Influence of intrinsic decoherence on nonclassical effects in the multiphoton Jaynes-Cummings model

Le-Man Kuang
Theoretical Physics Division, Nankai Institute of Mathematics, Tianjin 300071, People's Republic of China and Department of Physics and Institute of Physics, Hunan Normal University, Hunan 410006, People's Republic of China

Xin Chen
Institute of Acoustics and Department of Electrical Science and Engineering, Nanjing University, Nanjing 210093, People's Republic of China

Mo-Lin Ge
Theoretical Physics Division, Nankai Institute of Mathematics, Tianjin 300071, People's Republic of China

(Received 28 November 1994)

In this paper, we study the multiphoton Jaynes-Cummings model (JCM) governed by the Milburn equation [G. J. Milburn, Phys. Rev. A **44**, 5401 (1991); **47**, 2415 (1993)], which models the decoherence of a quantum system as the quantum system evolves through intrinsic mechanisms beyond conventional quantum mechanics. We give an exact solution of this equation for the multiphoton Jaynes-Cummings Hamiltonian and apply it to investigate the influence of the intrinsic decoherence on nonclassical effects (atomic inversion, oscillations of the photon-number distribution, squeezing of the cavity field, and photon antibunching) in the JCM. It is shown that during the time evolution, the intrinsic decoherence in the atom-field interaction suppresses these nonclassical behaviors in the JCM.

PACS number(s): 03.65.Bz, 42.50.Dv

I. INTRODUCTION

In the past few years there has been considerable interest in studying the decoherence problem of a quantum system by means of modifying the Schrödinger equation [1–7], called the intrinsic-decoherence approach. In particular, Milburn [8,9] proposed a simple intrinsic-decoherence model. This model yields a modification of the von Neumann equation for the density operator of a quantum system through a simple modification of the usual Schrödinger evolution. In Milburn's model, the off-diagonal elements of the density operator are intrinsically suppressed in the energy eigenstate basis because the quantum system evolves under a stochastic sequence of identical unitary transformation, thereby the intrinsic decoherence is realized.

It is well known that the Jaynes-Cummings model (JCM) [10] in quantum optics can describe many pure quantum phenomena, called nonclassical effects, such as collapses and revivals of the atomic inversion, oscillations of photon number distribution, photon antibunching, and squeezing of the cavity field. It has been generally accepted that these nonclassical effects originate from quantum coherences. Therefore, it is an interesting topic to investigate the influence of the intrinsic decoherence on the nonclassical effects in the JCM. The purpose of the present paper is to study the influence of the intrinsic decoherence on nonclassical effects in the multiphoton JCM [11,12] governed by the Milburn model. We will show that the intrinsic decoherence in the atom-field interaction modifies the time evolution of the atomic inversion and investigate oscillations of photon-number distribution, quadrature squeezing behaviors of the cavity field, and photon antibunching.

This paper is organized as follows. In Sec. II, we present the exact solution of the Milburn equation for the multiphoton Jaynes-Cummings Hamiltonian and give the explicit expression of this solution in the two-dimensional atomic basis. In Sec. III, we examine the effect of the intrinsic decoherence on the atomic inversion. Section IV is devoted to investigating the influence of the intrinsic decoherence on nonclassical effects of the field in the JCM. Concluding remarks are provided in the last section.

II. EXACT SOLUTION OF THE MILBURN EQUATION

Consider a quantum system described by the density operator $\hat{\rho}(t)$. Dynamics of the system is governed by the evolution operator $\hat{U}(t)=\exp(-i\hat{H}t/\hbar)$, where $\hat{H}$ is the Hamiltonian of the system. In standard quantum mechanics, the change in a state of the system in a time interval $(t,t+\tau)$ is given by

$$\hat{\rho}(t+\tau)=\exp\left[-\frac{i\hat{H}\tau}{\hbar}\right]\hat{\rho}(t)\exp\left[\frac{i\hat{H}\tau}{\hbar}\right], \tag{1}$$

which is independent of the size of τ. Milburn [8] replaced the above paradigm with some postulates. He assumed that on a sufficiently small time scale the change in the state of the system is stochastic, the probability that the state of the system is changed is $p(\tau)$, which reflects quantum jumps in the state of the system. Given that the state of the system is undergoing some changes, the density is changed according to the following equation:

$$\hat{\rho}(t+\tau)=\exp\left[-\frac{i}{\hbar}\theta(\tau)\hat{H}\right]\hat{\rho}(t)\exp\left[\frac{i}{\hbar}\theta(\tau)\hat{H}\right], \quad (2)$$

where $\theta(\tau)$ is some function of τ. In standard quantum mechanics we have $\rho(\tau)=1$ and $\theta(\tau)=\tau$. In the Milburn theory we only require that $p(\tau)\to 1$ and $\theta(\tau)\to\tau$ for values of τ which are sufficiently large. Milburn also assumed that $\lim_{\tau\to 0}\theta(\tau)=\theta_0$, which effectively introduces a minimum time step in the Universe [13]. The inverse of this time step is the mean frequency of the unitary step, $\gamma=1/\theta_0$.

