PHYSICAL REVIEW A VOLUME 52, NUMBER 3 SEPTEMBER 1995

Potential scattering T matrix in a strong static magnetic field and a collinear low-frequency radiation field

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(Received 25 January 1995)

Several approximations to the T matrix for collisions by a static short-range potential in the presence of a strong static magnetic field and a collinear polarized radiation field are considered. We discuss their different ranges of validity and compare the expressions of the T matrix in the different approximations.

PACS number(s): 34.80.Rb, 34.50.Rk

I. INTRODUCTION

For many years the physics of electron scattering in the presence of strong electromagnetic fields has been a topic of interest. Elementary processes occurring in the presence of strong laser fields and/or strong magnetic fields are expected to be important in several areas of physics (astrophysics, solid state physics, laser-atomic physics, etc.) [1,2]. In a number of cases, the influence on the scattering process of the assisting strong external fields is accounted for exactly (i.e., beyond any perturbation theory scheme). In fact a number of exact electron wave functions (both nonrelativistic and relativistic) in the presence of a laser and/or a magnetic field are available in the literature, and often the external field affects the scattered charged particle more strongly than the potential. This allows the description of the field-assisted collision as an event in which the field dressed states are the unperturbed states and the scattering potential, as in the conventional field-free theory, is the perturbation responsible for the transition from the one field dressed state to another. Although, as a rule, it is not difficult to derive formally the pertinent S or T matrix of the collision event in external fields, because of computational difficulties calculations are either restricted to first-order (in the scattering potential) treatments or to the use of *ad hoc* approximations. Thus, since the very beginning of the investigations on collision processes in strong fields, many efforts have been made to provide physically significant and simple approximations to the pertinent S and T matrices.

Several approximations covering different physical situations have been worked out on the S matrix of charged particle potential scattering in the presence of a laser field alone [3,4] (see below for details). Less work has been done up to now on the T matrix of charged particle potential scattering in a quantizing magnetic field. It is worthwhile to remember that in this case, the first-order cross section exhibits a divergent behavior, whenever the final momentum of the one-dimensional free motion is zero, and that such divergent behavior is expected to disappear in more accurate treatments, going, for instance, over first-order treatments. In this respect, a very useful approximation to the exact T matrix is the so-called higher-order modified Born approximation (HOMBA) [5,6].

It is the aim of this paper to present a number of approximations concerning the charged particle scattering T matrix when a radiation and a constant magnetic field are present. To the best of our knowledge, no contributions seem to be available in the literature concerning this argument, although several theoretical papers have been devoted to this elementary process. Moreover, some elementary atomic processes occurring in the presence of lasers and magnetic fields, which are presently of laboratory interest (as an instance, we quote the negative ion photodetachment [7,8]), still require a deeper understanding. We hope that some of the ideas and procedures reported below may also prove useful for such atomic processes.

In the following we will restrict our analysis to the simple case in which the em field is linearly polarized along the direction of the magnetic field. The magnetic field will be assumed to be homogeneous and constant along the incident beam direction and strong enough to quantize the electron motion in the plane perpendicular to it. We will analyze the Born series of the T matrix in cylindrical coordinates, considering several approximations to the T matrix and discussing their different ranges of validity. We will show that in the low-frequency approximation the T matrix may be factorized in terms of Bessel functions, accounting for the interaction with the radiation field times the T matrix for scattering in the presence of a quantizing magnetic field only. This result permits the use of the HOMBA for the radiation field-free T matrix, obtaining a closed form for the total cross section that does not diverge at the Landau resonances.

We also develop a quasistatic treatment of the collisional process, assuming that the collision event takes place in the presence of a quasistatic field with a fixed phase and then making an appropriate average of the total cross section. In this approximation, which may be considered a generalization of the result derived by Ferrante [9] for laser-assisted scattering, the effect of the radiation field becomes completely classical and corresponds to the absorption or emission of a large number of photons. Consequently the frequency of the radiation will not enter into the derived formula from the beginning. This approximation corresponds in practice to a low-frequency one. Atomic units are used throughout.

II. TOTAL CROSS SECTION

Our starting point is the unperturbed state of the charged particle in the presence of both fields. The magnetic field is assumed constant and homogeneous, directed along the z axis. The radiation field is linearly polarized along the magnetic field and is taken in the form $A_z = A_0 \cos\omega t$ (dipole approximation).

In cylindrical coordinates (ρ,φ,z) and in the Landau gauge, the unperturbed wave functions may be written in terms of radiation dressed Landau wave functions given by

$$|i\rangle = \sum_l (-)^l J_l(\lambda)|n_i s_i k_i\rangle \times \exp\left\{-\frac{i}{\hbar}\left[\frac{k_i^2}{2}+(n_i+\tfrac{1}{2})\omega_c + l\omega\right]t\right\} . \quad (1)$$

The role of the radiation field dressing the Landau state is played by the Bessel functions of order l; their argument $\lambda = \varepsilon k_i/\omega^2$ contains the parameters of the radiation field; ε is the intensity of the radiation field and ω the photon energy in a.u.; and $|n_i s_i k_i\rangle$ are the Landau wave functions given by [10]:

$$|nsk\rangle = \exp(ikz)\exp[i(n-s)\varphi]I_{ns}(\xi) \quad (2)$$

with

$$I_{ns}(\xi) = \begin{cases} \left[\dfrac{\gamma}{\pi}\right]^{1/2}(n!s!)^{-1/2}e^{-\xi/2}\xi^{(s-n)/2}L_n^{s-n}(\xi) & \text{for } s \geq n \ , \quad (3) \\ \left[\dfrac{\gamma}{\pi}\right]^{1/2}(n!s!)^{-1/2}e^{-\xi/2}\xi^{(n-s)/2}L_s^{n-s}(\xi) & \text{for } n \geq s \ ; \quad (4) \end{cases}$$

where $\xi = \gamma\rho^2$, $\gamma = \hbar\omega_c/2$ in a.u. and $L_n^{s-n}(\xi)$ are associated Laguerre polynomials. The energy eigenvalues are given by

$$E_n = \frac{k^2}{2} + \left[n + \frac{1}{2}\right]\omega_c \ , \quad (5)$$

where $n = 0,1,2,\ldots$ indicate the principal quantum number characterizing the nth Landau level, k is the particle momentum along z, and ω_c is the cyclotron energy in a.u. The energy eigenvalues are degenerate with respect to the quantum number s, which represents the distance of the center of the spiraling electron orbit along B. Then, the unperturbed wave function [Eq. (1)] has a factorized form given by the wave function of a particle in the presence of a magnetic field only (Landau states) times a Bessel function of order l. Such a simple structure, arising only in the case of a linear polarization of the radiation field along B, represents a superposition of states with energy $E_n + l\omega$, $l = 0, \pm 1, \pm 2, \ldots$ being the number of photons dressing the charged particle embedded in the magnetic field; the corresponding probability of each state is $J_f^2(\lambda)$. Actually these states are virtual because a free particle cannot exchange energy with an homogeneous plane-wave filling all the space.

