Nonclassical Bose-Einstein condensate

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A scheme is proposed to manipulate the quantum-statistical properties of a neutral atomic Bose-Einstein condensate in an ultracold alkali-atom-trap system with atoms in the ground hyperfine $F = 1$, $M_F = \pm 1, 0$ states (labeled respectively as $\mid g_{\pm,0}\rangle$). Initially, the atomic condensate is assumed to be prepared in a hyperfine sublevel $\mid g_-\rangle$. By effective two-photon excitations with two classical (σ^-) and quantum (σ^+) nonresonant copropagating traveling-wave light fields, we show that the atomic system can be settled into a coherent superposition of two atomic Bose-Einstein condensates corresponding to different ground hyperfine sublevels and different quantum statistics. Furthermore, with an appropriate choice of interaction time, atom-field coupling strengths, and quantum features of the quantized laser field, a nonclassical condensate can be prepared in the hyperfine sublevel $\mid g_+\rangle$.

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Recent advances in laser cooling and trapping have stimulated research on the system composed of ultracold atoms. Both experimental and theoretical aspects of achieving Bose-Einstein condensation (BEC) in a confined atomic gas have recently attracted great attention [1–9]. There seem to be many technical difficulties in reaching such a goal. Although the experimental achievements are beyond the ambitious goal, it is a general belief that BEC is possible in a confined atomic gas. Using such a belief, theoretical research on the interaction of a BEC with a light wave and other relative topics in quantum statistics of ultracold atomic ensembles made rapid progress in quantum optics [10–21]. In the present paper, we propose a scheme to manipulate an atomic BEC by a quantized light field, and show the possibility of preparing a novel condensate with a nonclassical density field.

Since many experiments attempting to realize gaseous BEC are carried out in some kinds of trapped geometry [1–9], it is plausible to adopt a conventional configuration with neutral atoms confined in a trap potential. In a harmonic trap, the minimum number of atoms (N_n) needed to obtain BEC is explicitly dependent on the oscillation frequency ν and ultimate temperature T. For instance, in an ultracold atom-trap system with $\nu = (2\pi) \times 100$ Hz and $T_c = 0.1$ μK, the maximum number of atoms in the normal-state fraction is $N_n = 10^4$. If the total number of atoms N exceeds N_n, the excess atoms ($N_c = N - N_n$) go to the vibrational ground state $\mid 0\rangle$. It has been shown that about 10^8 alkali atoms could be stored in a magneto-optical trap (MOT) [1–4] by the use of evaporative cooling techniques. Here, we will deal with moderate numbers of atoms and focus our attention on an atomic system of ^{7}Li [22–24] atoms in the ground hyperfine sublevels $F = 1, M_F = \pm 1, 0$, labeled, respectively, as $\mid g_{\pm,0}\rangle$. Initially the sublevels $\mid g_0\rangle$ and $\mid g_+\rangle$ are assumed to be depopulated by possible experimental techniques [1–4]. We make a further assumption that $N_c \simeq 10^4$ atoms are accumulated in the state $\mid g_-\rangle \otimes \mid 0\rangle$ to form a BEC. Two copropagating σ^+ and σ^- traveling-wave light fields are employed to excite the atomic condensate via two-photon excitations. The σ^- light wave is assumed to be strong enough to be treated as a classical field and the σ^+ light wave is a weak quantized field. In the processes of two-photon coupling, some intermediate states $\mid F' = 0, 1, 2, M_{F'} = 0\rangle$, labeled as $\mid j\rangle$, are involved. The schematic diagram for such a two-photon excitation is shown in Fig. 1. The transfer of atomic population between the ground-state sublevels $\mid g_-\rangle$ and $\mid g_+\rangle$ due to the two-photon excitations may create a new condensate in the sublevel $\mid g_+\rangle$. We shall show that the generated condensate can be settled into different quantum states by appropriately choosing the quantum states of the weak quantized light field. In this sense, the two-photon excitations result in a coherent superposition of two atomic Bose-Einstein condensates corresponding to different hyperfine sublevels and different quantum statistics [25]. Furthermore, if the interaction time for the two-photon coupling and strengths of the light fields are appropriately selected, and if the quantum light field is initially prepared in a certain nonclassical state, a nonclassical condensate can be generated by the use of quantum features interchanged from the quantized light field to the generated condensate.