In Milburn's theory, dynamics of the system are governed by the following evolution equation:

$$\frac{d}{dt}\hat{\rho}(t)=\gamma\left\{\exp\left[\frac{-i}{\hbar\gamma}\hat{H}\right]\hat{\rho}(t)\exp\left[\frac{i}{\hbar\gamma}\hat{H}\right]-\hat{\rho}(t)\right\}, \quad (3)$$

which is equivalent to the assumption that on a sufficiently short time the probability the system evolves is $p(\tau)=\gamma\tau$. Obviously, the generalized Eq. (3) alters the Schrödinger dynamics. It reduces to the ordinary von Neuman equation for the density operator in the limit $\gamma\to\infty$.

Expanding Eq. (3) to first order in γ^{-1}, Milburn obtained the following dynamical equation:

$$\frac{d}{dt}\hat{\rho}(t)=-\frac{i}{\hbar}[\hat{H},\hat{\rho}]-\frac{1}{2\hbar^2\gamma}[\hat{H},[\hat{H},\hat{\rho}]], \quad (4)$$

which is the Milburn equation which we will study below. Milburn discussed the solution of Eq. (4) for a harmonic oscillator and a precessing spin system. Authors in Refs. [14,15] gave the exact solution of the Milburn equation for a simple JCM. In what follows we shall consider the exact solution of the Milburn Eq. (4) for the multiphoton JCM in the resonant case.

The multiphoton Jaynes-Cummings Hamiltonian [11,12] describing an interaction of a two-level atom with a single-mode cavity field via an m-photon process in the rotating-wave approximation is given by

$$\hat{H}=\hbar\omega\left[\hat{a}^+\hat{a}+\frac{m}{2}\hat{\sigma}_3\right]+\frac{\hbar\Delta}{2}\hat{\sigma}_3+\hbar\lambda(\hat{\sigma}_-\hat{a}^{+m}+\hat{\sigma}_+\hat{a}^m),$$
$$\Delta=\omega_0-m\omega, \quad (5)$$

where ω is the frequency of the cavity field, ω_0 is the atomic transition frequency, λ is the atom-field coupling constant, $\hat{a}$ and $\hat{a}^+$ are the field annihilation and creation operators, respectively, $\hat{\sigma}_3$ is the atomic-inversion operator, and $\hat{\sigma}_\pm$ are the atomic "spin flip" operators which satisfy the relations $[\hat{\sigma}_+,\hat{\sigma}_-]=\hat{\sigma}_3$ and $[\hat{\sigma}_3,\hat{\sigma}_\pm]=\pm 2\hat{\sigma}_\pm$. For simplicity, in this paper we take $\hbar=1$ and the exact resonance between the field and atomic transition frequencies, i.e., $\omega_0=m\omega$.

We now start to find the exact solution for the density operator $\hat{\rho}(t)$ of the Milburn Eq. (4) applied to the Hamiltonian (5). For convenience, we introduce three auxiliary superoperators $\hat{R}$, $\hat{S}$, and $\hat{T}$ defined by

$$\exp(\hat{R}\tau)\hat{\rho}(t)=\sum_{k=0}^{\infty}\left[\frac{\tau}{\gamma}\right]^k\frac{1}{k!}\hat{H}^k\hat{\rho}(t)\hat{H}^k, \quad (6)$$

$$\exp(\hat{S}\tau)\hat{\rho}(t)=\exp(-i\hat{H}\tau)\hat{\rho}(t)\exp(i\hat{H}\tau), \quad (7)$$

$$\exp(\hat{T}\tau)\hat{\rho}(t)=\exp\left[-\frac{\tau}{2\gamma}\hat{H}^2\right]\hat{\rho}(t)\exp\left[-\frac{\tau}{2\gamma}\hat{H}^2\right], \quad (8)$$

where the Hamiltonian $\hat{H}$ is given by Eq. (5). From Eq. (6) to Eq. (8) it follows that

$$\hat{R}\hat{\rho}=\frac{1}{\gamma}\hat{H}\hat{\rho}\hat{H},\quad \hat{S}\hat{\rho}=-i[\hat{H},\hat{\rho}],\quad \hat{T}\hat{\rho}=-\frac{1}{2\gamma}\{\hat{H}^2,\hat{\rho}\}. \quad (9)$$

Substituting Eq. (9) into Eq. (4), we can obtain the formal solution of the Milburn equation [16,17] as follows:

$$\hat{\rho}(t)=\exp(\hat{R}t)\exp(\hat{S}t)\exp(\hat{T}t)\hat{\rho}(0), \quad (10)$$

where $\hat{\rho}(0)$ is the density operator of the initial atom-field system. We assume that initially the field is prepared in the coherent state $|z\rangle$ defined by

$$|z\rangle=\sum_{n=0}^{\infty}\exp(-\tfrac{1}{2}|z|^2)\frac{z^n}{\sqrt{n!}}|n\rangle\equiv\sum_{n=0}^{\infty}Q_n|n\rangle \quad (11)$$

and the atom was prepared in its excited state $|e\rangle$, so that the initial density operator is given in the form

$$\hat{\rho}(0)=\begin{bmatrix}|z\rangle\langle z| & 0\\ 0 & 0\end{bmatrix}. \quad (12)$$