Using the above wave functions [Eq. (1)] as unperturbed states and following usual procedures [2], the expression for the total cross section for collisions in the presence of a radiation field and a magnetic field with absorption of $l = l_f - l_i$ photons results:

$$\sigma_T = \sum_l \sigma_T^l(+) + \sigma_T^l(-) \ , \quad (6)$$

where

$$\sigma_T^l(\pm) = \frac{\pi}{\gamma}\sum_{n_f, s_f,}\sum_{s_i}\frac{|T(E)_\pm^l|^2}{k_i k_f} \quad (7)$$

and

$$k_f = \pm[k_i^2 + 2(n_i - n_f)\omega_c + 2l\omega]^{1/2} \ . \quad (8)$$

In the above expressions $(+)$ corresponds to forward scattering and $(-)$ to backward scattering. The expression for the total cross section [Eq. (7)] has a general meaning, and the T matrix $T(E)_\pm^l$ assumes different forms according to the particular scattering process considered. In the following we limit ourselves to the simple process of potential scattering.

It is worthwhile to note that when the free-motion energy of the incident particle $k_i^2/2$ matches exactly the difference of energy between two Landau levels plus the energy of the photon exchanged during the collision, the final momentum $k_f \to 0$, giving rise to the so-called Landau divergences in the cross section. To remove these unphysical divergences of the total cross section, it is necessary to calculate the T matrix beyond the first Born approximation; for instance, in the HOMBA [5].

III. *T* MATRIX

With the help of the unperturbed wave functions (1) we can construct the exact T matrix for the scattering of a particle embedded in both a radiation and a magnetic field by a static potential as

$$T(E)^l_\pm=\sum_{l_i} J_{l-l_i}(\lambda_f)\langle n_f s_f k_f|V|n_i s_i k_i\rangle_\pm J_{l_i}(-\lambda_i)$$
$$+\sum_{l_i,l_a}\sum_a \frac{J_{l-l_i}(\lambda_f)\langle n_f s_f k_f|V|n_a s_a k_a\rangle_\pm J_{l_a}(\lambda_a) J_{l_a}(\lambda_a)\langle n_a s_a k_a|V|n_i s_i k_i\rangle J_{l_i}(-\lambda_i)}{\frac{k_i^2}{2}+(n_i+\frac{1}{2})\omega_c+l_i\omega-\left[\frac{k_a^2}{2}+(n_a+\frac{1}{2})\omega_c+l_a\omega\right]}$$
$$+\sum_{l_i,l_a,l_b}\sum_{a,b}\frac{J_{l-l_i}(\lambda_f)\langle n_f s_f k_f|V|n_a s_a k_a\rangle_\pm J_{l_a}(\lambda_a)J_{l_a}(\lambda_a)}{\frac{k_i^2}{2}+(n_i+\frac{1}{2})\omega_c+l_i\omega-\left[\frac{k_a^2}{2}+(n_a+\frac{1}{2})\omega_c+l_a\omega\right]}$$
$$\times\frac{\langle n_a s_a k_a|V|n_b s_b k_b\rangle J_{l_b}(\lambda_b)J_{l_b}(\lambda_b)\langle n_b s_b k_b|V|n_i s_i k_i\rangle J_{l_i}(-\lambda_i)}{\frac{k_i^2}{2}+(n_i+\frac{1}{2})\omega_c+l_i\omega-\left[\frac{k_b^2}{2}+(n_b+\frac{1}{2})\omega_c+l_b\omega\right]}+\cdots \,. \tag{9}$$

An instructive diagrammatic representation of the exact T matrix is given in Fig. 1. The structure of the scattering series is similar to the one obtained when we consider collisions of particles with internal structure. In our case, the discrete structure (double line) represents the bound motion of the scattering particle in the plane perpendicular to the magnetic field. The single line represents the free motion of the particle, dressed by l_α photons, along the direction of the magnetic field. The Nth term of the Born series can be understood as a transition from the initial to the final Landau states, the result of a series of potential induced virtual transitions between Landau levels given by the quantum numbers $n_{ia}+n_{ab}+,\ldots,+n_{st}+n_{tf}=n_i-n_f=n_{if}$ and the exchange of l real photons as a result of a series of exchanges of virtual photons $l_{ia}+l_{ab}+,\ldots,+l_{st}+l_{tf}=l_i-l_f=l$. In the T matrix, the sum over the intermediate states labeled $a,b,\ldots$ must be intended as a sum over $n_a s_a, n_b s_b,\ldots$ and an integration over $dk_a, dk_b,\ldots$. From the above expression of the exact T matrix, it is easy to obtain the expressions of the T matrix for the case when only one field is present.

A. Laser field-free T matrix

We note that by putting $l_i=l_a=l_b=\cdots=l_f=0$ in the above expression for the T matrix [Eq. (9)], we recover the T matrix for the case when only the magnetic field is present, as [11]

$$T(E)_\pm=\langle n_f s_f k_f|V|n_i s_i k_i\rangle_\pm+\sum_a\frac{\langle n_f s_f k_f|V|n_a s_a k_a\rangle_\pm\langle n_a s_a k_a|V|n_i s_i k_i\rangle}{\frac{k_i^2}{2}+(n_i+\frac{1}{2})\omega_c-\left[\frac{k_a^2}{2}+(n_a+\frac{1}{2})\omega_c\right]}$$
$$+\sum_{a,b}\frac{\langle n_f s_f k_f|V|n_a s_a k_a\rangle_\pm\langle n_a s_a k_a|V|n_b s_b k_b\rangle\langle n_b s_b k_b|V|n_i s_i k_i\rangle}{\left\{\frac{k_i^2}{2}+(n_i+\frac{1}{2})\omega_c-\left[\frac{k_a^2}{2}+(n_a+\frac{1}{2})\omega_c\right]\right\}\left\{\frac{k_i^2}{2}+(n_i+\frac{1}{2})\omega_c-\left[\frac{k_b^2}{2}+(n_b+\frac{1}{2})\omega_c\right]\right\}}\,. \tag{10}$$