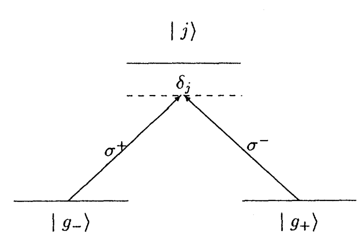

FIG. 1. Schematic diagram of two-photon excitation of atoms in a Bose-Einstein condensate.

To initiate our arguments, we begin with a systematic Hamiltonian for the noninteracting (ideal) boson alkali atoms confined in MOT in the following second quantized form [12–16,25–28] in a frame rotating at the frequency of the applied laser fields and in the rotating wave approximation (RWA):

$$\begin{aligned}\frac{H}{\hbar} = &\sum_{\mathbf{n}} \epsilon_{\mathbf{n}} \left(\phi^{\dagger}_{g+,\mathbf{n}}\phi_{g+,\mathbf{n}} + \phi^{\dagger}_{g-,\mathbf{n}}\phi_{g-,\mathbf{n}}\right) + \sum_{j,\mathbf{n}} (\epsilon_{\mathbf{n}} + \delta_j)\, \phi^{\dagger}_{j,\mathbf{n}}\phi_{j,\mathbf{n}} \\ &+\Bigg[\sum_{j,\mathbf{n}_1,\mathbf{n}_2} \Omega_j e^{-i\varphi_1}\phi^{\dagger}_{j,\mathbf{n}_1}\phi_{g+,\mathbf{n}_2}\langle \mathbf{n}_1 \mid e^{i\mathbf{k}\cdot\hat{\mathbf{r}}} \mid \mathbf{n}_2\rangle + \sum_{j,\mathbf{n}_1,\mathbf{n}_2} g_j e^{-i\varphi_2} a\phi^{\dagger}_{j,\mathbf{n}_1}\phi_{g-,\mathbf{n}_2}\langle \mathbf{n}_1 \mid e^{i\mathbf{k}\cdot\hat{\mathbf{r}}} \mid \mathbf{n}_2\rangle \\ &+\sum_{q} (\omega_q - \omega)\, a^{\dagger}_q a_q + \sum_{q,\mathbf{n}_1,\mathbf{n}_2} \xi_+(q) a_q \phi^{\dagger}_{j,\mathbf{n}_1}\phi_{g+,\mathbf{n}_2}\langle \mathbf{n}_1 \mid e^{i\mathbf{q}\cdot\hat{\mathbf{r}}} \mid \mathbf{n}_2\rangle \\ &+\sum_{q,\mathbf{n}_1,\mathbf{n}_2} \xi_-(q) a_q \phi^{\dagger}_{j,\mathbf{n}_1}\phi_{g-,\mathbf{n}_2}\langle \mathbf{n}_1 \mid e^{i\mathbf{q}\cdot\hat{\mathbf{r}}} \mid \mathbf{n}_2\rangle + \text{H.c.}\Bigg]\ , \end{aligned} \tag{1}$$

