Effects of temperature on the absorption line-shape function for driven two-level atoms: A non-Markovian treatment

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The description of a non-Markovian process is presented and the asymptotic solution of the *generalized master equation* is obtained for a two-level atom pumped externally by a monochromatic time-dependent electric field. We give the explicit expression for the absorption line-shape function and discuss its features as a function of the temperature and the memory parameter. We verified the following interesting results: (a) At a critical temperature the line shape undergoes a *phase transition*, the single-bump profile splits into two bumps; (b) for small values of the memory parameter k the linewidths broaden, becoming larger than the ones calculated under the Markovian approximation, and after reaching some maximum value they narrow monotonically with increasing values of k, this behavior being standard for any finite temperature.

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I. INTRODUCTION

In quantum irreversible processes the most familiar treatment that one encounters in the literature is the Markovian approximation, which assumes the time correlation τ of the enviroment variables is much smaller than the characteristic decay time γ^{-1} of the system of interest. Formally the analysis is performed by using a master equation, which is a Liouville equation containing additional terms that come from the coupling of the system with the reservoir and that answer for the irreversibility of the evolution of the system [1–3]. However, non-Markovian effects are important in optics and radiation-matter interaction phenomena, due to the necessity to go beyond the Markovian approximation in experiments involving memory times τ smaller but of the order of γ^{-1} [4–11].

In a previous paper [12] we analyzed and discussed the memory effects on the density-matrix coherence terms of a driven two-level atom in the stationary regime, and we also pointed out the importance of the entropy, as a function of the detuning, in order to discern between Markovian and non-Markovian relaxation processes. Continuing our work in this line the present paper is devoted to the study of effects of the temperature on the absorption line-shape function for a driven two-level atom in its steady state. We observed the existence of a phase transition in the line-shape function, characterized by its split into two components, occurring at a critical temperature, which depends on the memory parameter k. We also studied the linewidths for a continuous range of values of k at several different temperatures.

The paper is organized as follows. In Sec. II the generalized master equation is introduced and in Sec. III its asymptotic solution is obtained. In Sec. IV the absorption line-shape function is deduced. In Sec. V we present, analyze, and discuss the results for the line shape and the linewidth as functions of the temperature and the memory parameter. Finally, Sec. VI contains the summary of our work.

II. THE GENERALIZED MASTER EQUATION

The total Hamiltonian we considered is constituted by the terms

$$\hat{H}=\hat{H}_{0\mathscr{S}}+\hat{H}_{0\mathscr{R}}+\hat{V}^1+\hat{V}^2(t), \tag{1}$$

where $H_{0\mathscr{S}}$ and $H_{0\mathscr{R}}$ are the Hamiltonians of the system of interest ($\mathscr{S}$) and reservoir ($\mathscr{R}$), respectively, while V^1 is the system-reservoir interaction and $V^2(t)$ is a time-dependent external force acting only on the system. Concerning the nature of the whole system some hypotheses are made: (a) the interaction V^1 contains only nondiagonal terms in the representations that diagonalize $H_{0\mathscr{S}}$ and $H_{0\mathscr{R}}$; (b) the system and reservoir are initially uncorrelated, that is, $\rho(0)=\rho_{\mathscr{S}}(0)\rho_{\mathscr{R}}(0)$; (c) the reservoir is ideal, this means it remains undisturbed and it is always in thermal equilibrium, independently of the strength of V^1; therefore $\rho(t)=\rho_{\mathscr{S}}(t)\rho_{\mathscr{R}}(0)$, where

$$\rho_{\mathscr{R}}(0)=e^{-H_{0\mathscr{R}}/kT}/\mathrm{Tr}_{\mathscr{R}}e^{-H_{0\mathscr{R}}/kT}$$

is the canonical distribution of the reservoir and the reduced density operator is obtained from $\hat{\rho}_{\mathscr{S}}(t)=\mathrm{Tr}_{\mathscr{R}}\hat{\rho}(t)$.

With the above ingredients the generalized master equation (GME) in the interaction picture is written as

$$\frac{\partial\hat{\rho}_{\mathscr{S}\mathscr{I}}(t)}{\partial t}=-i[\hat{V}^2_{\mathscr{I}}(t),\hat{\rho}_{\mathscr{S}\mathscr{I}}(t)] - \int_0^t dt' \;\mathrm{Tr}_{\mathscr{R}}\{\hat{V}^1_{\mathscr{I}}(t),[\hat{V}^1_{\mathscr{I}}(t'),\hat{\rho}_{\mathscr{S}\mathscr{I}}(t')\hat{\rho}_{\mathscr{R}}(0)]\}. \tag{2}$$

The physical problem we are going to study is a two-level atom ($\mathscr{S}$) coupled to a reservoir ($\mathscr{R}$) and driven by a classi-