B. Magnetic field-free T matrix

The Landau radiation field dressed wave functions [Eq. (1)] do not reduce to the wave functions for a particle embedded in a radiation field in the limit of $B\to 0$. Then, to recover the T-matrix expression for the case when only the radiation field is present, we have to replace in Eq. (9) the Landau states $|n_f s_f k_f\rangle$, $|n_a s_a k_a\rangle$, $|n_b s_b k_b\rangle,\ldots$ with plane waves $|k_f\rangle, |k_a\rangle, |k_b\rangle,\ldots$ and the energy for the particle in the magnetic field E_n with the free-particle energy $E=k^2/2$, obtaining

$$T(E)^l_\pm=\sum_{l_i}J_{l-l_i}(\lambda_f)\langle k_f|V|k_i\rangle_\pm J_{l_i}(-\lambda_i)+\sum_{l_i l_a}\sum_a\frac{J_{l-l_i}(\lambda_f)\langle k_f|V|k_a\rangle_\pm J_{l_a}(\lambda_a)J_{l_a}(\lambda_a)\langle k_a|V|k_i\rangle J_{l_i}(-\lambda_i)}{\frac{k_i^2}{2}+l_i\omega-\left[\frac{k_a^2}{2}+l_a\omega\right]}$$
$$+\sum_{l_i,l_a,l_b}\sum_{a,b}\frac{J_{l-l_i}(\lambda_f)\langle k_f|V|k_a\rangle_\pm J_{l_a}(\lambda_a)J_{l_a}(\lambda_a)\langle k_a|V|k_b\rangle J_{l_b}(\lambda_b)J_{l_b}(\lambda_b)\langle k_b|V|k_i\rangle J_{l_i}(-\lambda_i)}{\left\{\frac{k_i^2}{2}+l_i\omega-\left[\frac{k_a^2}{2}+l_a\omega\right]\right\}\left\{\frac{k_i^2}{2}+l_i\omega-\left[\frac{k_b^2}{2}+l_b\omega\right]\right\}}+\cdots \tag{11}$$

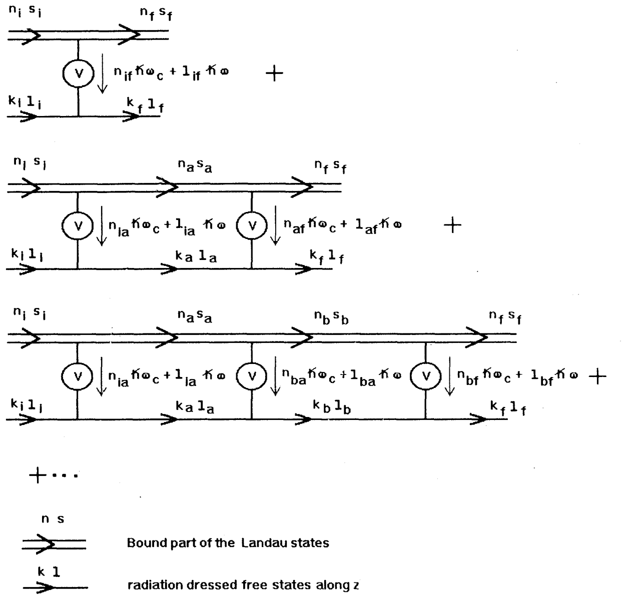

FIG. 1. Diagrammatic representation of the exact T-matrix series for potential scattering in the presence of a magnetic field and a radiation field.

IV. FBA T MATRIX

The first born approximation (FBA) for collisions in the presence of a magnetic field and a radiation field is obtained by taking only the first term of the expansion (9),

$$T(E)^l_\pm = \sum_{l_i} J_{l-l_i}(\lambda_f)\langle n_f s_f k_f|V|n_i s_i k_i\rangle_\pm J_{l_i}(-\lambda_i) \ . \quad (12)$$

Using the addition theorem of the Bessel functions

$$\sum_{l_i} J_{l-l_i}(\lambda_f) J_{l_i}(-\lambda_i) = J_l(\lambda_{fi}) \ , \quad (13)$$

with $\lambda_{fi} = (\varepsilon/w^2)(k_f - k_i)$, we get for the FBA T matrix the expression

$$T(E)^l_\pm = J_l(\lambda_{fi})\langle n_f s_f k_f|V|n_i s_i k_i\rangle_\pm \ . \quad (14)$$

The FBA T matrix, and consequently the total cross section also, show in this case a factorized structure, where the interaction with the radiation field is accounted for by the Bessel functions while the interaction with the magnetic field resides in the first-order matrix element between the initial and the final Landau levels $\langle n_f s_f k_f|V|n_i s_i k_i\rangle_\pm$.

V. LOW-FREQUENCY APPROXIMATIONS

Because the T-matrix series [Eq. (9)] contains sums over the discrete quantum numbers n,s,l and integrations over $dk_a, dk_b, \ldots$, it is too involved to be used beyond the first Born approximation. In this section we derive two different low-frequency approximations (LFA's) for the T matrix, generalizing the two approaches used by Kruger and Jung [3] and by Kroll and Watson [4] for a collision assisted by a laser radiation field. The collisional process in the presence of a radiation and a magnetic field is ruled by three characteristic times: (i) the collisional time $t_c \simeq d/v$, where d is the range of the potential and v the particle velocity; (ii) the characteristic time of the system involved in the collision, given in our case by $t_B = 1/\omega_c$; and (iii) the radiation characteristic time $t_R = 1/\omega$. In our analysis we have considered the magnetic field to be strong enough to quantize the motion of the scattering particle in the plane perpendicular to B. As shown in previous papers [6,12] this occurs if the magnetic field and the energy of the particle have values that allow transitions to the final Landau levels with the quantum number n_f is not very large, i.e., when the ratio of the energy of the free motion along B and the cyclotron energy $R = k^2/2\omega_c$ is a small number; this is consistent with the condition

$$t_B < t_c \ . \quad (15)$$

Moreover we can assume that

$$t_c \ll t_R \ ; \quad (16)$$

i.e., the time scale of the variation of the radiation field is much greater than the collisional time.