Following the approach in Refs. [14,18], we divide the Hamiltonian (5) into a sum of two terms which commute with each other, that is,

$$\hat{H}=\hat{H}_0+\hat{H}_I,\quad [\hat{H}_0,\hat{H}_I]=0, \quad (13)$$

where

$$\hat{H}_0=\omega\begin{bmatrix}\hat{n}+m/2 & 0\\ 0 & \hat{n}-m/2\end{bmatrix},\quad \hat{H}_I=\lambda\begin{bmatrix}0 & \hat{a}^m\\ \hat{a}^{+m} & 0\end{bmatrix}. \quad (14)$$

Similarly, the square of the Hamiltonian (5) can also be expressed as a sum of diagonal and off-diagonal terms in the form

$$\hat{H}^2=\hat{A}+\hat{B},\quad [\hat{A},\hat{B}]=0, \quad (15)$$

where

$$\hat{A}=\begin{bmatrix}\hat{S}(\hat{n},+) & 0\\ 0 & \hat{S}(\hat{n},-)\end{bmatrix},$$
$$\hat{B}=2\lambda\omega\begin{bmatrix}0 & \hat{a}^m(\hat{n}-m/2)\\ (\hat{n}-m/2)\hat{a}^{+m} & 0\end{bmatrix} \quad (16)$$

with

$$\hat{S}(\hat{n},+)=\omega^2\left[\hat{n}+\frac{m}{2}\right]^2+\lambda^2\hat{a}^m\hat{a}^{+m},$$
$$\hat{S}(\hat{n},-)=\omega^2\left[\hat{n}-\frac{m}{2}\right]^2+\lambda^2\hat{a}^{+m}\hat{a}^m. \quad (17)$$

For convenience, we introduce the auxiliary operator $\hat{\rho}_2(t)$ defined by

$$\hat{\rho}_2(t)=\exp(\hat{S}t)\exp(\hat{T}t)\hat{\rho}(0) . \tag{18}$$

Then, the exact solution of the Milburn equation for the multiphoton Jaynes-Cummings Hamiltonian in the resonant case is given in the following form (see the Appendix):

$$\hat{\rho}(t)=\sum_{k=0}^{\infty}\left[\frac{t}{\gamma}\right]^k\frac{1}{k!}\hat{H}^k\hat{\rho}_2(t)\hat{H}^k . \tag{19}$$

Although the above form of the solution of the Milburn equation is pleasant, it is inconvenient in use. In most cases of practical interest, one needs to know the explicit matrix elements of the density operator $\hat{\rho}(t)$. Therefore, in what follows we evaluate these elements of the density operator in the two-dimensional atomic basis.

Since $\hat{H}_0$ commutes with $\hat{H}_I$, from Eq. (13) we have

$$\hat{H}^k=\sum_{l=0}^{k}\begin{bmatrix}k\\ l\end{bmatrix}\hat{H}_0^{k-l}\hat{H}_I^l . \tag{20}$$

It can be proved that when l is an even number, the off-diagonal term of the operator $\hat{H}_0^{k-l}\hat{H}_I^l$ vanishes, then we find that

$$\sum_{l\in\text{even}}\begin{bmatrix}k\\ l\end{bmatrix}\hat{H}_0^{k-l}\hat{H}_I^l=\frac{1}{2}\begin{bmatrix}\hat{\varphi}_n^k(+)+\hat{\phi}_n^k(+) & 0\\ 0 & \hat{\varphi}_n^k(-)+\hat{\phi}_n^k(-)\end{bmatrix} , \tag{21}$$

while when l is an odd number, the diagonal term of the operator $\hat{H}_0^{k-l}\hat{H}_I^l$ vanishes, and we find that

$$\sum_{l\in\text{odd}}\begin{bmatrix}k\\ l\end{bmatrix}\hat{H}_0^{k-l}\hat{H}_I^l=\frac{1}{2}\begin{bmatrix}0 & \hat{a}^m\dfrac{[\hat{\varphi}_n^k(-)-\hat{\phi}_n^k(-)]}{\sqrt{\hat{a}^{+m}\hat{a}^m}}\\ \hat{a}^{+m}\dfrac{[\hat{\varphi}_m^k(+)-\hat{\phi}_n^k(+)]}{\sqrt{\hat{a}^m\hat{a}^{+m}}} & 0\end{bmatrix} , \tag{22}$$

where the operators $\hat{\varphi}_n^k(\pm)$ and $\hat{\phi}_n^k(\pm)$ are defined by, respectively,

$$\hat{\varphi}_n(+)=\omega\left[\hat{n}+\frac{m}{2}\right]+\lambda\sqrt{\hat{a}^m\hat{a}^{+m}},\quad \hat{\varphi}_n(-)=\omega\left[\hat{n}-\frac{m}{2}\right]+\lambda\sqrt{\hat{a}^{+m}\hat{a}^m} , \tag{23}$$

$$\hat{\phi}_n(+)=\omega\left[\hat{n}+\frac{m}{2}\right]-\lambda\sqrt{\hat{a}^m\hat{a}^{+m}},\quad \hat{\phi}_n(-)=\omega\left[\hat{n}-\frac{m}{2}\right]-\lambda\sqrt{\hat{a}^{+m}\hat{a}^m} . \tag{24}$$

