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Photon-hopping conduction and collectively induced transparency in a photonic band gap

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We study the dynamics of photon-hopping conduction between impurity atoms in a photonic band gap via resonance dipole-dipole interaction (RDDI) in the low-excitation density limit. This is done both in the absence and in the presence of a localized dielectric defect mode. The random impurity atom positions are modeled by means of a Gaussian random distribution of RDDI's with variance J and atomic line fluctuations with variance δ. By numerical calculation, we demonstrate the occurrence of a number of collective quantum features of photon-hopping conduction. These include collective enhancement of the energy-transfer rate and collectively induced transparency within the dielectric cavity mode. These effects are shown to depend sensitively on the variances J and δ. Our results suggest that RDDI mediated hopping conduction may protect atomic excitation energy from dissipation through nonradiative relaxation channels.

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I. INTRODUCTION

Photonic band-gap (PBG) materials constitute a fundamentally new class of dielectric materials in which the electromagnetic interaction is controllably altered, and in some cases completely removed over certain frequency and lengths scales. The existence of a PBG has been demonstrated both computationally [1] and experimentally [2]. It has been suggested that many new physical phenomena will occur in PBG materials, such as photon localization [3], suppression of spontaneous emission and fractionalized single atom inversion [4–6], and vacuum Rabi splitting and photon-atom bound states [6,7]. In the case of a collection of N impurity atoms, the PBG environment leads to fundamental changes in the collective time scale factor and superradiance rate. This leads to the potential for superradiant and lasing devices with ultrafast modulation speed and near zero threshold pumping energy [8].

In this paper we present results on photon-hopping conduction between impurity atoms inside a PBG via resonance dipole-dipole interaction (RDDI). Energy transfer between impurity atoms in solids has been studied for many years and continues to be an active area of fruitful research in physics [9]. Inside a PBG, where spontaneous emission is suppressed, the RDDI energy-transfer process becomes the dominant interaction mechanism between atoms. The photonic band gap protects this interaction from many incoherent effects. Conversely, RDDI hopping conduction may occur on time scales that are short compared to other nonradiative relaxation channels in the PBG material, thereby further enhancing the time scale of coherent quantum evolution. This leads to a number of interesting collective effects within the resulting photonic impurity band. In particular, the collective energy-transfer rate is shown to be strongly enhanced. The energy transfer from a localized dielectric defect mode to the impurity atoms is also considered. In particular, if randomness of atomic locations is ignored, collectively induced transparency occurs, i.e., there is almost no absorption of the resonant photon in the cavity mode by a large collection of unexcited impurity atoms.

II. PHOTON-HOPPING CONDUCTION

An excited atom in a PBG interacts strongly with its own localized radiation field, leading to the formation of a photon-atom bound state [7]. The photon emitted by the excited atom will exhibit tunneling on a length scale given by the localization length $\xi_{\rm loc}$ before being Bragg reflected back to the emitting atom. The result is a stationary-state superposition of a localized photon and partially excited atom. The localization length is on the scale of several optical wavelengths, and consequently the tunneling photon can be absorbed by another atom located within the distance $\xi_{\rm loc}$. In this manner, photon-hopping conduction takes place from one impurity atom to another.

In this section we investigate dynamical properties of the photon-hopping conduction between impurity atoms in a PBG. We consider a collection of N two-level atoms within a PBG and no nearby resonant radiation mode. The only interaction between atoms is the excitation transfer via the RDDI. The quantum dynamics of the two-level atoms is described by the set of 2×2 Pauli spin operators. σ_z describes the atomic inversion, $\sigma^\dagger$ and σ^- describe atomic excitation and deexcitation, respectively. The model Hamiltonian for interacting two-level atoms in a perfect photonic crystal (in the interaction picture) takes the form [10]

$$H=\sum_{j=1}^{N}\frac{\hbar\delta_j}{2}\sigma_j^z+\sum_{i\neq j}^{N}J_{ij}\sigma_i^\dagger\sigma_j\,. \tag{1}$$