A. KJM T matrix

In this section we follow the approach given by Kruger and Jung [3] for the simpler case of collisions in the presence of a low-frequency radiation field only. We call this the Kruger-Jung magnetic-field approximation (KJM).

We split the collision process into three stages. In the first stage, the electron is embedded in the magnetic field and in the radiation field and is allowed to exchange l_i virtual photons with the radiation field, acquiring an energy $E_i' = E_i + l_i\omega = k_i^2/2 + (n_i + \frac{1}{2})\omega_c + l_i\omega$. The probability that the electron will gain such energy is given by the squared Bessel function $J^2_{l_i}(-\lambda_i)$. In the second stage, the electrons collide with the potential in the presence of the magnetic field, and the radiation plays no role. In the third stage, the electrons, leaving the region of interaction with the potential, can again exchange photons with the radiation field, acquiring the final energy $E_f = E_i' + (l - l_i)\omega = E_i + l\omega$. The probability for the electron to exchange $(l - l_i)$ photons is given by $J^2_{l-l_i}(\lambda_f)$. In other words, the KJM consists in assuming that the field couples strongly with the scattering particle only in the initial and final states, so that we can neglect the radiation $\leftrightarrow$ (particle+magnetic field) interaction during the (particle+magnetic field) $\leftrightarrow$ potential scattering event. This assumption is supposed to be good if the energy of the particle embedded in the magnetic field $E_i = k_i^2/2 + (n_i + \frac{1}{2})w_c$ is much larger than the photon energy ω. The T matrix [Eq. (9)] will then be approximated, neglecting all the terms $l_a w, l_b w, \ldots$ appearing in the denominators. This allows us to sum the T matrix over all the intermediate dressing photon numbers $l_a, l_b, l_c, \ldots$.

Using the relation

$$\sum_{l_a} J_{l_a}(\lambda_a) J_{l_a}(\lambda_a) = J_0(0) = 1 , \tag{17}$$

we obtain for the KJM T matrix the expression

$$T(E)^l_{\pm} = \sum_{l_i} J_{l-l_i}(\lambda_f) \langle n_f s_f k_f | V | n_i s_i k_i \rangle_{\pm} J_{l_i}(-\lambda_i) + \sum_{l_i} \sum_{a} \frac{J_{l-l_i}(\lambda_f) \langle n_f s_f k_f | V | n_a s_a k_a \rangle_{\pm} \langle n_a s_a k_a | V | n_i s_i k_i \rangle J_{l_i}(-\lambda_i)}{\frac{k_i^2}{2} + (n_i + \frac{1}{2})\omega_c + l_i\omega - \left[\frac{k_a^2}{2} + (n_a + \frac{1}{2})\omega_c \right]}$$
$$+ \sum_{l_i} \sum_{a,b} \frac{J_{l-l_i}(\lambda_f) \langle n_f s_f k_f | V | n_a s_a k_a \rangle_{\pm} \langle n_a s_a k_a | V | n_b s_b k_b \rangle \langle n_b s_b k_b | V | n_i s_i k_i \rangle J_{l_i}(-\lambda_i)}{\left\{ \frac{k_i^2}{2} + (n_i + \frac{1}{2})\omega_c + l_i\omega - \left[\frac{k_a^2}{2} + (n_a + \frac{1}{2})\omega_c \right] \right\} \left\{ \frac{k_i^2}{2} + (n_i + \frac{1}{2})\omega_c + l_i\omega - \left[\frac{k_b^2}{2} + (n_b + \frac{1}{2})\omega_c \right] \right\}} + \cdots , \tag{18}$$

or in a more compact form

$$T(E)^l_{\pm} = \sum_{l_i} J_{l-l_i}(\lambda_f) T(E')_{\pm} J_{l_i}(-\lambda_i) , \tag{19}$$

where $T(E')_{\pm}$ is the T matrix [Eq. (10)] for collision in the presence of a magnetic field calculated only at the shifted initial energy $E' = E_{ni} + l_i\omega$ with $E_{ni} = k_i^2/2 + (n_i + \frac{1}{2})\omega_c$.

If we make further assumption that the T matrix $T(E')_{\pm}$ is a sufficient smooth function of E', it may be considered as essentially a constant. Then, using the additional theorem [Eq. (13)], we find that the KJM T matrix assumes, as in the FBA case, a completely factorized form:

$$T(E)^l_{\pm} = J_l(\lambda_{fi}) \langle n_f s_f k_f | T(E_{ni})_{\pm} | n_i s_i k_i \rangle_{\pm} . \tag{20}$$

B. KWM T matrix

We use now a different approach to the LFA, proposed first by Kroll and Watson [4], for electron collisions in the presence of a radiation field. We call this the Kroll Watson magnetic-field approximation (KWM).

Following Kroll and Watson [4], we assume that in the generic denominator of the exact T matrix [Eq. (9)]

$$\left| \frac{(l_i - l_m)\omega}{\frac{k_i^2}{2} - \frac{k_m^2}{2} + (n_i - n_m)\omega_c} \right| < 1 , \tag{21}$$

we can expand the denominators of the T matrix up to the first order in ω as

$$\frac{1}{\frac{k_i^2}{2} - \frac{k_m^2}{2} + (n_i - n_m)\omega_c + (l_i - l_m)\omega} = \frac{1}{\frac{k_i^2}{2} - \frac{k_m^2}{2} + (n_i - n_m)\omega_c} + \frac{(l_i - l_m)\omega}{\left[\frac{k_i^2}{2} - \frac{k_m^2}{2} + (n_i - n_m)\omega_c \right]^2} . \tag{22}$$

Defining the shifted momentum

$$k'_m = k_m - \frac{l\omega}{k_f - k_i} k_m \tag{23}$$

with $l = l_f - l_i$, we can rearrange the T matrix so that its denominator no longer depends on the quantum numbers $l_i, l_a, l_b, \ldots$.

