Index of refraction of dilute matter in atomic interferometry

Jacques Vigué*
Laboratoire Collisions Agrégats Réactivité, IRSAMC, Université Paul Sabatier and CNRS URA No. 770, 118, route de Narbonne; 31062 Toulouse Cedex, France
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When an atom propagates through a dilute medium, the wave that describes its propagation is modified by the interaction with the medium. In the present paper, we propose to treat this modification by introducing an index of refraction. This is a natural extension of this idea so familiar for light propagation, and such an extension has already been made in the case of neutrons in dense matter. Various interesting consequences of this idea are explored, most particularly in the case of ultracold atoms as the refraction index diverges in the quantum threshold limit.

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INTRODUCTION

The index of refraction is the natural way of describing the interaction of a light wave with matter. This idea is very old, even if its basic understanding requires the knowledge of Maxwell's equations, and its evaluation from first principles is possible only with quantum mechanics.

The propagation of any type of wave through a medium can also be described by an index of refraction. An index was thus introduced to describe neutron propagation through dense matter [1]. I propose here to extend this type of description to the propagation of atoms through a dilute medium. This description will have practical interests in the various types of atomic interferometry experiments [2]: the dilute medium can be the residual content of the vacuum chamber or atoms voluntarily introduced in some part of the interferometer. This description is also interesting as it introduces a more general analogy between light waves and atomic waves. For instance, it appears immediately that the index of refraction is a complex number: as usual the real part describes a modification of the phase velocity while the imaginary part describes an attenuation of the wave by scattering. The first effect has just been observed by Pritchard and co-workers [3] while the second effect is just the traduction of the mean free path in wave formalism.

The idea of a refraction index in atomic optics appears to be fruitful because it enhances the analogy with ordinary optics as well as neutron optics. This paper discusses several such interesting prospects appearing thanks to this analogy.

THE INDEX OF REFRACTION IN THE SIMPLEST CASE

Let us consider first the dilute medium as made of fixed scattering centers. The thermal motion of the atoms will be briefly considered in a second step. The incident wave is described as a plane-wave $\exp(i\vec{k}\cdot\vec{r})$ and we consider one scattering center located at the origin O. The asymptotic behavior of the wave is given by the well-known formula [4]:

$$\Psi \underset{r\to\infty}{\sim} e^{i\vec{k}\cdot\vec{r}} + f(\vec{k},\vec{k}')\frac{e^{ikr}}{r}, \tag{1}$$

where the second term describes the asymptotic part of the scattered wave in the direction $\vec{k}'$. The function $f(\vec{k},\vec{k}')$ contains all the information on the scattering process and is closely related to the S matrix. This formalism can be applied almost without modification to the scattering of light waves or of atomic waves.

In the case of light waves, ψ will be the electric field and its vectorial character complicates the problem slightly. However, the calculation is usually done in the electric dipole approximation which gives a great simplification: in the language of partial waves, only the $l=1$ terms are taken into account. In the case of atomic waves, ψ is the wave function. The calculation will be quite complex if both the atom and the scattering center have internal degrees of freedom described by spins. In this paper, we consider only the case of structureless atoms and scattering centers (e.g., rare gases in their ground state). The great difference with the electromagnetic case is that many partial waves with different l values, including $l=0$, contribute to f in the general case. The rare cases in which only one l value is important (usually $l=0$) belong to the quantum threshold regime [4,5].

In the electromagnetic case, the connection between the scattering properties of one center and the index of refraction is well established in the limit of a dilute medium. The simplest way is to define a macroscopic polarizability χ and to use Maxwell's equations. A less usual technique uses integral equations [6]. Finally, this connection can be done in a very simple way by summing the scattered amplitudes [7]. All these treatments give the same result

$$n=1+\frac{\chi}{2}, \tag{2}$$

where

$$\chi=\frac{4\pi}{k^2}f(\vec{k},\vec{k})n_0. \tag{3}$$

*Fax: 33 61 55 83 17. Electronic address: jacques at yosemite.ups-tlse.fr

Here $f(\vec{k},\vec{k})$ is the forward-scattering amplitude and n_0 is the density of the scattering centers.