Here $\delta_j=\omega_j-\omega_a$ is the atomic frequency shift (from its average value ω_a) caused, in general random, by the static field in the photonic crystal. J_{ij} denotes the RDDI between the atoms ith and jth atoms. The dipole-dipole interaction has been shown to play an important role in Van der Waals dephasing of the symmetrical superradiant states in vacuum [11]. Far inside a PBG, spontaneous emission is almost totally suppressed [4,6] while RDDI remains strong as a result of the exchange of high-energy virtual photons, which lie outside the PBG. In this case, RDDI plays quite the opposite role from that in vacuum: it is a coherent rather than a dephasing interaction. In a PBG, we will show that the randomness of the dipole-dipole interaction plays an important role in photon-hopping conduction and lossless energy transfer between impurity atoms. Near the band edge where spontaneous emission becomes strong [6,8], competition between superradiance and photon-hopping conduction will take place. In particular, dephasing of superradiance near the band edge by random RDDI at high atomic densities may occur. In this paper, we consider effect of RDDI on a single atomic excitation and the competition between RDDI and nonradiative relaxation. The influence of RDDI on superradiance will be discussed elsewhere.

The order of magnitude of the RDDI in vacuum is given by the dipole energy scale

$$|J_{ij}|\sim|\vec{\mu}_i|\;|\vec{\mu}_j|/(R_{ij})^3, \tag{2}$$

where μ_i and μ_j are the induced dipole moments on atoms i and j, and R_{ij} is the interatomic separation. Using the fact that $|\vec{\mu}_i|\sim ea_0$, where e is the electronic charge and a_0 is the atomic Bohr radius, it follows that

$$|J_{ij}|\sim\hbar\omega_a(a_0/R_{ij})^3. \tag{3}$$

Here we have used the fact that $e^2/a_0\sim\hbar\omega_a$. The sign of J_{ij} depends on the orientation of the vectors $\vec{\mu}_i$ and $\vec{\mu}_j$ relative to the separation vector $\vec{R}_{ij}$. In a realistic description, the RDDI term in (1) would be replaced by a short-ranged, traceless tensor interaction for atomic excitation transfer between two sets of nearly degenerate atomic orbitals. We simulate the effects of this traceless tensor interaction with the simpler two-level atom system by allowing J_{ij} to be a Gaussian random variable with zero mean value. The effect of the PBG environment is to induce some phase shifts in the RDDI matrix elements, as well as to introduce an exponential damping factor $\exp(-R_{ij}/\xi_{\rm loc})$. It follows that the model Hamiltonian (1) describes N impurity atoms which are separated by a distance $R_{ij}\leqslant\xi_{\rm loc}$. The detail evaluation of J_{ij} as a function of atomic distance R_{ij} and atomic dipole configurations in a PBG may be found in Refs. [7,12]. The analogous expression for J_{ij} in free space can be found in [13,15]. For simplicity we consider the single excitation case in which only one atom, say, for instance, the first atom, is initially excited and $N-1$ atoms are initially unexcited. There are N states which span the subspace of interest. These base states we define as follows:

$$|j\rangle\equiv|-,\ldots,\underset{j}{+},-,\ldots,-\rangle \tag{4}$$

represents the jth atom excited and other atoms that are unexcited. The wave function of the system can be written as

$$|\psi(t)\rangle=\sum_{j=1}^{N}c_j(t)|j\rangle. \tag{5}$$

The time-dependent Schrödinger equation, projected on to the single excitation subspace of the N-atom Hilbert space, takes the form (setting $\hbar=1$)

$$\frac{d}{dt}c_j(t)=-i\delta_jc_j(t)-i\sum_{i(\neq j)}^{N}J_{ji}c_i(t), \tag{6}$$

where $j=1,2,\ldots,N$. Equation (6) has the initial condition

$$c_j(0)=\delta_{1j}, \tag{7}$$

that is, the first atom is initially excited and all others unexcited.

For comparison purposes, we start with the analytical solution of (6) in the absence of disorder. In this case all $\delta_j=0$ and $J_{ij}=J$. Such a model may be relevant to impurity atoms located at equal distances [16]. This actually can be realized just for $N\leqslant4$ in three-dimensional space. For a larger collection of atoms, it serves as an approximate model for specific atomic configurations in which J_{ij} has small variance compared to its mean value. This may occur if configuration averaging over the atomic positions is incomplete and nonergodic. Equation (6) then reduces to