Moreover, using Eq. (13), we can again get a completely factorized expression for the KWM T matrix as

$$T(E)^l_{\pm} = J_l(\lambda_{fi}) \langle n_f s_f k_f | T(E^*) | n_i s_i k_i \rangle_{\pm} , \tag{24}$$

where $\langle n_f s_f k_f | T(E^*) | n_i s_i k_i \rangle_{\pm}$ is the T matrix for potential scattering in the presence of a magnetic field evaluated at the energy

$$E^* = \frac{k_i^2}{2} + \frac{l\omega}{k_f - k_i} k_i + \frac{(l\omega)^2}{2(k_f - k_i)^2} + [n_i + \tfrac{1}{2}]\omega_c . \tag{25}$$

In both the KJM and KWM low-frequency approximations, the T matrix is completely factorized. We note that by considering only the first term in the expansion (22), the KWM expression reduces to the KJM. In the KWM, we consider the shift of the momentum of the scattering particle to first order in ω, as well as for the intermediate states of the T-matrix series, and then it may be considered a more accurate approximation. However, the KWM may be used only for the cases in which the scattering amplitude is a sufficiently slowly varying func-

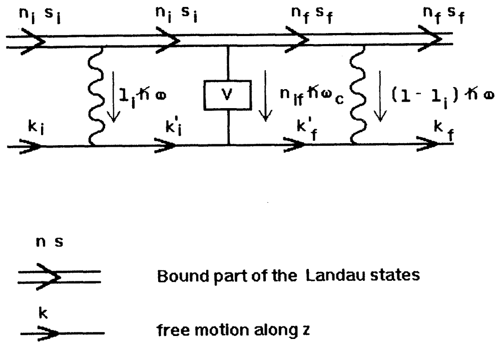

FIG. 2. Diagrammatic representation of the LFA T-matrix series with $k_i'^2/2=k_i^2/2+l_i\omega$ and $k_f'^2/2=k_f^2/2+(l-l_i)\omega$.

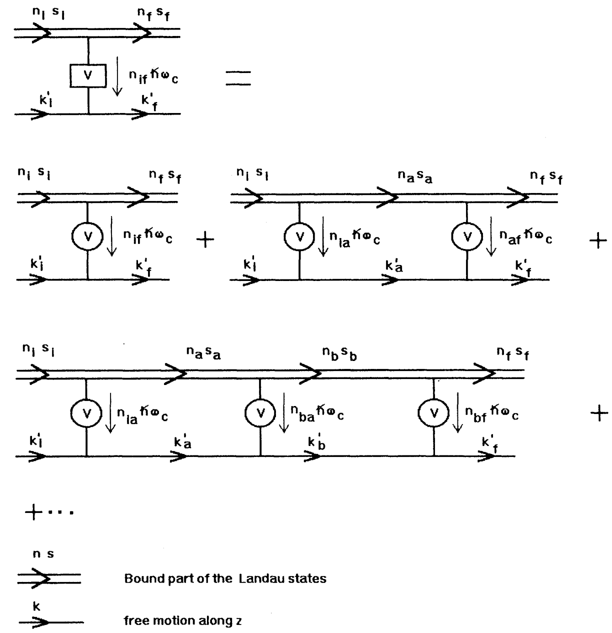

FIG. 3. Diagrammatic representation of the contracted symbol introduced in Fig. 2 with $k_m'=k_m$ for the KJ LFA T-matrix series and $k_m'=k_m-[l\omega/(k_f-k_i)]k_m$ for the KWLFA T-matrix series.

tion of the energy. This means that Eq. (24) does not hold if the radiation field-free process in the absence of the radiation field presents sharp resonances. The KJM in the form given by Eq. (19) can be used also in the presence of resonances. A diagrammatic representation of the LFA T matrix is given in Fig. 2. The contracted symbol introduced in Fig. 2 is expanded in Fig. 3 and represents the exact T matrix for potential scattering in the presence of a strong magnetic field, where k_m' is the effective intermediate momentum in the KJM and has the form given in Eq. (23) for the KWM.

VI. LFA-HOMBA

Once we have obtained the two factorized expressions for the KJM and KWM T matrices we are left with the evaluation of a suitable expression for the T matrix of electrons scattering in the presence of a static magnetic field. As we already anticipated in the Introduction, the total cross section presents unphysical infinities at Landau thresholds at least in the FBA. To avoid the infinities it is necessary to go beyond the FBA, using, for example, the so-called higher-order modified Born approximation [5,6]. This approximation allows us to sum all orders of the scattering series for collisions in the presence of a quantizing magnetic field, obtaining a closed form for the total cross section not divergent at the Landau thresholds. In such an approximation the Nth term of the Born series contains only contributions due to the $N-1$ elastic transitions between the same Landau level and only one inelastic scattering from n_i to n_f, while all the other contributions are neglected. Using the T matrix derived in the HOMBA in the LFA expressions for the T matrix [Eq. (19) or (24)], we obtain the LFA-HOMBA T matrix as [6]

$$T(E)'_{\pm}=J_l(\lambda_{fi})\langle n_f s_f k_f|T(E)|n_i s_i k_i\rangle_{\pm}^{\mathrm{HOMBA}}\,, \tag{26}$$

where

$$\langle n_f s_f k_f|T(E)|n_i s_i k_i\rangle_{\pm}^{\mathrm{HOMBA}}=\langle n_f s_f k_f|V|n_i s_i k_i\rangle_{\pm}\frac{k_f}{k_f-2\pi i\langle n_f s_f k_f|V|n_f s_f k_f\rangle}\,\frac{k_i}{k_i-2\pi i\langle n_i s_i k_i|V|n_i s_i k_i\rangle} \tag{27}$$

for $n_i\neq n_f$ and

$$\langle n_f s_f k_f|T(E)|n_i s_i k_i\rangle_{\pm}^{\mathrm{HOMBA}}=\langle n_i s_i k_i|V|n_i s_i k_i\rangle_{\pm}\frac{k_i}{k_i-2\pi i\langle n_i s_i k_i|V|n_i s_i k_i\rangle} \tag{28}$$

for $n_i=n_f$.

In this approximation the interaction with the magnetic field is taken into account exactly (the unperturbed wave functions are Landau wave functions), and the interaction with the potential in the presence of the magnetic field is considered at all orders. The derived T-matrix element may be substituted directly in the KJ FBA, while in the KWM it must be evaluated at the shifted momentum given in Eq. (23). The total cross section derived in the LFA HOMBA does not present infinities at the Landau thresholds [13].

VII. QUASISTATIC FIELD APPROXIMATION

In this section we derive the T matrix and the total cross section in the quasistatic field approximation

(QSA). We shall assume that in the time scale of the variation of the radiation field, the collision event takes place instantaneously. This approximation corresponds to a consideration of the collision event as taking place in the presence of a quasistatic field with a fixed particular phase. The fact that the field is actually not constant is recovered by an appropriate phase averaging. Accordingly, the frequency of the radiation field does not enter the final results. Then the QSA may be considered a simplification of the LFA at very low frequency.