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cal (c-number) monochromatic electric field. For this case the terms of the Hamiltonian (1) are ($\hbar=1$) (a) $\hat{H}_{0\mathscr{S}}=\omega_0/2(|\uparrow\rangle\langle\uparrow|-|\downarrow\rangle\langle\downarrow|)$, representing two-level atoms embedded in the reservoir and that are sufficiently diluted so their interactions can be ignored; $|\uparrow\rangle$ and $|\downarrow\rangle$ stand for the upper and lower levels, whose energies are $\varepsilon_1=\omega_0/2$ and $\varepsilon_2=-\omega_0/2$; (b) $\hat{H}_{0\mathscr{R}}=\Sigma_n\omega_n b_n^\dagger b_n$, describing the reservoir, constituted by a huge number of photons (or phonons) in thermal equilibrium at some temperature T; (c) $\hat{V}^1=\Sigma_n(K_n^* b_n^\dagger|\downarrow\rangle\langle\uparrow|+\text{H.c.})$ is the system-reservoir interaction, given in the rotating wave approximation (RWA); (d) $\hat{V}^2(t)=-\vec{\mu}\cdot\vec{E}=(F_0 e^{i\omega t}|\downarrow\rangle\langle\uparrow|+\text{H.c.})$ representing the pumping of the system by the external field; in this expression

$$\vec{E}(t)=(E_0\hat{\vec{\epsilon}}_- e^{i\omega t}+E_0^*\hat{\vec{\epsilon}}_+ e^{-i\omega t}),\quad F_0=-\vec{\mu}\cdot\vec{E}_0,$$

and

$$\vec{\mu}=\mu\hat{\vec{\epsilon}}_+|\downarrow\rangle\langle\uparrow|+\mu^*\hat{\vec{\epsilon}}_-|\uparrow\rangle\langle\downarrow|.$$

III. ASYMPTOTIC SOLUTION OF THE GME

The reduced density operator in Eq. (2) is expanded in terms of a complete set of operators $\hat{\rho}_{\mathscr{S}\mathscr{A}}(t)=\Sigma_{j=1}^4 W_j(t)\hat{O}_j$, where $\hat{O}_1=|\uparrow\rangle\langle\uparrow|$, $\hat{O}_2=|\downarrow\rangle\langle\downarrow|$, $\hat{O}_3=|\uparrow\rangle\langle\downarrow|$, and $\hat{O}_4=|\downarrow\rangle\langle\uparrow|$. W_1 and W_2 are the occupation probabilities of the upper and lower levels, respectively, while W_3 and W_4 are the coherence coefficients. Therefore, with this expansion, the q-number GME (2) goes to a c-number system of equations,

$$\dot{W}_j(t)=\sum_{k=1}^{2}\int_0^t dt' Q_{jk}(t,t')W_k(t')\;-(-1)^j[iF_0\bar{W}_3(t)-iF_0^*\bar{W}_4(t)],\quad j=1,2 \tag{3}$$

and

$$\dot{\bar{W}}_j(t)=\int_0^t dt'\bar{Q}_{jj}(t,t')\bar{W}_j(t')\;-(-1)^j[iF_0^*W_1(t)-iF_0^*W_2(t)-i\Delta\omega\bar{W}_j(t)],\quad j=3,4 \tag{4}$$

where $\bar{W}_3(t)=e^{-i\Delta\omega t}W_3(t)$ and $\Delta\omega=(\omega_0-\omega)$ is the detuning. We remind the reader that $W_1+W_2=1$ and $W_4=W_3^*$. The occupation probability kernels are

$$Q_{jk}(t,t')=-2(-1)^{j+k}\sum_m|K_m(\omega_m)|^2[\bar{n}(\omega_m)+\delta_{k1}]\times\cos(\omega_0-\omega_m)(t-t'),\quad j,k=1,2 \tag{5}$$

and the coherence kernels are

$$\bar{Q}_{33}(t,t')=-\sum_m|K_m(\omega_m)|^2[2\bar{n}(\omega_m)+1]e^{i(\omega-\omega_m)(t-t')} \tag{6}$$

and $\bar{Q}_{44}(t,t')=\bar{Q}_{33}(t,t')^*$. In the above expressions $\bar{n}(\omega_m)=(e^{\omega_m/k_BT}-1)^{-1}$ is the mean number of photons (or phonons) with frequency ω_m, k_B is the Boltzmann constant, and T is the absolute temperature.