In the case of atomic waves, the equivalent of Maxwell's equations is not available. The simplest way is to sum the scattered amplitudes, as has been done for neutrons [1] (see also Ref. [7]). It does not appear that there is any new feature and a fully analogous treatment is possible: therefore Eqs. (2) and (3) should have general validity.

SOME SIMPLE CONSEQUENCES

The propagation of an atomic wave in the medium described by the index of refraction n occurs with a modified wave-vector $\vec{k}_m$ given by

$$\vec{k}_m = \vec{k}n. \tag{4}$$

If the imaginary part of n is nonvanishing, the flux Φ transported by the wave is attenuated along the propagation. Assuming $\vec{k}$ parallel to the z axis, the flux Φ which behaves like $|\Psi|^2$ depends on z

$$\Phi = \Phi_0 \exp - [2kz \operatorname{Im}(n)],$$

$$\Phi = \Phi_0 \exp - \left\{ n_0 \frac{4\pi}{k} \operatorname{Im}[f(\vec{k},\vec{k})] z \right\}. \tag{5}$$

One recognizes the total scattering cross section σ given by the optical theorem [4]

$$\sigma = \frac{4\pi}{k} \operatorname{Im}[f(\vec{k},\vec{k})], \tag{6}$$

and Eq. (5) is just the usual attenuation of an atomic beam by diffusion $\Phi = \Phi_0 \exp(-n_0 \sigma z)$. Let us recall that for any interatomic potential which decreases faster than r^{-2} when r goes to infinity, the total cross section calculated by quantum mechanics is finite [4].

The real part of the refraction index modifies the phase velocity. The sign of the effect is not easily predictable. In the general case, the orders of magnitude of the real part and of the imaginary part of $f(\vec{k},\vec{k})$ should be comparable. This means that the attenuation and the phase shift should be related. Recently, Pritchard *et al.* [3] have observed simultaneously an attenuation of the fringe contrast and a phase shift in an atomic interferometer, by putting neon gas in one of the two interfering paths.

The order of magnitude of the modulus of f will give the order of magnitude of $|n-1|$. An interesting case is the quantum threshold regime where only the s wave contributes to f [4]. In this limiting case, the index of refraction is given by

$$n = 1 + \frac{2\pi}{k^2} n_0(-a + ika^2), \tag{7}$$

where a is the scattering length [4]. The index of refraction diverges when the wave-vector k goes to zero, the imaginary part being smaller than the real part. (This divergence is very useful for neutrons as it permits total external refraction of slow neutrons.) Moreover, because of a possible resonance near threshold, the scattering length may be very large as in ^{4}He-^{4}He scattering [8]. Values of the scattering length for heavier atoms have become recently available [9]. This low-energy behavior of the refraction index means that it will play a very important role in dense ultracold gases which are now commonly produced.

In the more common case of ground-state atoms at thermal energies, we will evaluate $|n-1|$ from the values of the total cross section (of the order of 10^{-18} m^2) and of the wave-vector k (of the order of 10^{10} m^{-1}); the index n is roughly given by

$$|n-1| \approx 10^{-28} n_0, \tag{8}$$

when n_0 is the atomic density in m^{-3}.

DOPPLER EFFECT

The thermal motion of the scattering center is easily treated by changing from the laboratory frame to the center of mass frame of the two particles. The forward-scattering amplitude is then evaluated for the relative collision energy. The main effect is to replace in Eqs. (2) and (3) the forward-scattering amplitude by some type of thermal average. Because in usual conditions the velocities of the atom and of the scattering center are of comparable magnitudes, this averaging effect is important and washes out all the sharp resonant structures [4,7] of the function f. This makes a large difference with the scattering of light waves by atoms: usually, the atomic velocity is very small with respect to the velocity of light and the Doppler broadening of the resonant atomic lines remains small.

