Reply to "Comment on 'Perturbation expansion of closed-time-path Green's functions'"

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Controversial points in this Comment concerning the general picture transformations which may give rise to a variety of interaction pictures are analyzed.

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The authors of this Comment [1] criticize the statement in our paper that the correct interaction picture prescribed by

$$|a,t\rangle_I = U(t,-\infty)|a\rangle_H, \tag{1}$$

$$U(t,t_0) = T\,\exp\left[-i\int_{t_0}^{t} H_{int}(t')dt'\right] \tag{2}$$

is the incoming one because they argued that the interaction picture in conformity with Eqs. (1) and (2) is not unique. Their opinion should be appreciated, although what we meant by "the correct interaction picture" was in contrast to the usual one as can be seen from the context of our paper [2]. In spite of this, their general expressions [(8a)–(8c)] to define a set of interaction pictures seems to be controversial. There are two main points to be clarified. First, an interaction picture is physically meaningful only when it can be used to properly define a perturbation expansion. This requires that when a full propagator defined in terms of the Heisenberg fields is transformed into the interaction picture in terms of the free fields the transformation matrices can be contracted successively under the time-ordering operator to form a single S matrix. This means that the transformation matrices should behave as the group elements. It is easy to see that the transformation matrix in (8c) does not generally meet this requirement. For example, for the initial conditions $R_H(0) = 1$ and $R_I(0) = U^{-1}(0,-\infty)$ taken in this comment, it does not satisfy the above-mentioned condition except that the Hamiltonian is time independent which leads to the incoming interaction picture. Another example is that t_0 in (8a)–(8c) cannot be arbitrary, because if one requires $U(t,t_0) = U(t,0)U(0,t_0)$, then t_0 must be zero due to $U(t,t_0) = U_{0S}^{-1}(t,t_0)U_S(t,t_0)$. Thus this set of picture transformations is not as general as the authors claimed.

Second, the introduction of another interaction picture specified by (10a)–(10c) involves changing the fundamental meaning of the Heisenberg picture, even though the validity of this interaction picture is still open. To show this, let us examine (10a)–(10c) for the time-independent case where they have the forms

$$|\Psi_S(t)\rangle = e^{-iHt}U(0,-\infty)|\Psi_H\rangle, \tag{3}$$

$$|\Psi_S(t)\rangle = e^{-iH_0t}|\Psi_I(t)\rangle, \tag{4}$$

$$|\Psi_I(t)\rangle = U(t,-\infty)|\Psi_H\rangle. \tag{5}$$

Transformation (3) defines an unusal Heisenberg operator $A'_H(t)$ which is related to the Shrödinger operator A_S and the usual Heisenberg operator $A_H(t)$ through

$$A'_H(t) = U^{-1}(0,-\infty)e^{iHt}A_S e^{-iHt}U(0,-\infty) \tag{6}$$

$$= U^{-1}(0,-\infty)A_H(t)U(0,-\infty)\ . \tag{7}$$

Let us consider a closed-time path Green function

$$G_p(t_1,...,t_n) = \mathrm{Tr}[\rho T_p A_H(t_1)...A_H(t_n)] \tag{8}$$

with $\rho = \exp(-\beta H)/\mathrm{Tr}[\exp(-\beta H)]$, which is then transformed, according to relations (3)–(5), by using $HU(0,-\infty) = U(0,-\infty)H_0$, into

$$G_p(t_1,...,t_n) = \mathrm{Tr}[e^{-\beta H_0}T_p A_I(t_1)...A_I(t_n)S_p]/\mathrm{Tr}[e^{-\beta H}]. \tag{9}$$

The controversial point of this perturbation expansion lies in the fact that

$$\mathrm{Tr}[e^{-\beta H}] \neq \mathrm{Tr}[e^{-\beta H_0}S_p]. \tag{10}$$

As a consequence, the denominator of (9) cannot be expressed as a sum of thermal vacuum diagrams to cancel the corresponding ones in its numerator which is necessary for a self-consistent perturbation expansion. This shows that not all the solutions satisfying Eq. (9) in the Comment are physically permissible.

[1] Stéphane Vago and Thierry Toutain, Phys. Rev. A **51**, 3377 (1995).

[2] Hong-Hua Xu and Chien-Hua Tsai, Phys. Rev. **41**, 53 (1990).