$$\frac{d}{dt}c_j(t)=-iJ\sum_{i(\neq j)}^{N}c_i(t). \tag{8}$$

Defining

$$\sum_{i(\neq j)}^{N}c_i(t)\equiv A_j(t), \tag{9}$$

one can write Eq. (8) as

$$\frac{d}{dt}c_j(t)=-iJA_j(t), \tag{10}$$

$$\frac{d}{dt}A_j(t)=-i(N-1)Jc_j(t)-i(N-2)JA_j(t). \tag{11}$$

The solution of Eqs. (10) and (11) can be easily found in the form

$$c_j(t)=a_1e^{iJt}+a_2e^{-iJ(N-1)t}, \tag{12}$$

where a_1 and a_2 are defined by the initial condition (7). The probability of the jth atom being excited, $P_j(t)=|c_j(t)|^2$, follows from Eq. (12):

$$P_j(t)=\begin{cases}1-4(N-1)\sin^2(JNt/2)/N^2, & j=1\\ 4\sin^2(JNt/2)/N^2, & j=2,\ldots,N.\end{cases} \tag{13}$$

This simple but illustrative result suggests that an unexcited atom never gets a fraction greater than $4N^{-2}$ of the energy of the excited atom. As N gets very large, the first atom essentially never transfers its energy at all. A similar energy trapping phenomenon has been predicted in the system of N two-level atoms in an ideal resonant cavity and without dipole-dipole interaction between atoms [16]. In contrast to this, energy trapping in a PBG occurs without a cavity mode as the interaction mediator between atoms. In contrast to Ref. [17], the absence of excitation transfer does not result from atomic coherence induced by an external field but results from strongly correlated RDDI. This result underscores the important role of disorder in enabling coherent energy transport in a PBG.

We now present results for a more realistic, disordered system in which atomic positions in the photonic crystal are random. For simplicity δ_j and J_{ij} can be taken in this case as

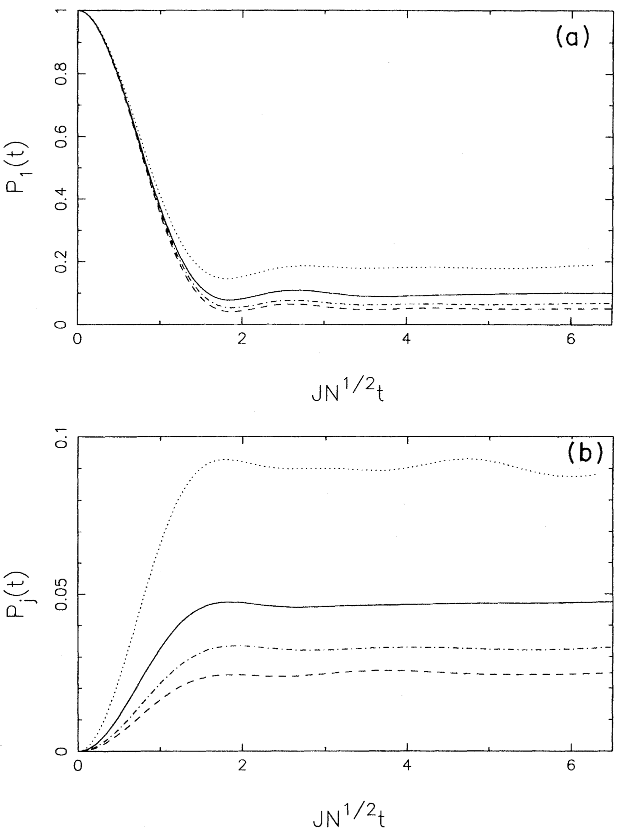

FIG. 1. Atomic population on the excited state, $P_1(t)$ (a) and $P_j(t)$ (b) as a function of the scaled time $N^{1/2}Jt$ for detuning variance $\delta=0$ and for number of atoms $N=10$ (dotted curves), 20 (solid curves), 30 (dashed-dotted curves), and 40 (dashed curves).

Gaussian random variables with zero mean values and with variances equal to δ and J, respectively [10]. For each set of the Gaussian random numbers δ_j and J_{ij}, we integrate the system of Eq. (6) using the fourth-order Runge-Kutta method [18]. The configurational average is taken over a large number ($\sim 2\times 10^3$) of sets of the random numbers δ_j and J_{ij}. The probability of the jth atom being excited can be written as

$$P_j(t)=[|c_j(t)|^2]_c, \tag{14}$$

where the square bracket $[\ ]_c$ denotes the configuration average over the random atomic positions.