where $\mid j\rangle$ represent intermediate atomic internal states with energies $\hbar\omega_j$ and $\delta_j = \omega_j - \omega$ denote the associated laser detunings (ω is the laser frequency). Atoms in a MOT are usually trapped near a local magnetic field strength extremum [1–9], where the ground hyperfine levels are Zeeman split by a small amount. We may neglect the Zeeman shifts here for the sake of simplicity. $\phi^{\dagger}_{i,\mathbf{n}}$ and $\phi_{i,\mathbf{n}}$ are atomic creation and annihilation operators for atoms with the atomic internal structure and center-of-mass motion being in the states $\mid i\rangle =\mid g_{\pm}, j\rangle$ and Fock states $\mid \mathbf{n}\rangle$ of vibrational energies $\epsilon_{\mathbf{n}}$, respectively. $\mathbf{n}$ is used to label the nth vibrational state and it is actually a triplet index (n_x, n_y, n_z). $\varphi_{1,2}$ are the phases of the applied laser fields. The first three terms in the above Hamiltonian are the energies of the ground-state sublevels $\mid g_{\pm}\rangle$ and intermediate states $\mid j\rangle$. Note that the ground hyperfine sublevel $\mid g_0\rangle$ is not involved in the questions considered here. The fourth and fifth terms describe the interaction of the atomic ensemble with the strong σ^--polarized laser and weak σ^+-polarized quantized laser fields. In terms of the conservation of angular momentum, the intermediate states are those with magnetic quantum number $M_{F'=0,1,2} = 0$. The strong σ^--polarized laser only couples to the internal states $\mid j\rangle$ and $\mid g_+\rangle$, and the weak σ^+-polarized quantum laser only to the states $\mid j\rangle$ and $\mid g_-\rangle$. The coupling strengths are, respectively, denoted as Ω_j and g_j. The quantized laser is described by the Hermitian conjugate operators a and $a^{\dagger}$. The last three terms are related to vacuum light fields and their interaction with the atomic ensemble, which may give rise to some incoherent effects, such as atom-atom interactions due to photon exchange, spontaneous emissions, and collective spontaneous emissions [13–18,25–28]. Previous analysis [13–15] has demonstrated that the rates Γ_j of the collective spontaneous emissions from the intermediate states $\mid j\rangle$ scale as $\Gamma_j \sim 2N_c\gamma_j$, with $2\gamma_j$ being the spontaneous emission rate of an individual atom. On the other hand, the many-body interactions originated from the photon exchange are proportional to $N_c\gamma_j/\mid \delta_j \mid$ [16–18,26,27]. Hence only under conditions with very large laser frequency detunings ($\mid \delta_j \mid \gg N_c\gamma_j$) can we neglect these incoherent processes. Typically, the optical linewidth for an individual atom is of the order of $\gamma_j \leq 10^7$ Hz. For the parameters considered here, the effective spontaneous emission rates, as was indicated in a recent calculation performed by Lewenstein and co-workers [16,26] and Javanainen [18], are of the order of hundreds of GHz. We may thus select the laser frequency detunings to be of the order of $\mid \delta_j \mid \geq 10^{11}$ Hz to avoid the incoherent influence from the vacuum fields. Moreover, a recent self-consistent analysis [16,26] has shown that, if the laser-atom interaction time is selected as short as 10 ps, the collective spontaneous emission can be legitimately neglected. Hereafter, we will assume that the two applied light waves are pulses of the same temporal envelope with duration 10 ps and width $\gamma_L \simeq 10^{11}$ Hz. Then, in the interaction Hamiltonian, $\exp(i\mathbf{k}\cdot\hat{\mathbf{r}})$ should be substituted by $f[\gamma_L(t-\mathbf{k}_L\cdot\hat{\mathbf{r}}/\omega)]\exp(i\mathbf{k}_L\cdot\hat{\mathbf{r}})$, which corresponds to an assumption that the pulses have forms of plane wavepackets propagating in the $\mathbf{k}_L$ direction with the center frequency ω and polarizations $\sigma^{\dagger}$ and σ^-, respectively. $f(\gamma_L t)$ is the temporal envelope of the pulses chosen to be real. In the trap system concerned, the characteristic length is the size of the vibrational ground-state wave function $a \sim 10^{-5}$ m, while the σ^+- and σ^--polarized photon momenta are in the range $\hbar k_L \pm \hbar\gamma_L/c$, whose changes are negligible in comparison with $1/a \sim 10^5\ \text{m}^{-1}$. We may thus safely set $\mathbf{k} \simeq \mathbf{k}_L$ [16,26]. Moreover, since the widths of the pulses are much smaller than the laser frequency detunings, the σ^+-polarized (weak) laser pulse can be approximately viewed as a single quantum electromagnetic mode with time-dependent amplitude. In such assumptions and approximations, the Hamiltonian can be rewritten as