The system of first order differential equations, Eqs. (3) and (4), is solved by Laplace transform technique, which leads to a system of algebraic equations for the transforms $\tilde{W}_j(p)$ and $\tilde{\bar{W}}_j(p)$ and whose solution yields

$$\tilde{W}_j(p)=\sum_{k=1}^{2}X_{jk}(p)\left[W_k(0)-(-1)^k\left(iF_0\frac{W_3(0)}{p+i\Delta\omega-\tilde{\bar{Q}}_{33}(p)}+\text{c.c.}\right)\right],\quad j=1,2 \tag{7}$$

with

$$X_{jk}(p)=\frac{p\,\delta_{jk}+(\tilde{Q}_{12}\delta_{j1}+\tilde{Q}_{21}\delta_{j2})+|F_0|^2h(p)}{p[p+\tilde{Q}_{12}(p)+\tilde{Q}_{21}(p)+2|F_0|^2h(p)]} \tag{8}$$

and

$$\tilde{\bar{W}}_3(p)=iF_0^*\frac{\tilde{W}_1(p)-\tilde{W}_2(p)+W_3(0)}{p+i\Delta\omega-\tilde{\bar{Q}}_{33}(p)}, \tag{9}$$

with $\tilde{\bar{W}}_4(p)=\tilde{\bar{W}}_3(p)^*$.

The quantities $\tilde{Q}_{jk}(p)$ and $\tilde{\bar{Q}}_{33}(p)$ are the transforms of the memory kernels $Q_{jk}(t)$ and $\bar{Q}_{33}(t)$ and they are given by the expressions

$$\tilde{Q}_{12}(p)=\int_0^\infty d\omega' g(\omega')|K(\omega')|^2\bar{n}(\omega')\frac{2p}{p^2+(\omega_0-\omega')^2}, \tag{10}$$

$$\tilde{Q}_{21}(p)=\int_0^\infty d\omega' g(\omega')|K(\omega')|^2[\bar{n}(\omega')+1]\times\frac{2p}{p^2+(\omega_0-\omega')^2}, \tag{11}$$

$$\tilde{\bar{Q}}_{33}(p)=-\int_0^\infty d\omega' g(\omega')|K(\omega')|^2[2\bar{n}(\omega')+1]\times\frac{1}{p-i(\omega-\omega')}, \tag{12}$$

in which the density distribution function $g(\omega)$ was introduced for the frequencies of the reservoir's bosons. In Eq. (8) $h(p)=\{1/[p+i\Delta\omega-\tilde{\bar{Q}}_{33}(p)]+\text{c.c.}\}$.

The asymptotic solutions are obtained by calculating the limit $W_j^\infty=\lim_{p\to0}p\tilde{W}_j(p)$, $j=1,2$; $\bar{W}_j^\infty$, $j=3,4$, come from a relation equivalent to the previous one. So we have in the Schrödinger picture

$$W_j^\infty=\frac{[\tilde{Q}_{12}(0)\delta_{j1}+\tilde{Q}_{21}(0)\delta_{j2}]+|F_0|^2h(0)}{\tilde{Q}_{12}(0)+\tilde{Q}_{21}(0)+2|F_0|^2h(0)},\quad j=1,2 \tag{13}$$

and

$$W_3^\infty = e^{-i\omega t}\frac{iF_0^*[\tilde{Q}_{12}(0)-\tilde{Q}_{21}(0)]}{[\tilde{Q}_{12}(0)+\tilde{Q}_{21}(0)+2|F_0|^2h(0)][+i\Delta\omega-\tilde{\tilde{Q}}_{33}(0)]}. \tag{14}$$

It is worth noting that the quantities W_j^∞ do not depend on the initial conditions, given by $W_j(0)$, as one can see from these equations.

In order to get analytical results for the asymptotic diagonal and nondiagonal elements of $\hat{\rho}_{\mathcal{S}}$, we have to introduce shapes for $g(\omega)$. For any choice of the product $g(\omega')|K(\omega')|^2$, the stationary kernels $\tilde{Q}_{12}(0)=\gamma\bar{n}(\omega_0)$ and $\tilde{Q}_{21}(0)=\gamma[\bar{n}(\omega_0)+1]$ remain the same, where $\gamma=2\pi g_0|K(\omega_0)|^2$ is the damping constant of the system. Contrarily, the kernel $\tilde{\tilde{Q}}_{33}(0)$ is sensitive to the shape of the product $g(\omega')|K(\omega')|^2$. For example, (a) $g(\omega')|K(\omega')|^2=g_0|K_0|^2$, constant, characterizes a Markovian correlation for the reservoir operators; the time-dependent kernels have null memory time, $Q_{jk}(t-t')\sim\delta(t-t')$; (b) any other choice leads to a non-Markovian process; we shall adopt in this work the Cauchy distribution

$$g(\omega')=g_0\frac{1}{1+(\omega'-\omega_0)^2\tau^2}, \tag{15}$$

where the memory time τ is a characteristic of the reservoir. Equation (15) is equivalent to considering an exponential time decay in the kernels, $Q_{jk}(t-t')\sim e^{-|t-t'|/\tau}$.

