expected by analogy with the capture of the laser mode by an external injected signal and it is found to occur in the numerical simulations of the system (1) and (2). So the stability limit is determined by

$$\Psi(\Delta_m)=\frac{\pi}{2},\quad \Delta_m=\frac{\Delta_0}{2}. \tag{22}$$

The constant C can be determined from Eq. (19), which in the continuous limit takes the form

$$CA(0)=\frac{1}{2\Delta_m}\int_{-\Delta_m}^{\Delta_m}A(\Delta)\left[1-\left(\frac{A(\Delta)\Delta}{A(0)MC}\right)^2\right]^{1/2}d\Delta. \tag{23}$$

The solution can be obtained in the limit $g_0-1\gg M$ for which the laser intensities are approximately the same. Thus the phase unlocking condition in the zeroth-order approximation $[A(\Delta)\equiv A(0)]$ is given by

$$\Delta_m>\frac{\pi}{4}M. \tag{24}$$

In the first-order approximation the laser intensity function is approximated by the second power polynomial function of the eigenfrequency

$$I(\Delta)=I(0)\left[1-\beta\left(\frac{\Delta}{\Delta_m}\right)^2\right], \qquad (25)$$

where β is a coefficient to be evaluated. From (20) and (21) it follows

$$I(\Delta)\{\Delta^2+[1+M-g(\Delta)]^2\}=(MC)^2I(0); \qquad (26)$$

the value of $I(0)$ can be obtained from (3) and (21) at $\Delta=0$ as

$$I(0)=\frac{g_0}{1+M(1-C)}-1. \qquad (27)$$

Substituting (25) and (27) into (26) and keeping the term $\beta(\Delta/\Delta_m)^2$ only at first order, we obtain for β

$$\beta=\left[\frac{\Delta_m}{MC}\right]^2\left[1+\frac{2g_0I(0)}{MC[1+I(0)]^2}\right]^{-1}. \qquad (28)$$

$I(0)$ and β at first order still contain the two unknowns C and Δ_m. In order to evaluate them, we go to the second-order approximation and obtain after substitution of $I(\Delta)$ into (23)

$$C=\frac{\pi}{4}\left(1-\frac{\beta}{4}\right). \qquad (29)$$

From (20) and (22) we have

$$\Delta_m=MC\sqrt{1-\beta}. \qquad (30)$$

We thus have a closed system (27)–(30) for the four unknowns $I(0)$, β, C, and Δ_m. In the limit of $g_0-1\gg M$ the system has a single root in the neighborhood of $\beta=0$, which can be obtained numerically.

The analytic criteria described above and the numerical simulations for a large number of lasers ($N\geqslant 100$) allow us to draw a map of the different regimes (Fig. 2). The solid lines show the analytical boundaries of the different domains (1 is the domain of the steady-state coherent lasing, 2 is the domain of the unsteady lasing, and 3 is the domain of the vanishing laser output). The analytical boundaries are in agreement with the numerical simulations. Indeed, integrating the system for several g_0 values, at Δ_0 values close to the boundaries, we are able to identify close pairs of Δ_0 parameters giving rise to different regimes, thus drawing the numerical boundary which is close to the analytical one. In the figure, $+$, Δ, and $\bigcirc$ refer, respectively, to domains 1, 2, and 3 for $N=100$. The same regimes are obtained for the same parameters and $N=1000$.

For the Lorentzian eigenfrequency distribution Eq. (23) has to be written

$$NCA(0)=\int f(\Delta)A(\Delta)\left[1-\left(\frac{A(\Delta)\Delta}{A(0)MC}\right)^2\right]^{1/2}d\Delta, \qquad (31)$$

where $f(\Delta)$ is given by Eq. (10).