$$\begin{aligned}\frac{H}{\hbar} = &\sum_{\mathbf{n}} \epsilon_{\mathbf{n}} \left(\phi^{\dagger}_{g+,\mathbf{n}}\phi_{g+,\mathbf{n}} + \phi^{\dagger}_{g-,\mathbf{n}}\phi_{g-,\mathbf{n}}\right) + \sum_{j,\mathbf{n}} (\epsilon_{\mathbf{n}} + \delta_j)\, \phi^{\dagger}_{j,\mathbf{n}}\phi_{j,\mathbf{n}} + \Bigg[\sum_{j,\mathbf{n}_1,\mathbf{n}_2} \Omega_j e^{-i\varphi_1} f(\gamma_L t)\phi^{\dagger}_{j,\mathbf{n}_1}\phi_{g+,\mathbf{n}_2}\langle \mathbf{n}_1 \mid e^{i\mathbf{k}_L\cdot\hat{\mathbf{r}}} \mid \mathbf{n}_2\rangle \\ &+\sum_{j,\mathbf{n}_1,\mathbf{n}_2} g_j e^{-i\varphi_2} f(\gamma_L t) a\phi^{\dagger}_{j,\mathbf{n}_1}\phi_{g-,\mathbf{n}_2}\langle \mathbf{n}_1 \mid e^{i\mathbf{k}_L\cdot\hat{\mathbf{r}}} \mid \mathbf{n}_2\rangle + \text{H.c.}\Bigg]\ . \end{aligned} \tag{2}$$

The coherent transitions described in the above Hamiltonian include atomic excitations from ground hyperfine sublevel $| g_+, \mathbf{n}_2\rangle$ to intermediate states $| j, \mathbf{n}_1\rangle$ by absorption of σ^--polarized light photons, and transitions between $| g_-, \mathbf{n}_2\rangle$ and $| j, \mathbf{n}_1\rangle$ driven by the quantized light field. In view of the large laser frequency detunings, we may adiabatically eliminate the intermediate states. As will be shown in the following, the applied laser fields produce two effects: two-photon transitions between $| g_\pm, \mathbf{n}\rangle$ and $| g_\mp, \mathbf{n}\rangle$, and ac Stark shifts, respectively. The atomic excitations of the condensate create a new condensate in the sublevel $| g_+\rangle$ if more than one atom is left in $| g_+, 0\rangle$, and the excitations of atoms in the normal-state fraction ($| g_-, \mathbf{n} \neq 0\rangle$) give rise to atoms in the normal-state fraction ($| g_+, \mathbf{n} \neq 0\rangle$) for the newly generated condensate. In our case, the atomic and quantized light fields satisfy the following Heisenberg equations of motion:

$$
\begin{aligned}
i\frac{\partial}{\partial t}\phi_{g_+,\mathbf{n}} &= \sum_{j,\mathbf{n}_1} \Omega_j^* e^{i\varphi_1} f(\gamma_L t)\phi_{j,\mathbf{n}_1}\langle \mathbf{n} \mid e^{-i\mathbf{k}_L\cdot\hat{\mathbf{r}}} \mid \mathbf{n}_1\rangle \ , \\
i\frac{\partial}{\partial t}\phi_{j,\mathbf{n}} &= \delta_j\phi_{j,\mathbf{n}} + \sum_{\mathbf{n}_2} \Omega_j e^{-i\varphi_1} f(\gamma_L t)\phi_{g_+,\mathbf{n}_2}\langle \mathbf{n} \mid e^{i\mathbf{k}_L\cdot\hat{\mathbf{r}}} \mid \mathbf{n}_2\rangle + \sum_{\mathbf{n}_2} g_j e^{-i\varphi_2} f(\gamma_L t) a\phi_{g_-,\mathbf{n}_2}\langle \mathbf{n} \mid e^{i\mathbf{k}_L\cdot\hat{\mathbf{r}}} \mid \mathbf{n}_2\rangle \ , \\
i\frac{\partial}{\partial t}a &= \sum_{j,\mathbf{n}_1,\mathbf{n}_2} g_j^* e^{i\varphi_2} f(\gamma_L t)\phi^\dagger_{g_-,\mathbf{n}_2}\phi_{j,\mathbf{n}_1}\langle \mathbf{n}_2 \mid e^{-i\mathbf{k}_L\cdot\hat{\mathbf{r}}} \mid \mathbf{n}_1\rangle \ .
\end{aligned} \tag{3}
$$