IV. THE ABSORPTION LINE-SHAPE FUNCTION

The rate at which quanta are absorbed from the external field is expressed by [13,14]

$$F=\frac{1}{\varepsilon_1-\varepsilon_2}\{\varepsilon_1[\dot{W}_1(t)]_{EF}+\varepsilon_2[\dot{W}_2(t)]_{EF}\}, \tag{16}$$

where $[\dot{W}_1(t)]_{EF}$ and $[\dot{W}_2(t)]_{EF}$ are the rates of change of the occupation probabilities of the levels $|\uparrow\rangle$ and $|\downarrow\rangle$ induced by the external field (EF). These quantities are given by the terms containing the coupling parameters to the field, F_0 and F_0^* in Eq. (3). Then in the Schrödinger picture they are

$$[\dot{W}_1(t)]_{EF}=iF_0W_3(t)e^{i\omega t}-iF_0^*W_4(t)e^{-i\omega t}$$

and

$$[\dot{W}_2(t)]_{EF}=-iF_0W_3(t)e^{i\omega t}+iF_0^*W_4(t)e^{-i\omega t},$$

which yield $[\dot{W}_1(t)]_{EF}=-[\dot{W}_2(t)]_{EF}$. Inserting this result in Eq. (16) one writes for the absorption line-shape function F

$$F=iF_0W_3(t)e^{i\omega t}-iF_0^*W_4(t)e^{-i\omega t}$$

and finally

$$F=-2|F_0|^2\mathrm{Im}\left(\frac{W_3(t)}{F_0^*}e^{i\omega t}\right), \tag{17}$$

which shows the dependence of the line-shape function on the coherence coefficient $W_3(t)$. The asymptotic value of F is obtained by taking W_3^∞ in Eq. (17).

Our aim is to investigate the asymptotic behavior of the temperature-dependent absorption line-shape function with the detuning for non-Markovian processes. As we have already mentioned, we took for the density of levels function a Cauchy distribution, Eq. (15), and this provides the following expression for the dimensionless line-shape function $F_{dim}=2F/\gamma$:

$$F_{dim}=\frac{2|F_0|^2/\gamma^2}{[\bar{n}(\omega_0)+1/2]^2+\zeta^2(1+k^2\zeta^2)-2k\zeta^2[\bar{n}(\omega_0)+1/2]+2|F_0|^2/\gamma^2}, \tag{18}$$

where $k=\gamma\tau$ is the memory parameter and $\zeta=\Delta\omega/\gamma$.

V. RESULTS AND DISCUSSION

The function (18) presents two distinct line shapes that depend on a relation between k and $\bar{n}(\omega_0)$. For $k[2\bar{n}(\omega_0)+1]<1$ the line shape presents a single-bump profile whereas for $k[2\bar{n}(\omega_0)+1]>1$ two symmetric bumps are present. This can be viewed as a kind of a phase transition of the line shape that depends on the temperature (through $\bar{n}$) and the memory parameter k. To visualize this behavior Fig. 1 exhibits F_{dim} as a function of the detuning for $|F_0|/\gamma=5$, $\bar{n}=5$ and three values of k, $k=0$ (Markovian case), 0.05, 0.08, while Fig. 2 corresponds to the same values of $\bar{n}$ and $|F_0|/\gamma$ and also three values of k, $k=0,0.10,0.20$.

The height of the twin-bump line shape is

$$\Lambda=\frac{8k^2|F_0|^2/\gamma^2}{\{k[2\bar{n}(\omega_0)+1]-1\}^2+8k^2|F_0|^2/\gamma^2+1}, \tag{19}$$

whereas for the single-bump line shape the height is the same as one obtained in the Markovian approximation,

$$\lambda=\frac{8|F_0|^2/\gamma^2}{[2\bar{n}(\omega_0)+1]^2+8|F_0|^2/\gamma^2}. \tag{20}$$

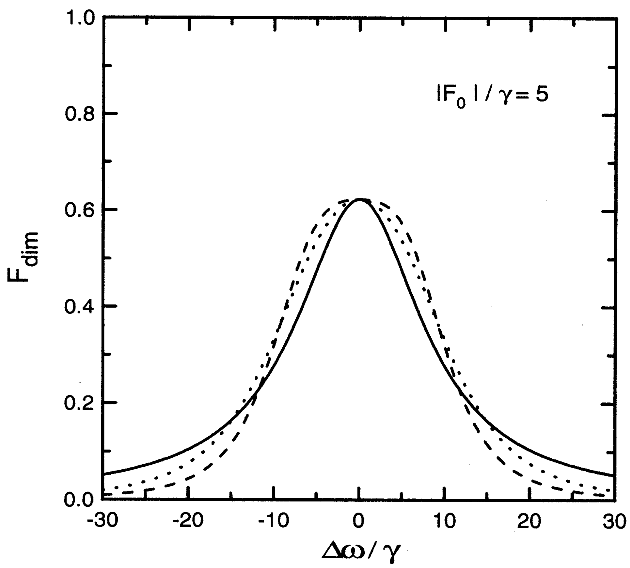

FIG. 1. The absorption line-shape function F_{dim} as a function of the detuning for $\bar{n}=5$ and three values of the memory parameter k, satisfying the condition $k(2\bar{n}+1)<1$. The solid line corresponds to $k=0$ (Markovian case), the dotted line corresponds to $k=0.05$, and the dashed line corresponds to $k=0.08$.