We start again from the unperturbed wave function for the scattering particle in the presence of both fields [Eq. (1), which can be written in the equivalent form

$$|a\rangle=|nsk\rangle\exp\left\{-\frac{i}{\hbar}\left[\frac{k^2}{2}+(n+\tfrac{1}{2})\omega_c\right]t+i\lambda\sin(\omega t+\alpha)\right\}, \quad (29)$$

where $|nsk\rangle$ are Landau wave functions given by Eq. (2) and α is the initial phase of the radiation field.

Assuming that the low-frequency conditions [Eqs. (15) and (16)] are satisfied and that the collisional event takes place instantaneously, we can put $\omega t \ll 1$ in the wave function [Eq. (29)] so that

$$\sin(\omega t+\alpha)\simeq\omega t\cos\alpha+\sin\alpha \quad (30)$$

and the unperturbed wave function becomes

$$|a\rangle=|nsk\rangle\exp\left[-\frac{i}{\hbar}\tilde{E}_n t\right], \quad (31)$$

representing a Landau wave function with energy eigenvalues shifted by $\lambda\omega\cos\alpha$:

$$\tilde{E}_n=\left[\frac{k_n^2}{2}+(n+\tfrac{1}{2})\omega_c\right]-\lambda\omega\cos\alpha . \quad (32)$$

Then, in the QSA approximation, the colliding electron is affected by a static electric field characterized by the field phase α.

Proceeding in the usual way, we find that the probability transition per unit time will be

$$P_{if}=2\pi|\langle n_f s_f k_f|\tilde{T}|n_i s_i k_i\rangle_\pm|^2\delta(\tilde{E}_f-\tilde{E}_i) , \quad (33)$$

which apart from the definition of the energy of the colliding particle [Eq. (32)] is the same as that obtained in the radiation field-free case.

The T matrix is given by

$$\langle n_f s_f k_f|\tilde{T}|n_i s_i k_i\rangle_\pm=\langle n_f s_f k_f|V|n_i s_i k_i\rangle_\pm+\sum_a\frac{\langle n_f s_f k_f|V|n_a s_a k_a\rangle_\pm\langle n_a s_a k_a|V|n_i s_i k_i\rangle}{\tilde{E}_i-\tilde{E}_a+i\eta}+\sum_{a,b}\frac{\langle n_f s_f k_f|V|n_a s_a k_a\rangle_\pm\langle n_a s_a k_a|V|n_b s_b k_b\rangle\langle n_b s_b k_b|V|n_i s_i k_i\rangle}{(\tilde{E}_i-\tilde{E}_a+i\eta)(\tilde{E}_i-\tilde{E}_b+i\eta)}+\cdots \quad (34)$$

and is dependent through the energies $\tilde{E}_m$ on the field phase α. The expression for the total cross section is then obtained by finding an average over all the values of the radiation field phase

$$\langle\sigma_T\rangle=\frac{1}{\pi}\int^{\pi}\sigma_T d\alpha \quad (35)$$

with

$$\sigma_T=\frac{\pi}{\gamma}\frac{1}{k_i}\sum_{n_f s_f}\sum_{s_i}\int dk_f|\langle n_f s_f k_f|\tilde{T}|n_i s_i k_i\rangle_\pm|^2\times\delta(\tilde{E}_f-\tilde{E}_i) , \quad (36)$$

where

$$\delta(\tilde{E}_f-\tilde{E}_i)=\frac{k_f^2}{2}-\frac{k_i^2}{2}-\omega_c n_{if}-\frac{\varepsilon}{\omega}(k_f-k_i)\cos\alpha . \quad (37)$$

The phase averaging is required to remove any reference to the particular field phase at which the collision event takes place.

To obtain the total cross section, we are left in Eq. (35) with two integrations over $d\alpha$ and dk_f. Because of the presence of the δ function in Eq. (36), we can get rid of one of these two integrations by obtaining two equivalent expressions for the total cross section.

We first derive the expression for the total cross section obtained by integrating Eq. (36) over dk_f and leaving the integration over $d\alpha$.

Making use in Eq. (36) of the δ function property

$$\delta(\tilde{E}_f-\tilde{E}_i)=\frac{\delta(k_f\pm k_{f,0})}{\left|k_{f,0}-\frac{\varepsilon}{\omega}\cos\alpha\right|} , \quad (38)$$

where $k_{f,0}$ is the solution of the equation $\tilde{E}_f-\tilde{E}_i=0$ solved with respect to the unknown k_f,

$$k_{f,0}=\frac{\varepsilon}{\omega}\cos\alpha\pm\left[\left[\frac{\varepsilon}{\omega}\cos\alpha-k_i\right]^2+2\omega_c n_{if}\right]^{1/2} , \quad (39)$$

we get the total cross section as

$$\langle\sigma_T\rangle=\frac{1}{\gamma k_i}\sum_{n_f s_f}\sum_{s_i}\int^{\pi}\frac{1}{\left|k_{f,0}-\frac{\varepsilon}{\omega}\cos\alpha\right|}|\langle n_f s_f k_f|\tilde{T}|n_i s_i k_i\rangle_\pm|^2_{k_f=\pm k_{f,0}}d\alpha , \quad (40)$$

where we are left with an integration over $d\alpha$.

To perform such an integration, we have to keep in mind that the variable α can assume [see Eq. (39)] only the values for which

$$\cos\alpha = \left| \left[\frac{k_{f,0}^2 - k_i^2 - 2\omega_c n_{if}}{k_{f,0} - k_i} \right] \frac{\omega}{2\varepsilon} \right| \le 1 \ . \tag{41}$$