In Fig. 1 we plot the probability $P_1(t)$ [Fig. 1(a)] and $P_j(t)$ [Fig. 1(b)] [$P_j(t)$ is almost the same for any $j\neq 1$] as a function of the scaled time $N^{1/2}Jt$ for $\delta=0$ and various numbers of atoms N. Clearly each unexcited atom gets a fraction of the excited atom energy proportional to N^{-1}. This is in sharp contrast to the case when all $J_{ij}=J$, Eq. (13), where each unexcited atom can get only a fraction of the excited atom energy proportional to N^{-2} and the first atom essentially does not transfer its energy at all. Here, we see the important role of randomness of atomic locations in enabling coherent energy transport in a PBG. Clearly from Fig. 1(a) the transfer rate is proportional to $\sqrt{N}$ over the major (early stage) decay process until $P_j(t)$ reaches their steady-state values proportional N^{-1}. As a result of rapid energy transfer, other nonradiative decay mechanisms, such as multiphonon transition, will be ineffective. This suggests that the PBG can be used as an ultrafast and nearly lossless device for energy transfer (photon-hopping conduction). In Fig. 2 we plot $P_1(t)$ for $N=20$ and for various values $\delta/J=1$, 5, 10, and 20. Clearly, energy transfer from the first atom to the unexcited atoms is reduced with inhomogeneous line broadening caused by the random static field in the lattice.

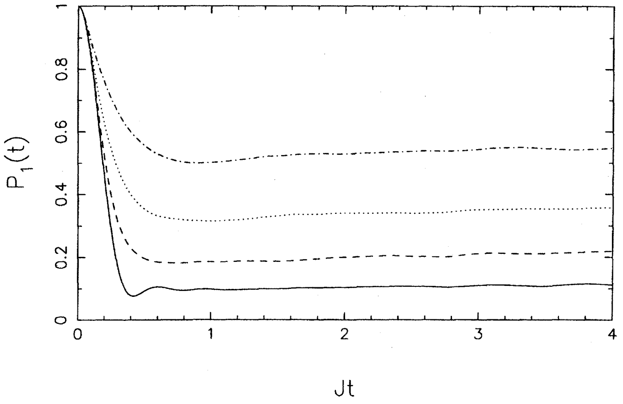

FIG. 2. Atomic population on the excited state $P_1(t)$ as a function of the scaled time Jt for $N=20$ and for $\delta/J=0$ (solid curve), 5 (dashed curve), 10 (dotted curve), and 20 (dashed-dotted curve).

III. COLLECTIVELY INDUCED TRANSPARENCY

In this section we investigate energy transfer from a resonant dielectric defect mode to N two-level impurity atoms inside a PBG. We assume that the Q factor of the localized defect mode is sufficiently high that we describe it by an extended Jaynes-Cummings model [10,16] with the Hamiltonian

$$H=\sum_{j=1}^{N}\frac{\hbar\delta_j}{2}\sigma_j^z+\sum_{i\neq j}^{N}J_{ij}\sigma_i^\dagger\sigma_j+g\sum_{j=1}^{N}(e^{i\vec{k}\cdot\vec{r}_j}\sigma_j^\dagger a+e^{-i\vec{k}\cdot\vec{r}_j}a^\dagger\sigma_j). \tag{15}$$

Here, a and $a^\dagger$ are the annihilation and the creation operators for photons in the defect mode, $\vec{k}$ is the wave vector of the defect mode, $\vec{r}_j$ is the position of the jth atom, and g is the atom-radiation mode coupling constant. Here, the magnitude of coupling constant g is related to the volume of the cavity mode ξ_{loc}^3. In particular [19],

$$g=\hbar\left(\frac{\omega_a\mu}{\hbar c}\right)\left(\frac{2\pi\hbar c^2}{\omega_a\xi_{\mathrm{loc}}^3}\right)^{1/2}. \tag{16}$$