In order to characterize the phase transition we are going to analyze the distance between the points of maxima of the two bumps, given by

$$D=\frac{\sqrt{2}}{k}\,\{k[2\bar{n}(\omega_0)+1]-1\}^{1/2}. \tag{21}$$

Figure 3 shows D as a function of k for different values of

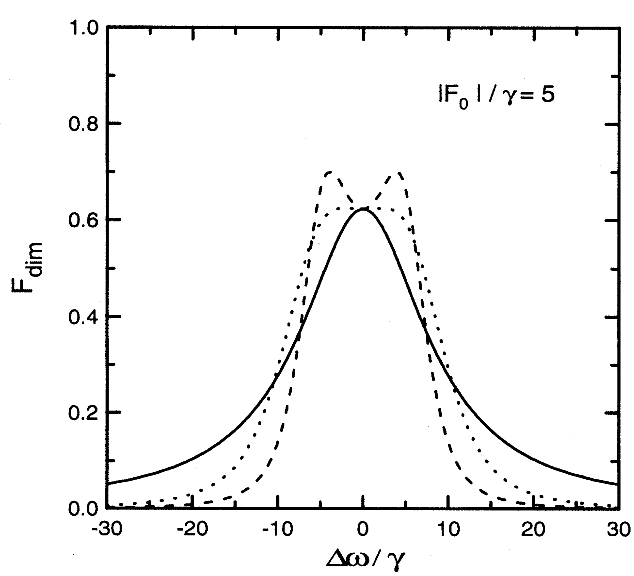

FIG. 2. The absorption line-shape function F_{dim} as a function of the detuning for $\bar{n}=5$ and two values of the memory parameter k, satisfying the condition $k(2\bar{n}+1)>1$. The solid line corresponds to $k=0$ (Markovian case), the dotted line corresponds to $k=0.10$, and the dashed line corresponds to $k=0.20$.

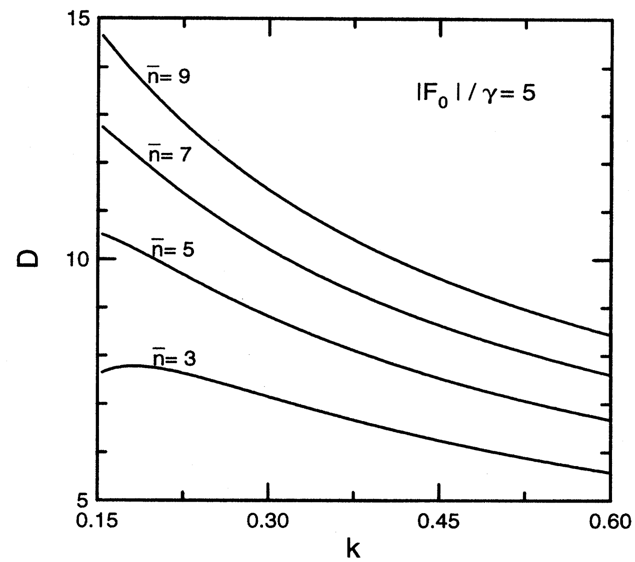

FIG. 3. The distance D between the maxima of the two bumps as a function of k and with $\bar{n}$ as parameter.

$\bar{n}$ that depends on the absolute temperature T. Remembering that the parameter k is characteristic of the reservoir, the critical temperature at which the transition occurs is determined from $k[2\bar{n}_c(\omega_0)+1]=1$ or

$$T_c=\frac{\hbar\omega_0}{k_B}\left[\ln\left(\frac{1+k}{1-k}\right)\right]^{-1}. \tag{22}$$

For small values of k the behavior of T_c is inversely proportional to k, $T_c\sim\hbar\omega_0/(2k_B)(1/k)[1-1/3k^2+O(k^4)]$. Now, the temperature dependence of the distance D around the critical temperature is

$$D=\frac{\sqrt{2}}{k}\,\{k[2\bar{n}_c(\omega_0)+2(T-T_c)\bar{n}_c'(\omega_0)+1]-1\}^{1/2}, \tag{23}$$

where $\bar{n}_c(\omega_0)=(e^{\omega_0/k_BT_c}-1)^{-1}$, and a direct calculation leads to

$$D\simeq\begin{cases}\dfrac{2k_BT_c}{\hbar\omega_0}\,\dfrac{T-T_c}{T_c}^{1/2}, & T>T_c\\ 0, & T<T_c\end{cases} \tag{24}$$

so the critical exponent 1/2 is characteristic of a phase transition that occurs due to the presence of a fourth power polynomial in the detuning ζ in the denominator of the line-shape function, Eq. (18). Meanwhile, in the Markovian approximation the phase transition is absent because the line-shape function is inversely proportional to a quadratic polynomial in the detuning.