Substituting Eqs. (21) and (22) into Eq. (20), we obtain the expression of the kth power of the Hamiltonian in the two-dimensional atomic basis as follows:

$$\hat{H}^k=\begin{bmatrix}\hat{f}_n^{(k)}(+) & \hat{a}^m\dfrac{1}{\sqrt{\hat{a}^{+m}\hat{a}^m}}\hat{g}_n^{(k)}(-)\\ \hat{a}^{+m}\dfrac{1}{\sqrt{\hat{a}^m\hat{a}^{+m}}}\hat{g}_n^{(k)}(+) & \hat{f}_n^{(k)}(-)\end{bmatrix} , \tag{25}$$

where these operators $\hat{f}_n^{(k)}(\pm)$ and $\hat{g}_n^{(k)}(\pm)$ are defined by, respectively,

$$\hat{f}_n^{(k)}(\pm)=\tfrac{1}{2}[\hat{\varphi}_n^k(\pm)+\hat{\phi}_n^k(\pm)],\quad \hat{g}_n^{(k)}(\pm)=\tfrac{1}{2}[\hat{\varphi}_n^k(\pm)-\hat{\phi}_n^k(\pm)] . \tag{26}$$

Making use of Eq. (A18) in the Appendix and Eq. (25), we find that

$$\hat{\mathcal{M}}^{(k)}(t)\equiv\hat{H}^k\hat{\rho}_2(t)\hat{H}^k=\begin{bmatrix}\hat{\mathcal{M}}_{11}^{(k)}(t) & \hat{\mathcal{M}}_{12}^{(k)}(t)\\ \hat{\mathcal{M}}_{21}^{(k)}(t) & \hat{\mathcal{M}}_{22}^{(k)}(t)\end{bmatrix} , \tag{27}$$

where the matrix elements are given by

$$\begin{aligned}\hat{\mathcal{M}}_{11}^{(k)}(t)=&\hat{f}_n^{(k)}(+)\hat{\Psi}_{11}(t)\hat{f}_n^{(k)}(+)+\hat{a}^m\hat{g}_n^{(k)\prime}(-)\hat{\Psi}_{21}(t)\hat{f}_n^{(k)}(+)\\ &+\hat{f}_n^{(k)}(+)\hat{\Psi}_{12}(t)\hat{g}_n^{(k)\prime}(-)\hat{a}^{+m}+\hat{a}^m\hat{g}_n^{(k)\prime}(-)\hat{\Psi}_{22}(t)\hat{g}_n^{(k)\prime}(-)\hat{a}^{+m} ,\end{aligned} \tag{28}$$

$$\begin{aligned}\hat{\mathcal{M}}_{22}^{(k)}(t)=&\hat{g}_n^{(k)\prime}(-)\hat{a}^{+m}\hat{\Psi}_{11}(t)\hat{a}^m\hat{g}_n^{(k)\prime}(-)+\hat{f}_n^{(k)}(-)\hat{\Psi}_{21}(t)\hat{a}^m\hat{g}_n^{(k)\prime}(-)\\ &+\hat{g}_n^{(k)\prime}(-)\hat{a}^{+m}\hat{\Psi}_{12}(t)\hat{f}_n^{(k)}(-)+\hat{f}_n^{(k)}(-)\hat{\Psi}_{22}(t)\hat{f}_n^{(k)}(-) ,\end{aligned} \tag{29}$$

$$\hat{\mathcal{M}}_{21}^{(k)}(m,t)=(\hat{\mathcal{M}}_{12}^{(k)}(t))^{+}$$
$$=\hat{g}_n^{(k)'}(-)\hat{a}^{+m}\hat{\Psi}_{11}(t)\hat{f}_n^{(k)}(+)+\hat{f}_n^{(k)}(-)\hat{\Psi}_{21}(t)\hat{f}_n^{(k)}(+)$$
$$+\hat{g}_n^{(k)'}(-)\hat{a}^{+m}\hat{\Psi}_{12}(t)\hat{g}_n^{(k)'}(-)\hat{a}^{+m}+\hat{f}_n^{(k)}(-)\hat{\Psi}_{22}(t)\hat{g}_n^{(k)'}(-)\hat{a}^{+m}, \tag{30}$$

with

$$\hat{g}_n^{(k)'}(\pm)=\frac{1}{\sqrt{\hat{a}^{+m}\hat{a}^{m}}}\hat{g}_n^{(k)}(\pm). \tag{31}$$

From Eqs. (19) and (27) we finally arrive at the explicit expression of the of the exact solution of the Milburn equation for the resonant multiphoton JCM Hamiltonian (5) in the following form:

$$\hat{\rho}(t)=\begin{bmatrix} \sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{t}{\gamma}\right]^k\hat{\mathcal{M}}_{11}^{(k)}(t) & \sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{t}{\gamma}\right]^k\hat{\mathcal{M}}_{12}^{(k)}(t) \\ \sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{t}{\gamma}\right]^k\hat{\mathcal{M}}_{21}^{(k)}(t) & \sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{t}{\gamma}\right]^k\hat{\mathcal{M}}_{22}^{(k)}(t)\end{bmatrix}. \tag{32}$$

