Reshaping, path uncertainty, and superluminal traveling

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It is shown that when the wave packet of a photon passes through a region where more than one path mode can be chosen, the interference between these modes of the probability waves of the photon causes a reshaping process of the wave packet. In some conditions this reshaping gives rise to an apparent superluminal traveling, as observed by Steinberg, Kwiat, and Chiao [Phys. Rev. Lett. **71**, 708 (1993)]. The present model also leads to some predictions that can be tested experimentally.

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Tunneling is a peculiar phenomena that can be understood only in quantum mechanics. As early as the 1930s, MacColl [1] pointed out that tunneling was characterized not only by a tunneling rate, but also by a tunneling time that the particle spent in the barrier. Since then many approaches to the tunneling time have been presented. Most of them imply that particles may move faster in barriers than in free space, but the physical nature of this high-speed traveling is not clear [2,3]. Recently Steinberg, Kwiat, and Chiao [4] demonstrated an excellent experiment that directly measured the time for a single photon tunneling across a barrier. Their experiment indicated an apparently superluminal transport in a barrier. They suggested [4,5] that the reshaping of the wave packet was responsible for this superluminal phenomenon, but they left an open question of why the reshaping should occur.

It should be noted that the barrier used in the Steinberg-Kwiat-Chiao (SKC) experiment was obviously different from the conventional one (for example, the square potential barrier) where the tunneling photons have pure imaginary components of the wave vector. In the SKC experiment, the tunnel barrier was a multilayer dielectric mirror that actually formed a one-dimensional photon crystal with a "photonic band gap" [6]. The tunneling process in this photon crystal is an analog of the Zener tunneling [7] in solid-state physics. The wave vector of a photon in the gap region is $k=\frac{1}{2}G+i\kappa$, where both G and κ are real and G is a reciprocal-lattice vector. Since G is a nonzero real number, k is a complex quantity. In what follows we will show that when the wave packet of a photon travels in a path-uncertain region, the interference between different path modes of the probability wave of the photon causes a reshaping process of the wave packet. The picture not only can explain the SKC results, but also leads to some predictions that can be tested experimentally.

The starting point of the present discussion is that when a photon beam travels across a region where many paths can be chosen, the output is a superposition of the waves along all the path modes, whether the incident beam is composed of a large number of photons or only of a single photon. Suppose the time-dependent incident wave function at initial point x_b is $\varphi_0(t)$. The output wave function at the final point x_e would be

$$\varphi(t)=\sum_i \eta_i\varphi_0(t-\tau_i), \tag{1}$$

where $|\eta_i|^2$ is the probability that the photon takes the ith path and τ_i the corresponding traveling time. From Eq. (1) we see that $\varphi(t)$ does not have a one-to-one time correspondence with the incident wave function. Rather $\varphi(t)$ depends on the distribution of $\varphi_0(t-\tau_i)$ in the time axis. Assuming that the incident beam is a wave packet with a Gaussian shape, the center of which is at point x_b when $t=0$,

$$\varphi_0(t)=f_0(t)\exp(i\omega t)=A\ \exp\left[-\frac{1}{2}\left(\frac{t}{\delta t}\right)^2\right]\exp(i\omega t), \tag{2}$$

where ω is the frequency centroid, A the normalizing factor, and δt the width of the wave packet ($\delta t\gg 2\pi/kc$). Combining Eqs. (1) and (2), we have

$$\begin{aligned}\varphi(t)&=\sum_i \eta_i f_0(t-\tau_i)\exp(-i\omega\tau_i)\exp(i\omega t)\\&=A\ \eta_i\exp\left[-\frac{1}{2}\left(\frac{t-\tau_i}{\delta t}\right)^2\right]\exp(-i\omega\tau_i)\exp(i\omega t).\end{aligned} \tag{3}$$

In order to give a deep insight into the physics, yet without losing the generality of the results, we discuss a simple case in which there are only two paths connecting the two points x_b and x_e. The wave function at the end point x_e would be

$$\begin{aligned}\varphi(t)=&\ \eta_1 f_0(t-\tau_1)\exp(-i\omega\tau_1)\exp(i\omega t)\\&+\eta_2 f_0(t-\tau_2)\exp(-i\omega\tau_2)\exp(i\omega t),\end{aligned} \tag{4}$$

where τ_1 and τ_2 (suppose $\tau_2>\tau_1$) are, respectively, the traveling times of a photon in the two paths. If there is a phase difference of π between the phases of the probability waves from the two paths, the amplitude of $\varphi(t)$ can be written as