Now going to the analysis of the linewidth, defined at half height of the line shape, we have two different functions, one for each phase,

$$\Gamma=\frac{\sqrt{2}}{k}\begin{cases}(\{k[2\bar{n}(\omega_0)+1]-1\}+\sqrt{k^2\Gamma_M^2+\{k[2\bar{n}(\omega_0)+1]-1\}^2})^{1/2} & \text{for } k(2\bar{n}+1)<1\\ (\{k[2\bar{n}(\omega_0)+1]-1\}+\sqrt{k^2\Gamma_M^2-\{k[2\bar{n}(\omega_0)+1]-1\}^2})^{1/2} & \text{for } k(2\bar{n}+1)>1,\end{cases} \tag{25}$$

where Γ_M is the standard Markovian linewidth [14,15] given by

$$\Gamma_M=\{[2\bar{n}(\omega_0)+1]^2+8|F_0|^2/\gamma^2\}^{1/2}, \tag{26}$$

and at the transition $k[2\bar{n}(\omega_0)+1]=1$ one has $\Gamma=(2\Gamma_M/k)^{1/2}$. It is worth noting that Γ may be larger or smaller than Γ_M, depending on the values of $\bar{n}(\omega_0)$ and k.

Figure 4 presents the linewidth as a function of the memory parameter k and with $\bar{n}$ as parameter. It also shows a curve, the dashed one, that separates the phases corresponding to the conditions $k[2\bar{n}(\omega_0)+1]<1$ and $k[2\bar{n}(\omega_0)+1]>1$. We note that for small values of k there occurs a broadening of Γ, for any temperature, reaching a maximum value and then the line shape narrows monotonically, becoming for large values of k proportional to $k^{-1/2}$. This narrowing of the linewidth is a characteristic of a non-Markovian relaxation process, because as k increases the GME becomes closer to a reversible system of equations, for which the linewidth of the levels is zero. We also note that for a fixed value of k the linewidth narrows with the decreasing of the temperature.

In order to resolve the twin bumps of the line shape we define the parameter $\sigma=\Lambda/\lambda-1$ and impose the condition $\sigma>1$, such that the height at the points of maximum is at least twice the value of the height at the point of minimum, at $\zeta=0$ (which also is the height of the Markovian line shape). The above condition leads to $k>1/(2\bar{n}+1-\Gamma_M/\sqrt{2})$ and for the denominator being a positive quantity one has the additional condition $\bar{n}>\sqrt{2}|F_0|/\gamma-1/2$. Under these requirements Fig. 5 exhibits the line shape for $\bar{n}=9$, $k=0.5$, and $|F_0|/\gamma=5$, and one perceives that the absorption line shape is constituted by two peaks located symmetrically with respect to $\Delta\omega=0$. At half height of the bumps the linewidth of each one is given by

$$\Gamma_{1B}=\frac{1}{\sqrt{2k}}(\{k[2\bar{n}(\omega_0)+1]-1\}+\sqrt{k^2\Gamma_M^2-\{k[2\bar{n}(\omega_0)+1]-1\}^2})^{1/2}-\frac{1}{\sqrt{2k}}(\{k[2\bar{n}(\omega_0)+1]-1\}-\sqrt{k^2\Gamma_M^2-\{k[2\bar{n}(\omega_0)+1]-1\}^2})^{1/2}. \tag{27}$$

The transition can be understood as due to the following mechanism: As long as the temperature of the reservoir is kept below T_c the atomic system absorbs energy mainly from the pumping field at frequencies around ω_0; now, for temperatures above T_c the number of photons of the reservoir increases and they compete with the pumping field for being absorbed by the system. The critical value N_{ph}^c of the number of photons of the reservoir can be obtained from a straightforward calculation: the number of photons is $N_{ph}=\int_0^\infty g(\omega')\bar{n}(\omega')d\omega'=\pi g_0\bar{n}(\omega_0)/\tau$; then at the phase transition, $k[2\bar{n}(\omega_0)+1]=1$, one has