Starting with the above solution, through taking the trace over the part of the field in the JCM we can obtain the reduced density operator of the atom in this form,

$$\hat{\rho}_A(t)=\mathrm{Tr}_{\text{field}}\hat{\rho}(t)$$
$$=\begin{bmatrix} \sum_{k,n=0}^{\infty}\frac{1}{k!}\left[\frac{t}{\gamma}\right]^k\langle n|\hat{\mathcal{M}}_{11}^{(k)}(t)|n\rangle & \sum_{k,n=0}^{\infty}\frac{1}{k!}\left[\frac{t}{\gamma}\right]^k\langle n|\hat{\mathcal{M}}_{12}^{(k)}(t)|n\rangle \\ \sum_{k,n=0}^{\infty}\frac{1}{k!}\left[\frac{t}{\gamma}\right]^k\langle n|\hat{\mathcal{M}}_{21}^{(k)}(t)|n\rangle & \sum_{k,n=0}^{\infty}\frac{1}{k!}\left[\frac{t}{\gamma}\right]^k\langle n|\hat{\mathcal{M}}_{22}^{(k)}(t)|n\rangle\end{bmatrix}. \tag{33}$$

For late use, we write down the expectation values of the operators $\hat{\mathcal{M}}_{ij}^{(k)}$ $(i,j=1,2)$ with respect to the number-state basis,

$$\langle n|\hat{\mathcal{M}}_{11}^{(k)}(t)|n\rangle=(f_n^{(k)}(+))^2|\psi_1(n,t)|^2+f_n^{(k)}(+)g_{n+m}^{(k)}(-)\psi_1^*(n,t)\psi_2(n+m,t)$$
$$+g_{n+m}^{(k)}(-)f_n^{(k)}(+)\psi_2^*(n+m,t)\psi_1(n,t)+(g_{n+m}^{(k)}(-))^2|\psi_2(n+m,t)|^2, \tag{34}$$
$$\langle n|\hat{\mathcal{M}}_{22}^{(k)}(t)|n\rangle=(g_n^{(k)}(-))^2|\psi_1(n-m,t)|^2+f_n^{(k)}(-)g_n^{(k)}(-)\psi_2(n,t)\psi_1^*(n-m,t)$$
$$+g_n^{(k)}(-)f_n^{(k)}(-)\psi_1(n-m,t)\psi_2^*(n,t)+(f_n^{(k)}(-))^2|\psi_2(n,t)|^2, \tag{35}$$
$$\langle n|\hat{\mathcal{M}}_{21}^{(k)}(t)|n\rangle=\langle n|\hat{\mathcal{M}}_{12}^{(k)}(t)|n\rangle^*$$
$$=f_n^{(k)}(+)g_n^{(k)}(-)\psi_1(n-m,t)\psi_1^*(n,t)+f_n^{(k)}(+)f_n^{(k)}(-)\psi_2(n,t)\psi_1^*(n,t)$$
$$+g_n^{(k)}(-)g_{n+m}^{(k)}(+)\psi_1(n-m,t)\psi_2^*(n+m,t)+f_n^{(k)}(-)g_{n+m}^{(k)}(m,-)\psi_2(n,t)\psi_2^*(n+m,t), \tag{36}$$

where these functions $f_n^{(k)}(\pm)$ and $g_n^{(k)}(\pm)$ can be obtained through replacing the number operator $\hat{n}$ in their corresponding operator forms $\hat{f}_n^{(k)}(\pm)$ and $\hat{g}_n^{(k)}(\pm)$ by the number n. The functions $\psi_1(n,t)$ and $\psi_2(n,t)$ are defined by, respectively,

$$\psi_1(n,t)\equiv\langle n|\Psi_1(t)\rangle$$
$$=Q_nR_n^{(+)}(t)\exp\left[-\frac{t}{2\gamma}S(n,+)\right]e^{-in\omega t}, \tag{37}$$
$$\psi_2(n,t)\equiv\langle n|\Psi_2(t)\rangle$$
$$=Q_{n-m}V_n^{(-)}(t)\exp\left[-\frac{t}{2\gamma}S(n-m,+)\right]e^{-i(n-m)\omega t}, \tag{38}$$

where $R_n^{(+)}(t)$, $V_n^{(-)}(t)$, and $S(n,+)$ are also obtained through replacing the number operator $\hat{n}$ in their corresponding operator expressions (see the Appendix) by the number n.

III. INFLUENCE OF THE INTRINSIC DECOHERENCE ON ATOMIC INVERSION

It is well known that in the JCM the quantum coherences which are built up during the interaction between the field and the atom significantly affect the dynamics of the atom [19–21]. The existence of the quantum coherences is the reason why one can observe collapses and revivals of the atomic inversion. Therefore the intrinsic decoherence should suppress the time evolution of the atomic inversion. To see this, in what follows we evaluate the atomic inversion in the resonant multiphoton JCM.