Here the atomic dipole moment $\mu\sim ea_0$. Again using the fact that $e^2/a_0\approx\hbar$, it follows that

$$g\sim\hbar\omega_a(a_0/\xi_{\mathrm{loc}})^{3/2}. \tag{17}$$

In Eq. (15) we further assume that the N atoms are confined within a region that is small compared to ξ_{loc}. This allows us to neglect the exponential decay of the mode amplitude functions. For simplicity we consider only the case when the defect frequency is in exact resonance with the atomic transition frequency ω_a. As in the preceding section, we consider the case of single excitation. Suppose that initially there is only one photon in the defect mode and all atoms are unexcited. The basis states can then be chosen as

$$|0\rangle=|-,-,\ldots,-;1\rangle, \tag{18a}$$

$$|j\rangle\equiv|-,\ldots,\underset{j}{+},\ldots,-;0\rangle. \tag{18b}$$

In the state $|0\rangle$, all the atoms are in ground state and there is one photon in the cavity mode. In the state $|j\rangle$, the jth atom is excited and there is no photon in the defect mode. The wave function of the system in this one photon sector of the Hilbert space is of the form

$$|\psi(t)\rangle=\sum_{j=1}^{N} c_j(t)|j\rangle+c_0(t)|0\rangle. \tag{19}$$

The projected time-dependent Schrödinger equation has the following form:

$$\frac{d}{dt}c_0(t)=-ig\sum_{j=1}^{N} e^{-i\vec{k}\cdot\vec{r}_j}c_j(t), \tag{20}$$

$$\frac{d}{dt}c_j(t)=-i\delta_j c_j(t)-i\sum_{i(\neq j)}^{N} J_{ji}c_i(t)-ge^{i\vec{k}\cdot\vec{r}_j}c_0(t). \tag{21}$$

Defining

$$\tilde{c}_j(t)\equiv e^{-i\vec{k}\cdot\vec{r}_j}c_j(t)$$

and

$$\tilde{c}_0(t)\equiv c_0(t),$$

Eqs. (20) and (21) become

$$\frac{d}{dt}\tilde{c}_0(t)=-ig\sum_{j=1}^{N}\tilde{c}_j(t), \tag{22}$$

$$\frac{d}{dt}\tilde{c}_j(t)=-i\delta_j\tilde{c}_j(t)-i\sum_{i(\neq j)}^{N}\tilde{J}_{ji}\tilde{c}_i(t)-igc_0(t), \tag{23}$$

where

$$\tilde{J}_{ji}\equiv J_{ji}e^{i\vec{k}(\vec{r}_i-\vec{r}_j)}. \tag{24}$$

The initial condition of Eqs. (22) and (23) is

$$\tilde{c}_0(0)=1, \quad \tilde{c}_j(0)=0. \tag{25}$$

As before, we begin by studying the case of uniform $\tilde{J}_{ij}=J$ for all (i,j) and $\delta_j=0$. Equations (22) and (23) then have the form

$$\frac{d}{dt}\tilde{c}_0(t)=-ig\sum_{j=1}^{N}\tilde{c}_j(t), \tag{26}$$

$$\frac{d}{dt}\tilde{c}_j(t)=-iJ\sum_{i=1}^{N}\tilde{c}_i(t)-ig\tilde{c}_0(t)+iJ\tilde{c}_j(t). \tag{27}$$

It is clear from Eqs. (25)–(27) that $\tilde{c}_j(t)$ is the same for any $j\neq 0$. Equations (26) and (27) can be written as

$$\frac{d}{dt}\tilde{c}_0(t)=-ig\tilde{c}(t), \tag{28}$$

$$\frac{d}{dt}\tilde{c}(t)=-iJ(N-1)\tilde{c}(t)-igN\tilde{c}_0(t), \tag{29}$$

where

$$\tilde{c}(t)\equiv\sum_{j=1}^{N}\tilde{c}_j(t). \tag{30}$$

The solution of Eqs. (28) and (29) can be easily found in the form

$$\tilde{c}_0(t)=a_1e^{-i\lambda_1 t}+a_2e^{-i\lambda_2 t}, \tag{31}$$

$$\tilde{c}(t)=b_1e^{-i\lambda_1 t}+b_2e^{-i\lambda_2 t}, \tag{32}$$

where

$$\lambda_{1,2}=\tfrac{1}{2}(N-1)J\pm\tfrac{1}{2}[(N-1)^2J^2+Ng^2]^{1/2}. \tag{33}$$