$$f_T(t)=|f_{T1}(t)-f_{T2}(t)|, \tag{5}$$

where

$$f_{T1}(t)=|\eta_1|f_0(t-\tau_1), \tag{6}$$

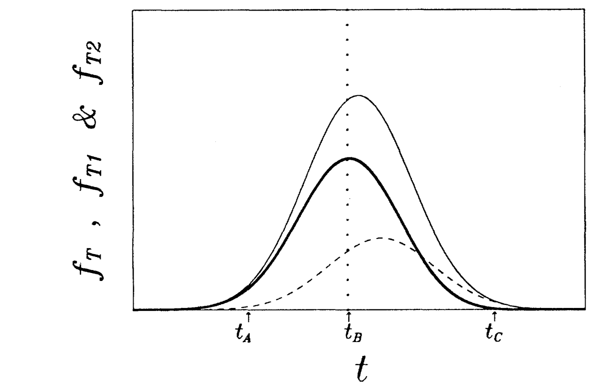

FIG. 1. Probability amplitudes $f_T(t)$ (thick solid curve), $f_{T1}(t)$ (thin solid curve), and $f_{T2}(t)$ (dashed curve) as a function of the time t for the waves arriving at the final point x_e for $|\eta_2/\eta_1|=\frac{1}{3}$ and $(\tau_2-\tau_1)/(\sqrt{2}\delta t)=0.3$. The vertical dotted line indicates the arrived time of the gravity center of the wave packet f_T.

$$f_{T2}(t)=|\eta_2|f_0(t-\tau_2) \tag{7}$$

are the transmitted amplitudes with only one path open. Then the average traveling time for those photons arriving at x_e from x_b can be obtained by

$$\tau=\frac{\int f_T^2(t)t\,dt}{\int f_T^2(t)dt}\,. \tag{8}$$

Because of the interference, only part of the photons can arrive at x_e, so the average time τ only applies for this part and not for all the photons that start from x_b. It is easy to prove that

$$\tau<\tau_1<\tau_2 \quad \text{if } |\eta_2|<|\eta_1|,\ \frac{|\eta_2|}{|\eta_1|}<\exp\left[-\left(\frac{\tau_2-\tau_1}{2\delta t}\right)^2\right],$$

$$\tau>\tau_2>\tau_1 \quad \text{if } |\eta_1|<|\eta_2|,\ \frac{|\eta_1|}{|\eta_2|}<\exp\left[-\left(\frac{\tau_2-\tau_1}{2\delta t}\right)^2\right],$$

$$\tau_1\leq\tau\leq\tau_2 \quad \text{otherwise}. \tag{9}$$

Therefore, in some particular conditions, the transmitted photons arrive before (or after) those traveled along any one definite path. Figure 1 shows a typical example of the first case given in the first of Eqs. (9). The leading part (for example, at time t_A) of the transmitted packet (f_T) is very similar to the leading part of the transmitted wave from the first path (f_{T1}). This means that the interference can be neglected for this part and the transmitted photon is, with very large probability, from the first path. At time t_B, f_T is much smaller than f_{T1}. In this case, the interference plays an important role and we cannot deduce which path the photon has passed through. In other words, the path uncertainty becomes larger. At time t_C, f_{T2} is comparable with f_{T1} and f_T is approximately equal to zero. In this case, the interference nearly completely holds back the transmission. From time t_A to t_C, the interference plays a more and more important role in holding back the transmission so that the transmission probability becomes smaller and smaller. Therefore, the leading part of the wave packet has larger transmission probability than the trailing part. In other words, if a photon starts early, it traverses more easily the path-uncertainty region than it does if it starts late. This is a simple example of the reshaping and the superluminal traveling. From this we can see how the reshaping and the superluminal traveling occur.

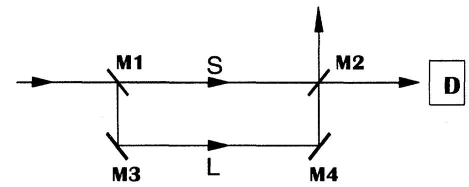

FIG. 2. Schematic diagram of a Mach-Zehnder-like interferometer. M_1 and M_2 are beam splitters. M_3 and M_4 are total reflection mirrors. Here the difference between L and S is greatly exaggerated. In practice, it is smaller than the coherent lengths of the probability waves.

The idea can be realized by a Mach-Zehnder-like interferometer. As shown in Fig. 2, first a photon wave is split by M_1 into two components that travel along either the shorter path S or the longer path L and later are recombined by M_2 into two waves, one of which travels toward the detector. For a traveling photon, there are two paths from the source to the detector. When a photon is detected by the detector, we cannot determine which path it travels along. The path difference between the longer and the shorter path is chosen to satisfy the condition that the corresponding phase difference is π. In this case, all three cases in Eq. (9) can be demonstrated easily when we regulate $|\eta_L/\eta_S|$ by changing the reflectances of M_1 and M_2, where $|\eta_L|^2$ and $|\eta_S|^2$ are, respectively, the probabilities for a photon to take the longer and the shorter path. Obviously, the first case in Eq. (9) implies a superluminal traveling from the source to the detector.