The atomic inversion is defined as the probability of the atom being in the excited state minus the probability of being in the ground state, i.e.,

$$\langle\hat{\sigma}_3(t)\rangle=\mathrm{Tr}[\hat{\rho}(t)\hat{\sigma}_3] \tag{39}$$

with the help of the identity $\hat{H}\hat{\sigma}_3=\hat{\sigma}_3(\hat{H}_0-\hat{H}_I)$, we can express Eq. (39) as the following form:

$$\langle\hat{\sigma}_3(t)\rangle=\mathrm{Tr}\left\{\exp\left[\frac{t}{\gamma}(\hat{H}_0^2-\hat{H}_I^2)\right]\hat{\rho}_2(t)\hat{\sigma}_3\right\}. \tag{40}$$

It is straightforward to write the operators $\exp[(t/\gamma)\hat{H}_0^2]$ and $\exp[-(t/\gamma)\hat{H}_I^2]$ in the two-dimensional atomic basis. The results are,

$$\exp\left[\frac{t}{\gamma}\hat{H}_0^2\right]=\begin{bmatrix}\exp\left[\frac{\omega^2 t}{\gamma}\left[\hat{n}+\frac{m}{2}\right]^2\right] & 0\\ 0 & \exp\left[\frac{\omega^2 t}{\gamma}\left[\hat{n}-\frac{m}{2}\right]^2\right]\end{bmatrix}, \tag{41}$$

$$\exp\left[-\frac{t}{\gamma}\hat{H}_I^2\right]=\begin{bmatrix}\exp\left[-\frac{\lambda^2 t}{\gamma}(\hat{a}^m\hat{a}^{+m})\right] & 0\\ 0 & \exp\left[-\frac{\lambda^2 t}{\gamma}(\hat{a}^{+m}\hat{a}^m)\right]\end{bmatrix}. \tag{42}$$

Taking account into $\hat{H}_0$ and $\hat{H}_I$ being commutable with each other, from Eqs. (41) and (42) we find that

$$\exp\left[\frac{t}{\gamma}(\hat{H}_0^2-\hat{H}_I^2)\right]=\begin{bmatrix}\exp\left[\frac{t}{\gamma}\hat{\theta}_n(+)\right] & 0\\ 0 & \exp\left[\frac{t}{\gamma}\hat{\theta}_n(-)\right]\end{bmatrix}, \tag{43}$$

where

$$\hat{\theta}_n(+)=\omega^2\left[\hat{n}+\frac{m}{2}\right]^2-\lambda^2\hat{a}^m\hat{a}^{+m},\quad \hat{\theta}_n(-)=\omega^2\left[\hat{n}-\frac{m}{2}\right]^2-\lambda^2\hat{a}^{+m}\hat{a}^m. \tag{44}$$

Substituting Eq. (A18) in the Appendix and Eq. (43) into Eq. (40), we have

$$\langle\hat{\sigma}_3(t)\rangle=\sum_{n=0}^{\infty}\left\{\langle n|\exp\left[\frac{t}{\gamma}\hat{\theta}_n(+)\right]\hat{\Psi}_{11}(t)|n\rangle+\langle n|\exp\left[\frac{t}{\gamma}\hat{\theta}_n(-)\right]\hat{\Psi}_{22}(t)|n\rangle\right\}. \tag{45}$$

Making use of Eqs. (A19) and (A20) in the Appendix and Eqs. (37), (38), and (44), through a lengthy but straightforward calculation we can obtain the expectation values in Eq. (45). The results are,

$$\begin{aligned}\langle n|\exp\left[\frac{t}{\gamma}\hat{\theta}_n(+)\right]\hat{\Psi}_{11}(t)|n\rangle&=|Q_n|^2\exp\left[-\frac{2\lambda^2 t}{\gamma}\frac{(n+m)!}{n!}\right]\\ &\times\left\{\cos^2\left[\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\cosh^2\left[\frac{\lambda\omega t}{\gamma}\left[n+\frac{m}{2}\right]\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\right.\\ &\left.+\sin^2\left[\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\sinh^2\left[\frac{\lambda\omega t}{\gamma}\left[n+\frac{m}{2}\right]\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\right\},\end{aligned} \tag{46}$$

$$\langle n|\exp\left[\frac{t}{\gamma}\hat{\theta}_n(-)\right]\hat{\Psi}_{22}(t)|n\rangle=|Q_n|^2\exp\left[-\frac{2\lambda^2 t}{\gamma}\frac{(n+m)!}{n!}\right]$$
$$\times\left\{\cos^2\left[\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\sinh^2\left[\frac{\lambda\omega t}{\gamma}\left[n+\frac{m}{2}\right]\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\right.$$
$$\left.+\sin^2\left[\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\cosh^2\left[\frac{\lambda\omega t}{\gamma}\left[n+\frac{m}{2}\right]\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\right\}. \tag{47}$$

Substituting these expectation values into Eq. (45), we arrive at the final result of the atomic inversion in the simple form

$$\langle\hat{\sigma}_3(t)\rangle=\sum_{n=0}^{\infty}|Q_n|^2\exp\left[-\frac{2\lambda^2 t}{\gamma}\frac{(n+m)!}{n!}\right]\cos\left[2\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]. \tag{48}$$

As expected, from the above expression we see that the revivals of the atomic inversion decay in the time evolution due to the appearance of the decay factor $\exp[-2\lambda^2 t/\gamma(n+m)!/n!]$ in Eq. (48). The decay becomes fast with the decrease of the decoherencing parameter γ. In particular, in the limit $\gamma\rightarrow+\infty$, the atomic inversion (48) reduces to

$$\langle\hat{\sigma}_3(t)\rangle=\sum_{n=0}^{\infty}|Q_n|^2\cos\left[2\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right], \tag{49}$$

which is the known expression for the atomic inversion in the multiphoton JCM governed by the von Neumann equation.