Here $a_{1,2}$ and $b_{1,2}$ are defined by the initial condition (25). The mean photon number $\langle n(t)\rangle=|c_0(t)|^2$ and the probability of the jth atom being excited, $P_j(t)=|c_j(t)|^2$, are obtained from solutions (31) and (32) as

$$\langle n(t)\rangle=1-\frac{Ng^2}{(N-1)^2J^2+Ng^2}\sin^2\{[(N-1)^2J^2+Ng^2]^{1/2}t\}, \tag{34}$$

$$P_j(t)=\frac{g^2}{(N-1)^2J^2+Ng^2}\sin^2\{[(N-1)^2J^2+Ng^2]^{1/2}t\}. \tag{35}$$

In the case of $J=0$, Eqs. (34) and (35) reduce to

$$\langle n(t)\rangle=1-\sin^2(\sqrt{N}gt), \tag{36}$$

$$P_j(t)=\frac{1}{N}\sin^2(\sqrt{N}gt). \tag{37}$$

That is, the photon is absorbed and reemitted by atoms, and the only difference from the single atom (Jaynes-Cummings) case is that the energy exchange rate is much faster (proportional to $\sqrt{N}$). This is in agreement with Ref. [16]. In the case of $J\neq 0$ and the number of atoms large enough to satisfy the condition

$$(N-1)^2J^2\gg Ng^2, \tag{38}$$

there is no photon absorption, $\langle n(t)\rangle\cong 1$, although all atoms are unexcited. Using the order-of-magnitude estimates for J and g, Eq. (38) becomes the condition that

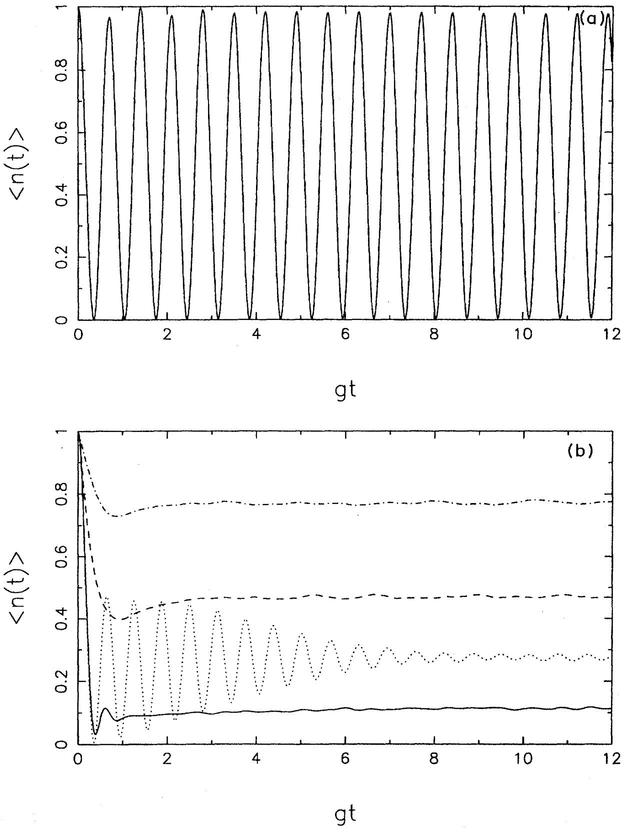

FIG. 3. Mean photon number $\langle n(t)\rangle$ as a function of the scaled time gt for $N=20$ atoms, detuning $\delta=0$; and for RDDI variance to cavity mode coupling ratio $J/g=0.1$ [Fig. 3(a)], 0.5 (dotted curve), 1 (solid curve), 5 (dashed curve), and 15 (dashed-dotted curve).

$$N \gg \frac{R^3}{(a_0\xi_{\rm loc})^{3/2}}, \tag{39}$$

where R^3 is the volume occupied by the atoms. By assumption $R^3 \leqslant \lambda^3$. The quantity $r \equiv (R^3/N)^{1/3}$ describes the average distance between nearest-neighbor atoms. It follows that transparency occurs when $r \ll (a_0\xi_{\rm loc})^{1/2}$. For an optical PBG we expect $\xi_{\rm loc} \gtrsim 10^4$ Å and $a_0 \simeq 1$ Å. This suggests that condition (39) is satisfied at typical gas densities ($r \lesssim 10^2$ Å). This transparency effect is a purely collective property analogous to the excitation trapping phenomenon discussed in the preceding section. That is, the atoms strongly (but randomly) correlated by RDDI do not absorb the localized, defect mode photon, and energy transfer from the defect mode to atoms does not occur.