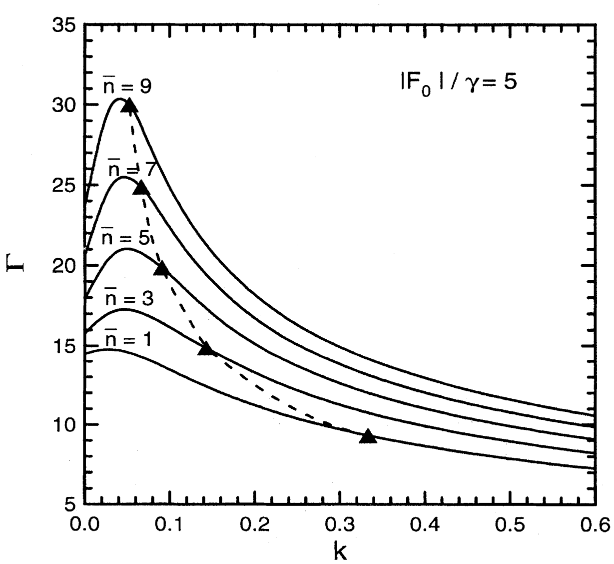

FIG. 4. The linewidth Γ as a function of k and with $\bar{n}$ as parameter. The dashed curve separates the phases corresponding to the conditions $k(2\bar{n}+1)<1$ and $k(2\bar{n}+1)<1$.

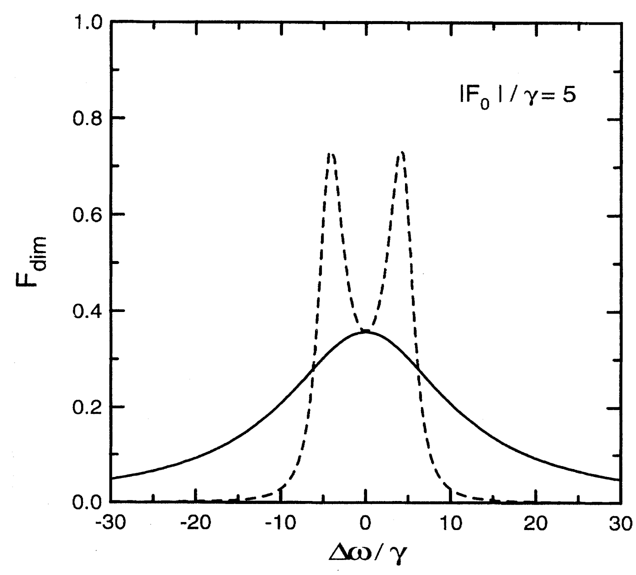

FIG. 5. The absorption line-shape function F_{dim} as a function of the detuning for $\bar{n}=5$. The solid line corresponds to $k=0$ (Markovian case) and the dashed one, corresponding to $k=0.5$, has two peaks whose linewidths at half height of the bumps are given by Eq. (27).

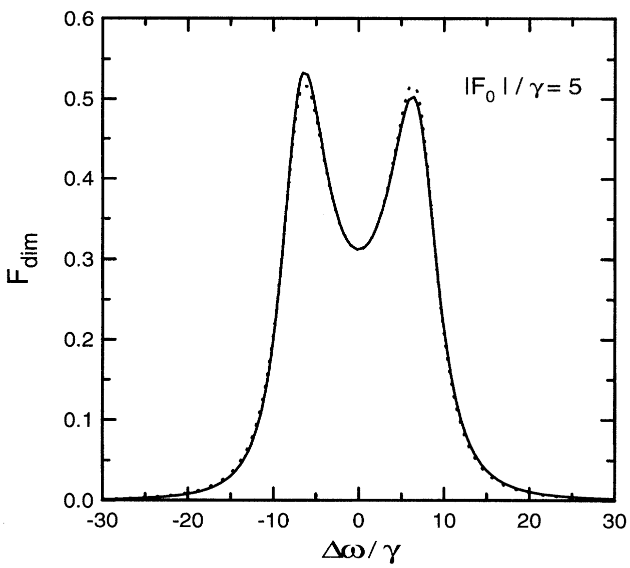

FIG. 6. The absorption line-shape function F_{dim} as a function of the detuning for $\bar{n}=10$, $k=0.2$, and $\omega/\gamma=130$. The dotted line corresponds to the first term in the expansion of $\bar{n}(\omega')$, Eq. (29), while the solid line includes the linear correction term that causes a symmetry breaking in the curve.

$$N_{ph}^c=\left(\frac{1}{2\tau|K|}\right)^2, \tag{28}$$

and for $k[2\bar{n}(\omega_0)+1]>1$, $N_{ph}>N_{ph}^c$. The critical number of photons is inversely proportional to the square root of the product of the reservoir memory time τ and the coupling between the system and reservoir. When $N_{ph}>N_{ph}^c$ the absorption from the pumping field is shifted to the neighboring frequencies $\omega=\omega_0\pm D/2$ while for frequencies ω around ω_0 the reservoir will furnish most of the absorbed energy, due to the shape of the frequency distribution $g(\omega)$.