IV. INFLUENCE OF INTRINSIC DECOHERENCE ON NONCLASSICAL EFFECTS OF THE FIELD

It is generally accepted that all nonclassical effects in quantum optics emerge as the consequence of quantum interference between components of superposition states of light, that is, nonclassical effects have their origin in quantum coherences. Therefore, the decay of quantum coherences should result in the deterioration of nonclassical effects. In those situations when it is difficult to observe directly nonclassical behavior of the light field, it is convenient to study the dynamics of other quantum systems coupled to the light field under consideration. In this section, we investigate the influence of the intrinsic decoherence on nonclassical effects of the field in the multiphoton JCM.

A. Oscillations of the photon-number distribution

As is well known, oscillation of the photon-number distribution in the JCM is a kind of nonclassical effect of the cavity field. The intrinsic decoherence in the field-atom interaction should result in the deterioration of the oscillating behavior. To see this, in what follows we discuss photon statistics in the radiation field in the JCM.

The reduced density operator of the cavity field can be obtained by taking the trace of the total density operator $\hat{\rho}(t)$ over the atomic states. That is, $\hat{\rho}_F=\mathrm{Tr}_A\hat{\rho}(t)$. Then, the probability $p(n,t)$ of finding n photons in the radiation is found to be

$$\begin{aligned}p(n,t)&=\langle n|\hat{\rho}_F(t)|n\rangle\\&=\sum_k\frac{1}{k!}\left[\frac{t}{\gamma}\right]^k[\langle n|\hat{\mathcal{M}}_{11}^{(k)}(t)|n\rangle+\langle n|\hat{\mathcal{M}}_{22}^{(k)}(t)|n\rangle]\,.\end{aligned} \tag{50}$$

Substituting the explicit expressions of function $f_n^{(k)}(\pm)$ and $g_n^{(k)}(\pm)$ into Eqs. (34) and (35), we find that

$$\begin{aligned}\langle n|\hat{\mathcal{M}}_{11}^{(k)}(t)|n\rangle=&\tfrac{1}{4}\varphi_n^{2k}(+)\{|\psi_1(n,t)|^2+|\psi_2(n-m,t)|^2+2\,\mathrm{Re}[\psi_1^*(n,t)\psi_2(n+m,t)]\}\\&+\tfrac{1}{4}\phi_n^{2k}(+)\{|\psi_1(n,t)|^2+|\psi_2(n+m,t)|^2-2\,\mathrm{Re}[\psi_1^*(n,t)\psi_2(n+m,t)]\}\\&+\tfrac{1}{2}\varphi_n^k(+)\phi_n^k(+)[|\psi_1(n,t)|^2-|\psi_2(n+m,t)|^2]\end{aligned} \tag{51}$$

$$\begin{aligned}\langle n|\hat{\mathcal{M}}_{22}^{(k)}(t)|n\rangle=&\tfrac{1}{4}\varphi_n^{2k}(-)\{|\psi_1(n-m,t)|^2+|\psi_2(n,t)|^2+2\,\mathrm{Re}[\psi_1^*(n-m,t)\psi_2(n,t)]\}\\&+\tfrac{1}{4}\phi_n^{2k}(+)\{|\psi_1(n-m,t)|^2+|\psi_2(n,t)|^2-2\,\mathrm{Re}[\psi_1^*(n-m,t)\psi_2(n,t)]\}\\&-\tfrac{1}{2}\varphi_n^k(-)\phi_n^k(-)[|\psi_1(n-m,t)|^2-|\psi_2(n,t)|^2]\,,\end{aligned} \tag{52}$$

where $\psi_1(n,t)$ and $\psi_2(n,t)$ are given by Eqs. (37) and (38). For later use we give their explicit expressions as follows:

$$\psi_1(n,t)=Q_n\left\{\cos\left[\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\cosh\left[\frac{\lambda\omega t}{\gamma}\left[n+\frac{m}{2}\right]\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\right.$$
$$\left.+i\sin\left[\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\sinh\left[\frac{\lambda\omega t}{\gamma}\left[n+\frac{m}{2}\right]\right]\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\right\}$$
$$\times\exp\left\{-\frac{t}{2\gamma}\left[\omega^2\left[n+\frac{m}{2}\right]^2+\lambda^2\frac{(n+m)!}{n!}\right]\right\}\exp(-in\omega t)\ , \tag{53}$$

$$\psi_2(n,t)=Q_{n-m}\left\{\cos\left[\lambda t\left[\frac{n!}{(n-m)!}\right]^{1/2}\right]\sinh\left[\frac{\lambda\omega t}{\gamma}\left[n-\frac{m}{2}\right]\right]\left[\frac{n!}{(n-m)!}\right]^{1/2}\right]\right.$$
$$\left.+i\sin\left[\lambda t\left[\frac{n!}{(n-m)!}\right]^{1/2}\right]\cosh\left[\frac{\lambda\omega t}{\gamma}\left[n-\frac{m}{2}\right]\right]\left[\frac{n!}{(n-m)!}\right]^{1/2}\right]\right\}$$
$$\times\exp\left\{-\frac{t}{2\gamma}\left[\omega^2\left[n-\frac{m}{2}\right]^2+\lambda^2+\lambda^2\frac{n!}{(n-m)!}\right]\right\}\exp[-i(n-m)\omega t]\ . \tag{54}$$