Since the c.m. energies $\epsilon_\mathbf{n}$ are very small compared to the spontaneous emission rates and laser frequency detunings, we have approximately set $\epsilon_\mathbf{n} \equiv 0$. Moreover, the atomic field operators $\phi_{j,\mathbf{n}}$ can be adiabatically eliminated,

$$
\phi_{j,\mathbf{n}} = -\frac{1}{\delta_j}\left[\sum_{\mathbf{n}_2} \Omega_j e^{-i\varphi_1} f(\gamma_L t)\phi_{g_+,\mathbf{n}_2}\langle \mathbf{n} \mid e^{i\mathbf{k}_L\cdot\hat{\mathbf{r}}} \mid \mathbf{n}_2\rangle + \sum_{\mathbf{n}_2} g_j e^{-i\varphi_2} f(\gamma_L t) a\phi_{g_-,\mathbf{n}_2}\langle \mathbf{n} \mid e^{i\mathbf{k}_L\cdot\hat{\mathbf{r}}} \mid \mathbf{n}_2\rangle\right] , \tag{4}
$$

where terms oscillating rapidly at the frequencies δ_j are neglected. We next substitute these results into the differential equations of the atomic field operators $\phi_{g_\pm,\mathbf{n}}$ and laser field operator a to derive

$$
\begin{aligned}
i\frac{\partial}{\partial t}\phi_{g_+,\mathbf{n}} &= -\sum_j \frac{\Omega_j\Omega_j^*}{\delta_j} f^2(\gamma_L t)\phi_{g_+,\mathbf{n}} - \sum_j \frac{\Omega_j^* g_j}{\delta_j} e^{i(\varphi_1-\varphi_2)} f^2(\gamma_L t) a\phi_{g_-,\mathbf{n}} \quad , \\
i\frac{\partial}{\partial t}\phi_{g_-,\mathbf{n}} &= -\sum_j \frac{g_j g_j^*}{\delta_j} f^2(\gamma_L t) a\phi^\dagger_{g_-,\mathbf{n}} a - \sum_j \frac{\Omega_j g_j^*}{\delta_j} e^{-i(\varphi_1-\varphi_2)} f^2(\gamma_L t)\phi^\dagger_{g_+,\mathbf{n}} a \quad , \\
i\frac{\partial}{\partial t}a &= -\sum_{j,\mathbf{n}} \frac{g_j g_j^*}{\delta_j} f^2(\gamma_L t)\phi^\dagger_{g_-,\mathbf{n}} a\phi_{g_-,\mathbf{n}} - \sum_{j,\mathbf{n}} \frac{\Omega_j g_j^*}{\delta_j} e^{-i(\varphi_1-\varphi_2)} f^2(\gamma_L t)\phi^\dagger_{g_-,\mathbf{n}}\phi_{g_+,\mathbf{n}} \quad .
\end{aligned} \tag{5}
$$

The influence of atomic excitations on the weak quantized field is originated from the two-photon Rabi oscillations of the condensed atoms ($| g_\pm, 0\rangle$) as well as normal-state atoms ($| g_\pm, \mathbf{n} \neq 0\rangle$). In this paper, we suppose that the condensated atoms produce an overwhelming counteraction on the weak quantized light field, and we neglect the effects from the atoms in the normal-state fraction. This is valid when the number of atoms in the condensate is large in comparison with that in the noncondensate fraction. We will give some further comments on this topic in what follows. On the other hand, since we are confronted with a situation of weak quantized light field and large laser frequency detunings, the ac Stark shift and time derivation of the ground-state sublevel $| g_-\rangle$ are negligible. In a customary way, the condensate is well described by a coherent state $| \alpha\rangle$ [29] defined as

$$
\phi_{g_-,0} \mid \alpha\rangle = \sqrt{N_c} e^{-i\vartheta} \mid \alpha\rangle \quad . \tag{6}
$$

The corresponding creation and annihilation operators can then be treated as c numbers. We introduce the ac Stark shifts

$$
\begin{aligned}
S_1 &= -\sum_j \frac{\Omega_j\Omega_j^*}{\delta_j}, \\
S_2 &= -\sum_j \frac{g_j g_j^* N_c}{\delta_j} \quad ,
\end{aligned}
$$

as well as effective two-photon coupling constant

$$
R = -\sum_j \frac{\Omega_j^* g_j \sqrt{N_c}}{\delta_j} e^{i(\varphi_1-\varphi_2-\vartheta)} \quad .
$$