With the presently available techniques of laser cooling and trapping, one can produce atomic gases with an almost negligible thermal motion. If one sends an atomic beam through such a target gas and measures the transmission of the beam as a function of the velocity, the resonances of the scattering amplitude will be observable. Such an experiment will be the perfect analog with atomic waves of an absorption experiment in atomic spectroscopy. To get the analog of a fluorescence experiment, one should detect the scattered particles. Such experiments are clearly feasible with present techniques, the only difficulty being that the column density of laser cooled gases are rather small (up to a few 10^{14} atoms/m^2). The contrast of a resonance will remain usually rather weak because many waves contribute to f and only one will contribute to the resonance. This is an example of a large difference with ordinary optics where the nonresonant part of the index of refraction of an atomic gas is usually considerably smaller than the resonant part, close to a resonance line.

PROSPECTS

As many efforts are devoted to develop atomic mirrors and cavities [10], I will discuss here only two points related to this type of experiment.

Diffusion of light by light is negligible up to extremely high intensities [11] and this means that the modes of the electromagnetic field are well described by harmonic oscillators. Because of the diffusion of atomic waves by themselves, the eigenmodes of an atomic cavity will depend on the atomic density in the cavity, and the atomic mode will noticeably differ from an harmonic oscillator. For a cavity roundtrip length L, the effect should be important as soon as

$$k\chi L \approx 1. \tag{9}$$

Depending if χ is mostly real or imaginary, the effect will be mainly a cavity damping term or a shift of the resonant k value. If the atom-atom interaction is further assumed to be in the quantum threshold regime, χ is mostly real [see Eq. (7)], the effect is mainly a shift of k, and condition (9) may be rewritten as

$$\frac{n_0 La}{k} \approx \frac{1}{2\pi}. \tag{10}$$

This can be considered as a type of collective quantum effect. We may compare the required density to the threshold density of Bose-Einstein condensation, given by

$$\frac{n_0^{\mathrm{BE}}}{k^3} \approx \frac{1}{8\pi^2}. \tag{11}$$

Because L is a macroscopic distance, if the scattering length a is not too small, the condition (10) can be fulfilled by densities n_0 considerably lower than condition (11).

Another fascinating issue is the development of stimulated emission of bosonic atoms. This phenomenon is explicitly taken into account in the quantum Boltzmann equation [12] and its existence is natural in quantum mechanics. A discussion with the present language directly derived from optics may help to understand how this would be feasible. I have shown above which processes are the atomic analogs of absorption and spontaneous emission of radiation: they are the formation and decay of a quasibound state of the atom A and the scattering center C. The resonances of the AC molecule can also give rise to stimulated emission, following the equation $AC^* + A \rightarrow 2A + C$. It is not clear that a population of AC molecules prepared in one of the possible resonances will be an atomic amplifier, because of the existence of competing processes. The resonance AC^* can emit an atom A in a spontaneous or stimulated manner, and this last effect is the basis of the expected atomic amplifier, but many other processes, such as scattering of A by AC^* or by C (in particular, involving other l waves) are authorized and they compete with the amplification process. All these processes induce losses and they will usually reduce or cancel the possible amplification. Finally, the AC molecular gas should be very cold in order to prevent a large Doppler dilution of the expected gain.

Clearly, an evaluation of all these effects is difficult and it is not easy to predict if this scheme of bosonic amplifier is efficient.

CONCLUSION

This paper has introduced the idea of describing the interaction of an atomic wave with dilute matter by an index of refraction. Such a generalization of the index of refraction has already been made for the propagation of neutrons in dense matter. This idea can be fruitful as it strengthens the analogy between usual optics and atomic optics. Finally, we have shown on some examples that this analogy can stimulate new experiments.

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