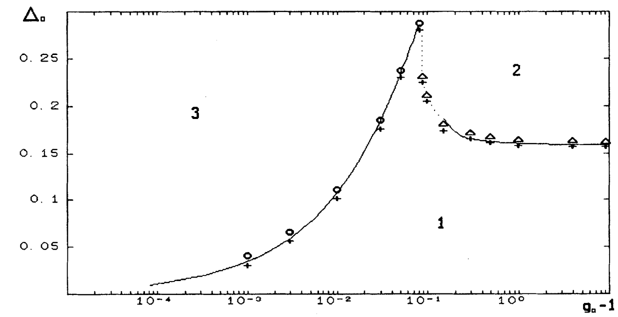

FIG. 2. The g_0 and Δ_0 domains at $M=0.1$ of the different lasing regimes: 1, coherent steady-state regimes; 2, dynamic regimes; 3, domain of the lasing dying out. The solid lines are the analytical criteria (8) and (27)–(30). The numerical simulations for $N=1000$: $+$, coherent steady-state regime; Δ, dynamic regime; $\bigcirc$, vanishing laser action.

For this integral equation in the limit of $g_0-1\gg M$ we can find for the order parameter

$$C=\left(1-\frac{2\gamma}{M}\right)^{1/2}. \qquad (32)$$

In the general case of a complex coupling parameter the expression (32) changes into

$$C=\left(1-\frac{2\gamma}{\mathrm{Re}(M)}\right)^{1/2}, \qquad (33)$$

with the additional condition that $\mathrm{Re}(M)>|\mathrm{Im}(M)|$ be satisfied.

To compare our analytical results with the numerical simulations of Ref. [6] we show in Fig. 3 the order-parameter dependence on the frequency detuning level for Lorentzian eigenfrequency distribution.

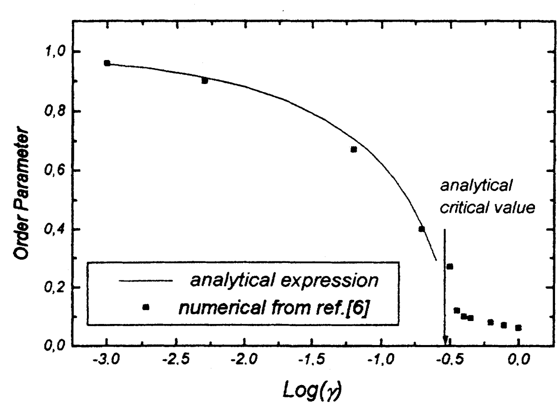

FIG. 3. Comparison of the order-parameter dependence on the eigenfrequency detuning level obtained by the analytical formula (32) and the numerical simulations of Ref. [6], for the same conditions. Log(γ) is the natural logarithm of the normalized frequency spread.

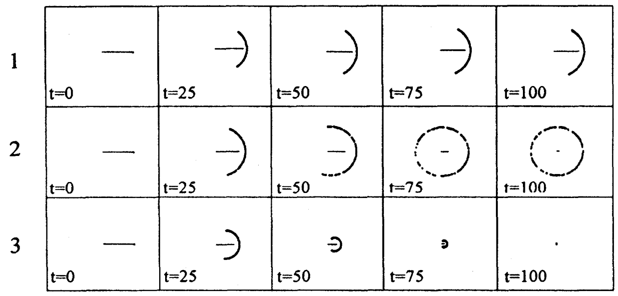

FIG. 4. Argand diagrams for $N=100$ starting from the in-phase locked initial state leading to 1, coherent steady-state regime ($M=0.1$, $g_0=1.5$, $\Delta_0=0.15$); 2, phase-unlocked regime ($M=0.1$, $g_0=1.5$, $\Delta_0=0.20$); 3, dying out of the lasing ($M=0.1$, $g_0=1.001$, $\Delta_0=0.20$).

III. THE DYNAMICS OF GLOBALLY COUPLED LASER ARRAY FIELDS

The temporal behaviors in the Argand diagrams (points represent the individual laser fields and the vectors represent the average field) are shown in Fig. 4. They are obtained starting from the zero-dephased initial state for the following values of the parameters in the three different domains: steady-state lasing (domain 1: $M=0.1$, $g_0=1.5$, $\Delta_0=0.15$), unsteady lasing (domain 2: $M=0.1$, $g_0=1.5$, $\Delta_0=0.20$), and vanishing laser action (domain 3: $M=0.1$, $g_0=1.001$, $\Delta_0=0.20$).