Now we consider a one-dimensional optical tunneling barrier made of alternative layers with refractive indexes $n_H>n_L$ (Fig. 3). Every interface serves as a scattering center that divides the wave packet into a reflected one and a transmitted one. Their wave functions are, respectively, proportional to the amplitude reflectance coefficient R and the

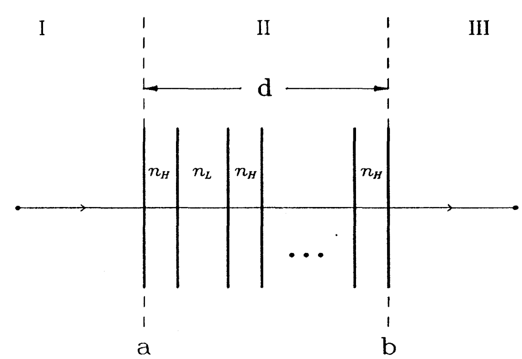

FIG. 3. Multilayer film geometry. Both regions I and III are free space. Region II is the multilayer film. The vertical solid lines indicate the interfaces.

transmission coefficient T. Taking the thickness of every layer to be one-fourth of the photon wave length in the media, the time traveling from one interface to the adjacent should be the same, independently of the velocity difference between different layers. Let τ_0 denote this time. Then the total time for the wave packet traveling across the multilayer from a to b without any reflection is $t_0 = m\tau_0$, where m is the number of layers of the barrier. The reflections of the wave packet at the interfaces create varieties of path modes. However, all these modes have their traveling time longer than t_0. Furthermore, it is easy to show that τ_i should take one of the values

$$\tau_i = t_0 + 2n\tau_0, \quad n = 1,2,3,\ldots. \tag{10}$$

Taking into account Eq. (3) and the fact that the phase of an optical wave shifts π when it is reflected at interface from the lower refractive index side, we get the transmitted wave function at point b (suppose the centroid of the packet is at point a when $t=0$)

$$\begin{aligned}\varphi(t) &= A \sum_{k_1,k_2=0}^{\infty} \xi_m(k_1,k_2) T^{m+2k_1+1} R^{2k_2} (-1)^{k_1+k_2} \exp(i\omega t - im\pi/2) \exp\left[-\frac{1}{2}\left(\frac{t-t_0-2k_1\tau_0-2k_2\tau_0}{\delta t}\right)^2\right] \\ &= A\exp(i\omega t - im\pi/2) \sum_{k_1,k_2=0}^{\infty} \xi'_m(k_1,k_2) \exp\left[-\frac{1}{2}\left(\frac{t-t_0-2k_1\tau_0-2k_2\tau_0}{\delta t}\right)^2\right],\end{aligned} \tag{11}$$

where $\xi_m(k_1,k_2)$ is the number of path modes corresponding to reflections of $2k_2$ times and transmissions of $m+2k_1+1$ times at the interfaces. From (11) the contributions of different modes are weighted by a time-dependent exponent. So it is natural to expect that the mixing of these modes should result in some reshaping of the wave packet at the barrier output.

An apparent superluminal velocity can be demonstrated by comparing the time lag of the wave-packet centroid across the barrier with the time lag of the wave-packet traveling the same distance in free space. The centroid of the wave packet can be represented either by its peak [8], by the peak of its Gaussian fit curve [4], or by its gravity center [9]. We find that, in the case $\delta t \gg d/c$, the wave packet keeps its original Gaussian distribution after the tunneling process and the centroids given by all three methods coincide. Using the third method, an apparent superluminal velocity is obtained when

$$\Delta t = \frac{\int |\varphi(t)|^2 t\, dt}{\int |\varphi(t)|^2 dt} - d/c = \tau_{\text{tunnel}} - d/c \tag{12}$$

takes a negative value. Here τ_{tunnel} is a function of the thickness d of the barrier. A typical result is illustrated in Fig. 4 for $n_H = 2.22$, $n_L = 1.41$, and $\delta t = 86.3(2\pi/kc)$. In Fig. 4, when m increases in intervals of 2, we can find that τ_{tunnel} first increases rapidly and becomes almost independent later as m is large enough. It is very clear that the apparent tunneling velocity v_{tunnel} $(=d/\tau_{\text{tunnel}})$ increases with an increase of d and the apparent superluminal traveling is reached when m is larger than a definite number. The obvious difference between odd and even multilayers shows that the boundary of the barrier plays an important role in the tunneling process. The apparent tunneling velocity, however, should reach a maximum and slow down again when the thickness of the multilayer film is comparable with or larger than the wave packet, but in the latter case the packet form would be quite distorted and would have more than one peak.