Then, the differential equations of the density field $\phi_+ = \phi_{g_+,0}$ can be solved in terms of the initial operators $a(0)$ and $\phi_+(0)$ [30],

$$
\begin{aligned}
\phi_+(\tau) = e^{-i\frac{S_1+S_2}{2}\tau}\Bigg\{&\phi_+(0)\cos(\xi\tau) \\
&-i\left[\frac{R}{\xi}a(0) + \frac{S_1-S_2}{2\xi}\phi_+(0)\right]\sin(\xi\tau)\Bigg\},
\end{aligned} \tag{7}
$$

where $\xi = \sqrt{(S_1-S_2)^2/4+ | R |^2}$ is the generalized two-photon Rabi frequency, and $\tau = \int_{-\infty}^{t} f^2(\gamma_L t)dt$ is defined as the pulse area. If the laser fields and pulse area are selected to satisfy

$$S_1 = S_2 \quad , \tag{8}$$

$$| R | \tau_0 = \left(N + \frac{1}{2}\right)\pi \quad , \tag{9}$$

where $\tau_0 = \int_{-\infty}^{+\infty} dt f^2(\gamma_L t)$, the density field $\phi_+(\tau_0)$ becomes

$$\phi_+(\tau_0) = -i(-)^N e^{i\varphi} a(0) \equiv e^{i\chi} a(0), \tag{10}$$

where the phase φ is defined as $e^{i\varphi} = e^{-i\frac{S_1+S_2}{2}\tau_0} R / | R |$, and $\chi = \varphi + (N - 1/2)\pi$. Hence, after the two-photon excitations, the condensate in the hyperfine sublevel $| g_+\rangle$ is only characterized by the initial features of the quantized laser field.

In the interaction picture, we set the wave function of the total system, including the radiation and atomic density fields, to be $| \varphi\rangle =| \varphi_+\rangle \otimes | \varphi_f\rangle$, in which $| \varphi_+\rangle$ and $| \varphi_f\rangle$ are the wave functions of the atomic density field ϕ_+ and quantized laser field, respectively. If the quantized laser field is initially in a vacuum state $| \varphi_f\rangle =| 0\rangle$, i.e., $a(0) | 0\rangle = 0$, no atoms will be populated in the atomic hyperfine sublevel $| g_+\rangle$, for the field ϕ_+ will also be in a vacuum state $| \varphi_+\rangle =| 0\rangle$ after the excitations. If the quantized laser field is initially in a Fock state $| \varphi_f\rangle =| n\rangle$, i.e., $a^\dagger(0)a(0) | n\rangle = n | n\rangle$, the field ϕ_+ will also be in a Fock state at $t = +\infty$, for $\phi_+^\dagger(\tau_0)\phi_+(\tau_0) | \varphi\rangle = a^\dagger(0)a(0) | \varphi\rangle = n | \varphi\rangle$. More interestingly, if the quantized laser field is initially in a squeezed coherent state $| \varphi_f\rangle = D(\alpha_1, t = -\infty)S(\theta, t = -\infty) | 0\rangle$, with $D(\alpha_1, t = -\infty) \equiv \exp\,[\alpha_1 a^\dagger(0) - \alpha_1^* a(0)]$ and $S(\theta, t = -\infty) \equiv \exp\left[\frac{1}{2}\theta^* a^2(0) - \frac{1}{2}\theta a^\dagger 2(0)\right]$, where θ is the squeezing parameter, squeezing occurs at $t = +\infty$ for the state of the quantum density field ϕ_+, because the state generated by the operator

$$D_+(\alpha_1, t = +\infty)S_+(\theta, t = +\infty) \equiv \exp\,[\alpha_1\phi_+^\dagger(\tau_0) - \alpha_1^*\phi_+(\tau_0)] \exp\left(\frac{1}{2}\theta^*\phi_+^2(\tau_0) - \frac{1}{2}\theta\phi_+^{\dagger 2}(\tau_0)\right)$$

is $D(\alpha_1 e^{-i\chi}, t = -\infty)S(\theta e^{-i2\chi}, t = -\infty) | 0\rangle$, which has the squeezing parameter $\theta e^{-i2\chi}$. That means the squeezing properties of the quantized laser field can be converted to the quantum atomic density field ϕ_+. This kind of quantum conversion [30] can be shown clearly in terms of the expectation values of the fluctuation operators. If the quantized laser field is squeezed in one of the quadrature phase operators defined as $X_1 = (a + a^\dagger)/2$ or $X_2 = (a - a^\dagger)/2i$, at $t = +\infty$, the atomic density field ϕ_+ will be squeezed with the squeezed quadrature phase operator defined as $Y_1 = (\phi_+ e^{-i\chi} + \phi_+^\dagger e^{i\chi})/2$ or $Y_2 = (\phi_+ e^{-i\chi} - \phi_+^\dagger e^{i\chi})/2i$, respectively, for