In the domain of the dynamic regimes (2 in Fig. 2) the temporal behavior of the fields depends upon the active medium relaxation time τ. If this time is negligible $[\tau \ll 2(g_0-1-M)/M^2C^2]$, then the unsteady regime with a time-dependent random distribution of phases and brightness proportional to $1/N$, that of the in-phase locked system, is found to be "statistically" stable, that is, stable in average (same result as in Ref. [6], where this concept was introduced).

In the opposite case of long τ the active medium gain relaxation from the nonequilibrium level in a single laser without coupling has the form of damped oscillations with the following frequency ω_c and damping rate γ:

$$\omega_c \sim \sqrt{2(g_0-1-M)/\tau}, \qquad (34)$$

$$\gamma \sim -g_0/2(1+M)\tau. \qquad (35)$$

This frequency ω_c is sometimes called Toda's frequency after the name of the effective potential of nonlinear oscillations in the laser system (see for instance [10,11].

A resonant swing at the relaxation oscillations frequency can be initiated by the average field fluctuations. The characteristic modulation period can be estimated by analogy with the two coupled laser system [7]:

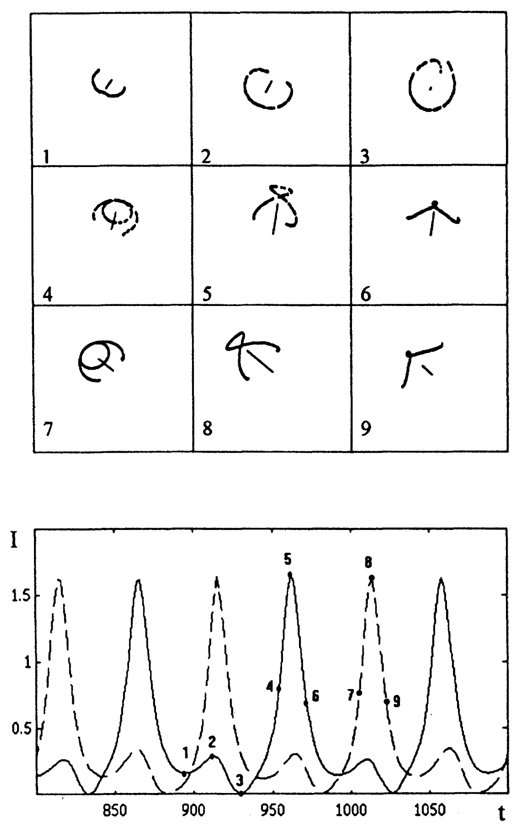

FIG. 5. The example of the cooperative phase-locking ($g_0=1.5$, $M=0.1$, $\Delta_0=0.21$, $\tau=50$). Solid line and Argand diagrams 1, . . . ,6 for $N=100$; dashed line and Argand diagrams 7, . . . ,9 for $N=1000$.

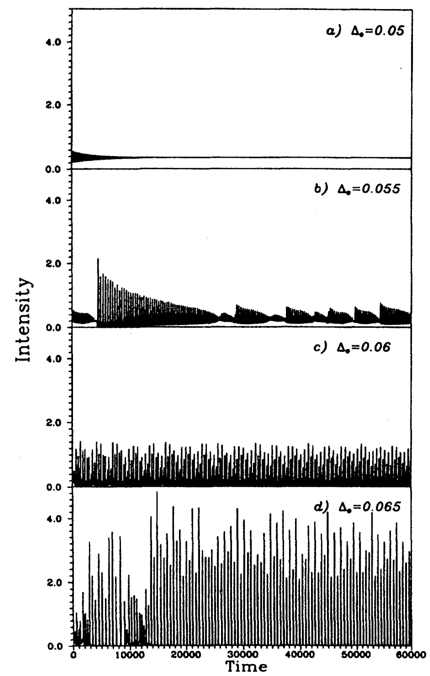

FIG. 6. Temporal evolutions of the axial far-field intensity in the case of a three-element array for different detuning levels ($g_0=1.5$, $M=0.03$, $\tau=3000$). Intensity is normalized to the saturation intensity.

$$T_{\text{in}} = 2\pi/\sqrt{\Delta_0^2 - M^2C^2(1-\beta)}. \tag{36}$$

the parameters C and β being roots of the system (27)-(30). The different correlation between these two characteristic times causes the different field dynamics.