In Eqs. (10)–(12) we have considered $\bar{n}(\omega')=\bar{n}(\omega_0)$ in order to get the above results that enabled us to analyze and discuss the effects of temperature on the phase transition in the absorption line shape in a transparent way. Now, a more accurate calculation needs to consider the expansion of $\bar{n}(\omega')$ around ω_0,

$$\bar{n}(\omega')=\bar{n}(\omega_0)+(\omega'-\omega_0)\bar{n}'(\omega_0)+\cdots. \tag{29}$$

The introduction of the linear correction term in the integrals (10)–(12) is discussed in the Appendix, where the dimensionless line-shape expression is given for the two-bump phase. Figure 6 shows F_{dim} and we note that the correction considered causes the symmetry breaking in the line-shape function. Obviously these line shapes should be closer to an actual situation if the temperature effects can be verified experimentally.

It is worth mentioning that the split of absorption line shapes was already observed, however, in a different context: In a resonant modulation experiment Autler and Townes [16] verified that at certain atomic frequencies an applied rf field affects the absorption lines of microwave radiation, leading to their split into two components. Otherwise the system we presented is subject to only one external radiation field and for a given k the split occurs when $T>T_c$.

V. SUMMARY

In this work we analyzed, in the non-Markovian approach, the effects of the temperature on the absorption line-shape function of a driven two-level atom in the steady state regime. We verified the existence of a phase transition characterized by a change on the profile of the line shape from one bump to two bumps. The critical temperature was determined and we verified that the distance between the two bumps near the critical temperature is proportional to $|T-T_c|^{1/2}$, with the critical exponent 1/2 being a consequence of the presence of a fourth power polynomial in the detuning in the denominator of the line-shape function. This criticality is absent in the Markovian approximation since the polynomial reduces to a second power one. Moreover we noted the symmetry of the line shapes since they are an even function of the detuning.

Concerning the linewidths, we verified that for small values of the memory parameter k they broaden, becoming larger than the ones calculated under the Markovian approximation, and after attaining some maximum value they narrow with increasing values of k and asymptotically they behave as $k^{-1/2}$, irrespective of the temperature.

We have calculated the line shapes with the usual approximation $\bar{n}(\omega')=\bar{n}(\omega_0)$, constant, in the integrals involving the memory kernels in order to give analytical expressions to our results, which facilitate the physical analysis. The introduction of the correction to $\bar{n}(\omega')$ breaks the symmetry of the line shapes, which is a situation closer to what can actually be observed experimentally.

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APPENDIX

Considering in Eq. (29) the linear correction term (second term) and introducing it in the integrals (10)–(12), the line-shape function (18), that we now write as $F_{dim}(\zeta,0)$, is modified to

$$F_{dim}(\zeta,b/a)=\frac{4|F_0|/\gamma^2(1+k^2\zeta^2)}{2a\left[a[1+(b/a)\zeta]+\frac{[(1+k^2\zeta^2)\zeta-ak\zeta+(a/k)(b/a)]^2}{a[1+(b/a)\zeta]}\right]+4|F_0|^2\gamma^{-2}(1+k^2\zeta^2)}, \tag{A1}$$

where

$$a=\bar{n}(\omega_0)+1/2 \quad \text{and} \quad b=-\gamma\bar{n}'(\omega_0).$$

Since the ratio $b/a \ll 1$, expanding Eq. (A1) up to first order correction in b/a, we obtain

$$F_{dim}(\zeta,b/a)=F_{dim}(\zeta,0)\left(1+g(\zeta)f(\zeta)\frac{b}{a}\right), \tag{A2}$$

where

$$f(\zeta)=\frac{F_{dim}(\zeta,0)}{4|F_0|^2\gamma^{-2}}$$

and

$$g(\zeta)=\frac{2\zeta}{k}[(1+k^2\zeta^2)(k\zeta^2-2a)+a^2k].$$

In Eq. (18), for $k[2\bar{n}(\omega_0)+1]>1$, the points of extrema are located at $r_0=0$ (minimum and $r_\pm=\pm(1/k)[(2ak-1)/2]^{1/2}$ (symmetric maxima). With the introduction of the correction term, these points are shifted, approximately, to $r'_0=\eta_0$ and $r'_\pm=r_\pm-\eta_1$, where

$$\eta_0=-\frac{b}{4k}\left(1-\frac{3}{2k^2r_+^2}\right)$$

and

$$\eta_1=\frac{1}{4}\frac{b}{a}\left[3r_+^2-\frac{a}{2k}\left(1-\frac{3}{2k^2r_+^2}\right)\right].$$

Therefore up to first order in b/a the distance between the points of maxima is not modified, although all three points are shifted from their original positions.

Besides the shifts in the location of the extrema points in the line-shape function, an asymmetry is present in the heights of the bumps, and their difference, calculated in the same approximation, is

$$F_{dim}(r_-,b/a)-F_{dim}(r_+,b/a) \approx 4F_{dim}(r_\pm,0)\frac{r_+}{k}\left(a+\frac{1}{4k}\right)\left(\frac{b}{a}\right),$$

also linear in b/a.

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