Another, more useful expression for the total cross section, which can be easily compared with the LFA cross section derived in the previous section, is derived by starting from Eqs. (35) and (36) and integrating first over $d\alpha$. To this aim we express the δ function as

$$\delta(\tilde{E}_f - \tilde{E}_i) = \frac{\delta(\alpha - \alpha_0)}{|(k_f - k_i)\frac{\varepsilon}{\omega}\cos\alpha_0|} \ , \tag{42}$$

where α_0 is the solution of the equation $\tilde{E}_f - \tilde{E}_i = 0$ solved with respect to the unknown α. Substituting Eq. (42) in Eq. (36), we get the new expression for the total cross section,

$$\langle \sigma_T \rangle = \frac{1}{\gamma k_i} \sum_{n_f s_f} \sum_{s_i} \int \frac{1}{\left| (k_f - k_i)\frac{\varepsilon}{\omega}\sin\alpha_0 \right|} \times |\langle n_f s_f k_f | \tilde{T} | n_i s_i k_i \rangle_{\pm}|^2 dk_f \ , \tag{43}$$

where we are left with the integration over dk_f; the variable k_f can assume [see Eq. (41)] only the values for which the following relation is verified:

$$\cos\alpha_0 = \left| \left[\frac{k_f^2 - k_i^2 - 2\omega_c n_{if}}{k_f - k_i} \right] \frac{\omega}{2\varepsilon} \right| \le 1 \ ; \tag{44}$$

this implies that the integration over dk_f must be performed between

$$k_f(\min) = \frac{\varepsilon}{\omega} - \left[\left[k_i - \frac{\varepsilon}{\omega} \right]^2 + 2\omega_c n_{if} \right]^{1/2} \tag{45}$$

and

$$k_f(\max) = \frac{\varepsilon}{\omega} + \left[\left[k_i - \frac{\varepsilon}{\omega} \right]^2 + 2\omega_c n_{if} \right]^{1/2} \ . \tag{46}$$

The two expressions for the total cross section [Eqs. (40) and (43)] are equivalent.

Total cross section in the QSA

The generic denominator of the T matrix in the QSA [Eq. (34)] is given by

$$\tilde{E}_i - \tilde{E}_m = \left[E_i + \frac{\varepsilon k_i}{\omega}\cos\alpha_0 \right] - \left[E_m + \frac{\varepsilon k_m}{\omega}\cos\alpha_0 \right] \ . \tag{47}$$

If the quantity $\varepsilon/\omega(k_i - k_m)\cos\alpha_0$ may be considered much smaller than $E_i - E_m$, the T matrix becomes equal to the T matrix obtained in the case of collisions in the presence of the magnetic field only, i.e., $\langle n_f s_f k_f | \tilde{T} | n_i s_i k_i \rangle \equiv \langle n_f s_f k_f | T | n_i s_i k_i \rangle$, and the cross section [Eq. (43)] may be written in a simple way as

$$\langle \sigma_T \rangle = \frac{1}{\gamma k_i} \sum_{n_f s_f} \sum_{s_i} \int \frac{1}{\left| (k_f - k_i)\frac{\varepsilon}{\omega}\sin\alpha_0 \right|} \times |\langle n_f s_f k_f | T | n_i s_i k_i \rangle_{\pm}|^2 dk_f \ . \tag{48}$$

This approximation can be thought of as a KJQSA because, as in the KJM, we have eliminated the parameters of the radiation field from the denominator of the T matrix.

Moreover, substituting in Eq. (47) the expression for $\cos\alpha_0$ [Eq. (44)], we can write the generic expression for the energy in the denominator of the T matrix as

$$\tilde{E}_m = \frac{k_m^2}{2} + \omega_c(n_m + \tfrac{1}{2}) - \left[\frac{k_f^2 - k_i^2 - \omega_c n_{if}}{2(k_f - k_i)} \right] k_m \ . \tag{49}$$

If we assume that the quantity

$$\frac{1}{2} \left[\frac{k_f^2 - k_i^2 - \omega_c n_{if}}{2(k_f - k_i)} \right] \ll \frac{k_m^2}{2} + \omega_c(n_m + \tfrac{1}{2}) \ , \tag{50}$$

we can add it to the expression for $\tilde{E}_m$ given in Eq. (49). Defining

$$\tilde{k}_m = \left[k_m - \left[\frac{k_f^2 - k_i^2 - \omega_c n_{if}}{2(k_f - k_i)} \right] \right] , \tag{51}$$

since $\int d\tilde{k}_m \equiv \int dk_m$, we find that the total cross section becomes

$$\langle \sigma_T \rangle = \frac{1}{\gamma k_i} \sum_{n_f s_f} \sum_{s_i} \int \frac{1}{|(\tilde{k}_f - \tilde{k}_i)\frac{\varepsilon}{\omega}\sin\alpha_0|} |\langle n_f s_f k_f | T(\tilde{E}(\alpha_0)) | n_i s_i k_i \rangle_{\pm}|^2 d\tilde{k}_f \tag{52}$$

with

$$\tilde{E}(\alpha_0) = \frac{\tilde{k}^2}{2} + \omega_c(n + \tfrac{1}{2}) \ , \tag{53}$$

and the T matrix has essentially the analytical structure of the radiation field-free result, with the momentum redefined through Eq. (51) and the energy redefined through Eq. (53). This approximation can be thought of as a KW QSA because, as in the KWM, the T matrix is evaluated at shifted energies and moments.

VIII. CONNECTIONS BETWEEN THE QSA AND THE LFA RESULTS

The QSA approximation may be considered a low-frequency approximation. In fact the basic assumption of the QSA is $t_c \ll t_R$; i.e., the time scale of the variation of the radiation field is much greater than the collisional time, or in other words, there are many collisions during a period of the radiation field. The above condition implies that $\omega \ll v/d$, which is the low-frequency condition. In the following we show how it is possible to recover the KJQSA expression for the cross section starting from the KJM.

Using Eq. (20), we can write the KJM cross section as

$$\sigma_T(\pm)=\frac{\pi}{\gamma}\sum_l \sum_{n_f s_f}\sum_{s_i}\frac{1}{k_f k_i}J_l^2(\lambda_{fi}) \times |\langle n_f s_f k_f|T(E)|n_i s_i k_i\rangle_\pm|^2 \quad (54)$$

with

$$\lambda_{fi}=\frac{\varepsilon}{\omega^2}(k_f-k_i)\ . \quad (55)$$

Starting from Eq. (54), we can recover the total cross section derived in the KJQSA [Eq. (48)] by following three steps.