However the regimes where the relaxation oscillation period is close to the average field modulation period seem to be of major interest. As the globally coupled array is similar to the nonlinear laser system driven by the injection of an external signal [12] or the laser with modulated losses [13], the nonlinear unsteady regime should be expected to exist. In particular typical phenomena for nonlinear systems are found such as spontaneous pulsing and chaos.

Examples of lasing regimes with spontaneous oscillations are shown in Fig. 5 both for arrays of 100 and 1000 elements. The resonant swing at the relaxation oscillations frequency causes the relatively short duration of the dominant pulse. The cooperative phase locking can take place regardless of the large spread of eigenfrequencies because of the short duration of the pulses.

The peak of brightness on the dashed curve ($\tau=50$, $N=100$) in Fig. 1 is explained by the cooperative field phase locking. The time-averaged brightness achieves up to 40% of the brightness of the in-phase locked system. This value weakly depends on the number of lasers and it is mainly determined by the M, Δ_0, g_0 and τ parameters.

Similar behaviors have been found integrating Eqs. (1) and (2) in the case of a small number of lasers (from 3 to 10) and a long active medium relaxation time. Figure 6 shows some temporal evolutions for a three-element array of lasers with $\tau=3000$, $M=0.03$, $g_0=1.5$. These parameter values are typical of CO_2 lasers arrays reported in some experiments [14,15].

IV. CONCLUSION

In the present work the lasing regimes of globally coupled laser arrays with time-independent spread eigenfrequencies have been analytically and numerically investigated. It is shown that the violation of the coherent phase-locked regime with the increase of the eigenfrequency detuning level can occur in two different manners.

The first type of coherence loss takes place when the small-signal gain slightly exceeds the threshold condition. In this case the lasing quenching occurs above a critical eigenfrequency detuning level.

The second one is realized at large gains. In this case the coherent regime is unstable above a critical value and different dynamic behaviors are obtained.

The dynamic behavior of the globally coupled laser depends upon the inertia of the active medium. In the case of large population decay times the regimes with resonant swing at the relaxation oscillations frequency have been found. In these regimes the average and peak value of the order parameter depends only weakly upon the number of array elements.

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[1] K. Kaneko, Phys. Rev. Lett. **41D**, 137 (1990).
[2] P. Alstrom and R. K. Ritala, Phys. Rev. A **35**, 300 (1987).
[3] V. V. Likhanskii and A. P. Napartovich, Usp. Fiz Nauk **160**, 101 (1990) [Sov. Phys. Usp. **160**, 101 (1990)].
[4] A. A. Golubentsev and V. V. Likhanskii, Quantum Electron. **17**, 592 (1990).
[5] S. N. Kozlov and V. V. Likhanskii, Laser Phys. **3**, 1067 (1993).
[6] Z. Jiang and M. McCall, J. Opt. Soc. Am. **10**, 155 (1993).
[7] A. V. Bondarenko, A. F. Glova, S. N. Kozlov, F. B. Lebedev, V. V. Likhanskii, A. P. Napartovich, V. D. Pysmenny, and V. P. Yartsev, Zh. Éksp. Teor. Fiz. **95**, 807 (1989) [Sov. Phys. JETP **68**, 461 (1989)].
[8] Ch. Kittel, *Introduction to the Solid State Physics* (Mir, Moscow, 1978).
[9] Sh. Ma, *The Modern Theory of the Critical Phenomena* (Mir, Moscow, 1980).
[10] G. L. Oppo and A. Politi, Z. Phys. B **59**, 111 (1985).
[11] T. Ogawa, Phys. Rev. A **37**, 4286 (1988); W. Lauterborn and R. Steinhoff, J. Opt. Soc. Am. B **5**, 1097 (1988).
[12] F. T. Arecchi, G. Lippi, G. Puccioni, and J. Tredicce, Acta Phys. **57**, 119 (1985).
[13] F. T. Arecchi, R. Meucci, G. Puccioni, and J. Tredicce, Phys. Rev. Lett. **49**, 1217 (1982).
[14] A. Lapucci and G. Cangioli, IEEE J. Quantum Electron. **29**, 2962 (1993).
[15] G. Lescroart, R. Muller, and G. L. Bourdet, Opt. Commun. **108**, 289 (1994).