Here let us discuss the SKC experiment [4]. In this case $m=11$, $n_H=2.22$, and $n_L=1.41$. The photon frequency $\omega = c/702$ nm. Taking $\delta t = 20$ fs, we find that $\Delta t = -2.16$ fs. The value is not far from that observed experimentally $(\Delta t = -1.47 \pm 0.21$ fs) [4]. One should not be surprised by the deviation of the two values, considering the experimental errors in Ref. [4] and that the parameters used in our calculation may not be quite consistent with the practical case of the experiment. In Ref. [4] it was claimed that the peak of the undistorted (but attenuated) single-photon wave packet appears on the far side of a tunnel barrier earlier than it would

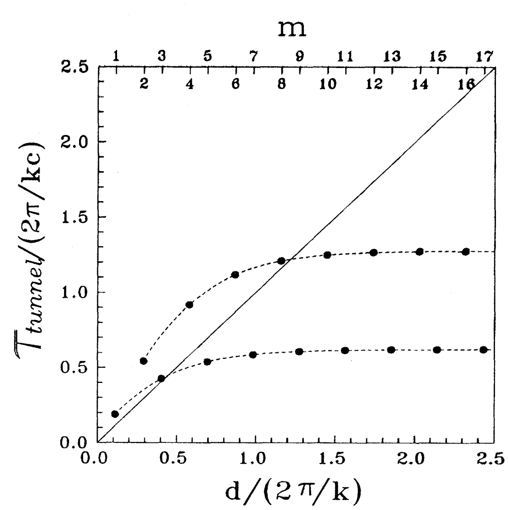

FIG. 4. Apparent tunneling time τ_{tunnel} as a function of the thickness d of the barrier for $n_H = 2.22$, $n_L = 1.41$, and $\delta t = 86.3(2\pi/kc)$. The solid line shows the traveling time for light speed c. Those dots show τ_{tunnel} for $m = 1,2,3,\ldots$ and can be classified into two groups: (i) m is an odd number. In this case, apparent superluminal tunneling is reached when $m \geq 5$ and τ_{tunnel} tends toward 0.62 as m increases. (ii) m is an even number. In this case, apparent superluminal tunneling is reached when $m \geq 10$ and τ_{tunnel} tends toward 1.28 as m increases.

if it were to propagate at c. However, from our discussion, the single-photon wave packet is "undistorted" only in the sense that it keeps about the same form (say, a Gaussian distribution). The existence of the exponents in Eq. (11) clearly shows that the interference effect is not symmetrical about the peak of the incident wave packet, which has been illustrated in Fig. 1, and should result in a shift of the peak in the process of tunneling. This shift contributes an excess speed to the motion of the wave-packet centroid and therefore is the cause for this type of apparent superluminal traveling.

In the SKC experiment, the leading part of a wave packet more possibly passes through a path-uncertain region than the other part, so the average start time of those transmitted photons is earlier than that of all incident photons. The traveling time through the region cannot be obtained by comparing a transmitted reshaped packet with an incident unreshaped packet. Therefore the very short tunneling time measured by Steinberg, Kwiat, and Chiao is not really the time the transmitted photons spend in the Zener barrier region. The real tunneling time would be longer than the SKC measured tunneling time. In Steinberg, Kwiat, and Chiao's paper [4], they showed that their measured tunneling time is consistent with the phase time [1–3,8]. This is not surprising. Their measured tunneling time is practically obtained by comparing the two peaks of the incident and the transmitted wave packets and this measuring method gives the phase time. However, the phase time is not the time a transmitted photon spends in the barrier region [2].

In fact, path uncertainty exists in a variety of traveling besides Zener tunneling and traveling in Mach-Zehnder-like interferometers. As long as there is nonhomogeneity in the traveling region, those traveling photons would be scattered by the nonhomogeneity. If there are two or more scattering bodies at different positions interacting with traveling photons and the scattering waves from these scattering bodies are coherent, the traveling path would be uncertain. This uncertainty in some conditions give rise to a superluminal traveling.

The present picture is somewhat phenomenological in the sense that it has not touched the question of how the interference should occur in a single-photon process. However, in addition to the explanation of the SKC result, the applicability of the model can be verified by further experiments. For example, we offer the following two predictions: (i) the superluminal phenomenon in Mach-Zehnder-like interferometers and (ii) the dependence of the apparent transmitted photon speed on the number of layers in optical Zener tunneling.

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