(i) The difference of energy between the initial and final states of a particle embedded in a magnetic field in the QSA is

$$E_f-E_i=\frac{\varepsilon}{\omega^2}(k_f-k_i)\cos\alpha\ , \quad (56)$$

and the corresponding energy difference used in the derivation of the LFA formula is

$$E_f-E_i=l\omega\ , \quad (57)$$

where ω is in a.u. ($\hbar=1$). Equating these two expressions for the energy difference, we find that the radiation field phase is related to l through

$$\cos\alpha=\frac{1}{x}\le 1\ , \quad (58)$$

where

$$x=\frac{\varepsilon}{\omega^2}(k_f-k_i)=\lambda_{fi}\ . \quad (59)$$

(ii) For large arguments, the squared Bessel function can be approximated as

$$J_N^2(x)\simeq\frac{2\cos^2\left[(x^2-N^2)^{1/2}-\left|N\cos^{-1}\left[\frac{N}{x}\right]\right|-\frac{1}{4}\pi\right]}{\pi(x^2-N^2)^{1/2}}\ . \quad (60)$$

In our case we have

$$J_l^2(\lambda_{fi})\simeq\frac{1}{\pi(\lambda_{fi}^2-l^2)^{1/2}}=\frac{1}{\pi\lambda_{fi}\sin\alpha}\ . \quad (61)$$

(iii) From the expression of the energy conservation we have

$$\frac{k_f^2}{2}+\omega_c(n_f+\tfrac{1}{2})=\frac{k_i^2}{2}+\omega_c(n_i+\tfrac{1}{2})+l\omega\ ; \quad (62)$$

we can differentiate the above expression by assuming that l is a continuous variable, obtaining

$$k_f dk_f=\omega dl\ . \quad (63)$$

With the help of (63) we can transform the sum over l into an integration over dk_f as

$$\sum_l \to \int dl=\int \frac{k_f}{\omega}dk_f\ . \quad (64)$$

Substituting Eqs. (61) and (64) in the expression for the LFA total cross section [Eq. (54)], we finally obtain the QSA total cross section already derived in Eq. (48).

IX. CONCLUDING REMARKS

We have derived the T-matrix series for collisions by a static short-range potential in the presence of a strong static magnetic field and a collinear polarized radiation field. We have shown how to recover from the T matrix in the presence of both fields the T matrix when it is present in only one field. We have provided different low-frequency approximations of the exact T matrix by generalizing the two approaches used by Kruger and Jung [3] and by Kroll and Watson [4] for a collision assisted by a laser radiation to the case in which quantizing magnetic field is also present. In this case the collision process is ruled by three characteristic times: (i) the collisional time $t_c \simeq d/v$; (ii) the characteristic time of the magnetic field $t_B=1/\omega_c$; and (iii) the characteristic time of the radiation field $t_R=1/\omega$. We have considered the magnetic field to be strong enough to quantize the motion of the scattering particle in the plane perpendicular to B. This occurs if during the collision only transitions to the final Landau levels are allowed, with a quantum number n_f that is not very large, i.e., when the ratio of the energy of the free motion along B and the cyclotron energy $R=k^2/2\omega_c$ is a small number; this is consistent with the condition $t_B<t_c$. Moreover, we have assumed that $t_c \ll t_R$, i.e., that the time scale of the variation of the radiation field is much greater than the collision time. In both low-frequency approximations (KJM and KWM) the T matrices are completely factorized. In the KWM, we consider the shift of the momentum of the scattering particle at the first order in ω, as well as for the intermediate states of the T-matrix series, and then it may be considered to be a more accurate approximation. However, the KWM may be used only for the cases in which the scattering amplitude is a sufficiently slowly varying function of the energy. This means that Eq. (24) does not hold if the radiation field-free process presents sharp resonances. The KJM in the form given by Eq. (19) can be used also in the presence of resonances. Combining the two factorized expressions for the KJM and KWM T matrices with the higher-order modified Born approxima-

tion [5,6] for collisions in the presence of a static magnetic field, we have obtained the LFA HOMBA T matrix that permits the calculation of the total cross section, removing the unphysical infinities at Landau thresholds present in the FBA.

We have also derived the T matrix and the total cross section in the quasistatic field approximation, assuming that in the time scale of the variation of the radiation field, the collision event takes place instantaneously. This approximation corresponds to a consideration of the collision event as taking place in the presence of a quasistatic field with a fixed particle phase; the fact that the field is actually not constant has been recovered by an appropriate phase averaging. Accordingly, the frequency of the radiation field does not enter the final results. Then the QSA may be considered a simplification of the LFA at very low frequency. Finally we have shown how it is possible to recover the QSA expression for the cross section by starting from the KJM approximation.

We conclude by saying that the different low-frequency approximations to the T matrix for collisions in the presence of a radiation and a quantizing magnetic field derived in this paper could be of interest for a series of elementary atomic processes. To this aim a generalization of our theory to any polarization of the radiation field would be desirable. Moreover, we remind the reader that the factorized forms of the T matrix derived in the LFA are obtained because of the factorized structure of the unperturbed wave functions of the electrons embedded in both the magnetic field and the radiation field linearly polarized along B. Such a simple structure of the T matrix cannot be recovered for an arbitrary polarization of the radiation field.

ACKNOWLEDGMENTS

The authors thank G. Ferrante for fruitful discussions. This work was supported by the European Community under Contract No. ERBCHRXCT 940552, by the Italian Ministry of University and Scientific Research, by the National Group of Structure of Matter, and by the Sicilian Committee for Nuclear and Structure of Matter Research.

[1] V. Canuto and J. Ventura, Fund. Cosmic Phys. **2**, 203 (1977).
[2] M. R. C. McDowell and M. Zarcone, Adv. At. Mol. Phys. **21**, 255 (1985).
[3] H. Krüger and Ch. Jung, Phys. Rev. A **17**, 1706 (1978).
[4] N. M. Kroll and K. M. Watson, Phys. Rev. A **8**, 804 (1973).
[5] H. Ohsaki, J. Phys. Soc. Jpn. **52**, 431 (1983).
[6] M. Zarcone, Nuovo Cimento D **13**, 749 (1991).
[7] W. A. M. Blumberg, W. M. Itano, and D. J. Larson, Phys. Rev. A **19**, 139 (1979).
[8] S. Bivona, R. Burlon, G. Ferrante, and C. Leone, Phys. Rev. A **50**, R1984 (1994).
[9] G. Ferrante, Phys. Rev. A **22**, 2529 (1980).
[10] J. Ventura, Phys. Rev. A **9**, 3021 (1973).
[11] G. Ferrante, S. Nuzzo, M. Zarcone, and S. Bivona, J. Phys. B **13**, 731 (1980).
[12] S. Nuzzo and M. Zarcone, Nuovo Cimento D **11**, 419 (1989).
[13] S. Nuzzo and M. Zarcone, J. Phys. B **22**, L627 (1989).