$$\begin{aligned}\langle(\Delta Y_j)^2\rangle|_{\tau_0} &= \langle\varphi \mid Y_j^2(\tau_0) \mid \varphi\rangle - [\langle\varphi \mid Y_j(\tau_0) \mid \varphi\rangle]^2 \\ &= \langle\varphi \mid X_j^2(t = -\infty) \mid \varphi\rangle \\ &\quad -[\langle\varphi \mid X_j(t = -\infty) \mid \varphi\rangle]^2 \\ &= \langle(\Delta X_j)^2\rangle|_{t=-\infty} \qquad (j = 1, 2) \ .\end{aligned}$$

On the other hand, the average number of atoms in the generated condensate is equal to the initial photon number of the σ^+-polarized quantum laser field, for $\langle\phi_+^\dagger(\tau_0)\phi_+(\tau_0)\rangle = \langle a^\dagger(0)a(0)\rangle$. Since the quantized light field is weak, only a small number of atoms are condensed in the hyperfine sublevel $| g_+, 0\rangle$. Furthermore, the two-photon excitations create coherence between the ground hyperfine sublevels $| g_\pm\rangle$; the atomic system is thus settled into a coherent superposition of two atomic Bose-Einstein condensates corresponding to different internal ground-state sublevels and different quantum statistics.

It is worthwhile emphasizing that the above discussions are carried out under the assumption that the condensate $\phi_- = \phi_{g_-,0}$ is in a coherent state. In a real condensate, the associated density field, which has Poissonian quanta statistics, may not be coherent. However, if the boson operators acquire very large macroscopic expectation values: $| \langle\phi_-\rangle | =| \langle\phi_-^\dagger\rangle | = \sqrt{N_c} \gg 1$, the commutators for the operators ϕ_- and $\phi_-^\dagger$ can be ignored, and the condensate in the state $| g_-\rangle$ can be approximately treated as a classical density. In such a limit, the operators for the density field ϕ_- can be replaced by the associated c numbers. This is analogous to that commonly used in quantum optics where large laser fields are treated classically. In the above considerations, we have neglected the interaction between particles. This assumption is only valid if the characteristic energy of elastic interaction between atoms is much smaller than the level spacing in the trap, i.e.,

$$n\tilde{U} \ll \hbar\nu \quad , \tag{11}$$

where n is the atom (condensate) density, $\tilde{U} = 4\pi\hbar^2 a_s/m$, m is the atom mass, and a_s is the scattering length. We consider 10^4 ^{7}Li atoms in a trap with $\nu = (2\pi) \times 100$ Hz, n is estimated to be of the order $n \sim 10^{14}$ cm^{-3}. With realistic number ~ 10 Å for the scattering length, we obtain $n\tilde{U} \sim 10^{-4}$ K $[n\tilde{U}/\hbar \sim (2\pi) \times 10^4$ Hz], i.e., much larger than $\hbar\nu$. This means that the interaction between (ground-state) atoms cannot be neglected. One may consider the interaction along with the following strategy: An ideal Bose gas can be well described with a macroscopic wave function $\psi_0(\mathbf{r}) = \sqrt{N_c}\langle\mathbf{r} \mid 0\rangle$. For any interacting condensate an analogous $\psi_I(\mathbf{r})$ should exist. In our theory, $\psi_0(\mathbf{r})$ has in effect been coupled with light fields as a rigid entity. It is reasonable to postulate that in an interacting system $\psi_I(\mathbf{r})$ is simply used instead of $\psi_0(\mathbf{r})$. Although the macroscopic wave function $\psi_I(\mathbf{r})$ is to be determined from the theory of an interacting Bose condensate, the results will be qualitatively similar to the results from the theory of an ideal Bose gas. As for the interaction between excited- and ground-state atoms, it takes the following form in the coordinate representation [31] and in the shape-independent approximation [27]:

$$H = 4\pi \sum_j \left\{ \int d\,\mathbf{R} |d_{j,+}|^2 \psi^\dagger_{g_+}(\mathbf{R}) \psi^\dagger_j(\mathbf{R}) \psi_j(\mathbf{R}) \psi_{g_+}(\mathbf{R}) + \int d\,\mathbf{R} |d_{j,-}|^2 \psi^\dagger_{g_-}(\mathbf{R}) \psi^\dagger_j(\mathbf{R}) \psi_j(\mathbf{R}) \psi_{g_-}(\mathbf{R}) \right\} , \tag{12}$$

where the atomic fields $\psi_{g_\pm}(\mathbf{R})$ and $\psi_j(\mathbf{R})$ are introduced as

$$\psi_{g_\pm}(\mathbf{R}) = \sum_{\mathbf{n}} \langle \mathbf{R}|\mathbf{n}\rangle \phi_{g_\pm,\mathbf{n}} , \tag{13}$$

$$\psi_j(\mathbf{R}) = \sum_{\mathbf{n}} \langle \mathbf{R}|\mathbf{n}\rangle \phi_{j,\mathbf{n}} , \tag{14}$$

and $|d_{j,\pm}|^2$ is the absolute value of the dipole moment between atomic internal states $|g_\pm\rangle$ and $|j\rangle$, respectively. For the density we consider in this paper, $n \sim 10^{14}$ cm^{-3}, the mean interparticle separation is much larger than the optical wavelength $\lambda/2\pi$:

$$n\lambda^3/8\pi^3 \ll 1 \quad . \tag{15}$$

In such a situation, the integration in the expression (12) is very small, and therefore the contact interaction between ground-state and excited-state atoms will play a negligible role. Also the collective optical excitations will be absent [10,11]. On the other hand, the number of condensate atoms is dependent on the temperature T of the system as

$$N_c = N \left[1 - \left(\frac{T}{T_c} \right)^3 \right] \quad , \tag{16}$$

where T_c is the critical temperature $[N_n = 1.2202(k_B T_c/\hbar\nu)^3]$. So far we have mostly ignored the atoms in the normal-state (noncondensate) fraction. From the experimental point of view, what seems realistic is that only a not-too-large part of atoms is in the condensate, and that the gas temperature is just a little below the critical value. As was shown [32], for magnetically trapped hydrogen only a small fraction of atoms can be in the condensate, mainly because of strongly enhanced spin relaxation. One may expect the same for lithium atoms in MOT, in this case because of the strongly enhanced three-body recombination. Once the condensate is formed in the trap, it is strongly compressed by the inhomogeneous trapping potential. The corresponding increase of density leads to strong enhancement of inelastic collisional processes, and the trapped gas sample decays very rapidly [32]. In this sense, the approximation, in which the effects from the noncondensate atoms are neglected, may become questionable. As is shown in the above analyses, the two-photon Raman processes consist of transitions within the condensates or normal-state fractions in the different hyperfine sublevels. Transitions within the normal-state fractions may produce significant counteraction on the weak quantum light field, and thereby may mask the quantum feature interchange between the condensate and quantum laser field. Quantitative analyses on the effects from the normal-state fraction are somewhat difficult, but we can make a few qualitative arguments. We note that the matrix element for the two-photon Raman coupling between the condensates in the hyperfine sublevel $|g_\pm\rangle$ is proportional to a numerical factor $\sqrt{N_c}$, while the maximum thermal occupation number of a noncondensate state in our situation is $k_B T/\hbar\nu \sim 20$. As a result, collective effects are less important for atoms in the normal-state fraction, especially in a system with small atomic density: $n\lambda^3/8\pi^3 \ll 1$. For example, we assume that the gas temperature is $T \sim 0.75T_c(N_c/N \simeq 37/64 > 1/2)$, then the influence on the weak quantum laser field is mainly caused by transitions within the condensates, and hence the abovementioned quantum conversion will not be masked by the transitions between the thermally occupied noncondensate states.

In summary, we emphasize that by the use of the quantum conversion between the quantized laser and atomic density fields demonstrated here, some novel kinds of BEC with nonclassical features for the associated density field can be achieved. It may be of theoretical interest to investigate the quantum statistics of that novel kind of condensate. The experimental exploration of the predicted effect has to await laser cooling and trapping technology capable of achieving the normal BEC with large degree of condensation.

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