We now study the influence of the random atomic positions on energy transfer from the defect mode to the impurity atoms. For simplicity we assume that δ_j and $\tilde{J}_{ij}$ are Gaussian random variables with zero mean values and with variances δ and J, respectively. As in the preceding section, for each set of the Gaussian random numbers δ_j and $\tilde{J}_{ij}$, we integrate Eqs. (22) and (23) using the fourth-order Runge-Kutta method [18]. The configurational average is taken over a large number ($\sim 2\times 10^3$) of sets of the random numbers δ_j and $\tilde{J}_{ij}$.

In Fig. 3 we plot the mean photon number as a function of gt for the case of $N=20$, $\delta=0$, and for various values of J. At small J [Fig. 3(a)], $\langle n(t)\rangle$ exhibit almost sinusoidal Rabi oscillation in agreement with the analytical solution (36). At $J=0.5g$ [dotted curve in Fig. 3(b)] the sinusoidal Rabi oscillation collapses. In this case the collapse of the Rabi oscillation is a result of random RDDI interactions between atoms. This leads to an effective off-diagonal inhomogeneous line broadening that damps out the single atom Jaynes-Cummings oscillations [20] and [21]. This is distinct from the diagonal inhomogeneous line broadening due to the random static field in the photonic crystal. At the value $J \cong g$[solid line in Fig. 3(b)], the maximum amount of photon energy is transferred to the atoms. In the long-time limit, each atom and the defect mode can get a fraction of the photon energy proportional to $1/N$. Further increase of J/g (dashed and dashed-dotted curves) leads once again to suppression of the energy transfer and the system remains more and more in the form of the defect photon. Large J/g may be realized by operating close to a photonic band edge where the localization length $\xi_{\rm loc}$ becomes considerably larger than the optical wavelength.

The process of energy transfer from an excited atom to $N-1$ unexcited atoms and a defect mode can be studied in a similar fashion. The only change is that the initial condition (28) becomes

$$\tilde{c}_0(0)=\tilde{c}_{j\neq 1}(0)=0, \quad \tilde{c}_1(0)=1. \tag{40}$$

We find that the defect mode, in general, absorbs only a fraction proportional to $1/N$ of the energy of the excited atom, in agreement with Ref. [16]. In the case of large δ or J, energy transfer from the excited atom to the defect mode decreases with increasing δ and J.

IV. DISCUSSION

We have studied photon-hopping conduction between impurity atoms in a PBG via RDDI, both in the absence and in the presence of a defect mode. The randomness of atomic positions was described in terms of inhomogeneous line broadening, as well as Gaussian random RDDI between pairs of atoms. Many collective quantum features of photon-hopping conduction were manifest. In particular, collective enhancement of the energy-transfer rate, excitation trapping, and collectively induced transparency have been predicted. These results suggest that photon-hopping conduction provides immunity for the resulting photonic impurity band from nonradiative extinction processes such as multiple phonon emission. Although our discussion focused on photon-hopping conduction in a PBG, the above results apply also for a system of Rydberg atoms in a near-resonant and resonant cavity [16,22–24]. Our results apply to the single excitation and low-excitation density limits. In the high-excitation density case we expect many different quantum features. In particular, we note that the model Hamilton (1) is similar to the spin-glass model in condensed matter physics [25]. For the single excitation it is apparent from Eq. (5) that

$$[\langle\sigma_j^\dagger\rangle]_c = 0, \tag{41}$$

$$[\langle\sigma_j^\dagger\sigma_j\rangle]_c = [|c_j(t)|^2]_c = P_j(t) > 0, \tag{42}$$

and

$$[\langle\sigma_j^\dagger\rangle\langle\sigma_j\rangle]_c=0. \tag{43}$$

Equation (43) describes the Edward-Anderson order parameter, which appears in the definition of a spin glass [25]. We have shown previously [8] that an incoherently excited collection of atoms near a photonic band edge can exhibit spontaneous symmetry breaking $|\langle\sigma_j^\dagger\rangle|>0$ in the steady-state (long-time) limit as a result of photon localization. It is possible that macroscopic coherence may likewise occur for the model Hamiltonian (1) with Gaussian random J_{ij} if a large finite density of excitations is present. Deep within the PBG, the above system may then tend to a new collective state, an optical analog of the quantum spin-glass state.

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