With the help of Eqs. (23) and (24) and making use of Eqs. (53) and (54) we can reduce Eqs. (51) and (52) to the simple form

$$\langle n|\hat{\mathcal{M}}_{11}^{(k)}(t)|n\rangle=\tfrac{1}{4}|Q_n|^2\left\{\varphi_n^{2k}(+)\exp\left[-\frac{t}{\gamma}\varphi_n^2(+)\right]+\phi_n^{2k}(+)\exp\left[-\frac{t}{\gamma}\phi_n^2(+)\right]\right.$$
$$+2\varphi_n^k(+)\phi_n^k(-)\cos\left[2\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\exp\left[-\frac{t}{\gamma}\varphi_n(+)\phi_n(+)\right]$$
$$\left.\times\exp\left[-\frac{2\lambda^2 t}{\gamma}\frac{(n+m)!}{n!}\right]\right\}, \tag{55}$$

$$\langle n|\hat{\mathcal{M}}_{22}^{(k)}(t)|n\rangle=\tfrac{1}{4}|Q_{n-m}|^2\left\{\varphi_n^{2k}(-)\exp\left[-\frac{t}{\gamma}\varphi_n^2(-)\right]+\phi_n^{2k}(-)\exp\left[-\frac{t}{\gamma}\phi_n^2(-)\right]\right.$$
$$-2\varphi_n^k(+)\phi_n^k(-)\cos\left[2\lambda t\left[\frac{n!}{(n-m)!}\right]^{1/2}\right]\exp\left[-\frac{t}{\gamma}\varphi_n(-)\phi_n(-)\right]$$
$$\left.\times\exp\left[-\frac{2\lambda^2 t}{\gamma}\frac{n!}{(n-m)!}\right]\right\}. \tag{56}$$

Substituting Eqs. (55) and (56) into Eq. (50), after summing over k, we arrive at the expression of the probability of finding n photons as follows:

$$p(n,t)=\tfrac{1}{2}|Q_n|^2\left\{1+\exp\left[-\frac{2\lambda^2 t}{\gamma}\frac{(n+m)!}{n!}\right]\cos\left[2\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\right\}$$
$$+\tfrac{1}{2}|Q_{n-m}|^2\left\{1-\exp\left[-\frac{2\lambda^2 t}{\gamma}\frac{n!}{(n-m)!}\right]\cos\left[2\lambda t\left[\frac{n!}{(n-m)!}\right]^{1/2}\right]\right\}. \tag{57}$$

As expected, from the above expression we can see that the oscillatory behavior of the photon-number distribution is weakened with the decrease of the parameter γ. In particular, in the limit $\gamma\to+\infty$, Eq. (57) reduces to the known result for the photon number distribution in the resonant multiphoton JCM governed by the von Neumann equation, that is,

$$p(n,t)=|Q_n|^2\cos^2\left[\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]+|Q_{n-m}|^2\sin^2\left[\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right], \tag{58}$$

where Q_n is given by Eq. (11).

The mean number of photons in the optical field can be calculated from Eq. (57) or from the expression $\langle\hat{n}(t)\rangle=\mathrm{Tr}_{\mathrm{field}}(\hat{\rho}_F\hat{a}^+\hat{a})$, and is found to be

$$\langle \hat{n}(t)\rangle = \bar{n} + \frac{m}{2} - \frac{m}{2}e^{-\bar{n}}\sum_{n=0}^{\infty}\frac{\bar{n}^n}{n!}\exp\left[-\frac{2\lambda^2 t}{\gamma}\frac{(n+m)!}{n!}\right]\cos\left[2\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right], \tag{59}$$

where $\bar{n}=|z|^2$ is the initial mean photon number in the field. Equation (59) means that the oscillatory behavior of the mean number of photons in the optical field is weakened with the decrease of the decoherence parameter γ. In the limit $\gamma \to +\infty$, the usual result given by the von Neumann equation is recovered, i.e.,

$$\langle \hat{n}(t)\rangle = \bar{n} + \frac{m}{2} - \frac{m}{2}e^{-\bar{n}}\sum_{n=0}^{\infty}\frac{\bar{n}^n}{n!}\cos\left[2\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]. \tag{60}$$

From the above calculation, we see that in the time evolution the additional term in the Milburn equation, which destroys quantum coherences, leads to the appearance of decay factors in Eqs. (57) and (59) which are responsible for the destruction of the oscillations of the photon-number distribution. With the decrease of the parameter γ, i.e., with a more rapid decoherencing, we can observe rapid deterioration of the oscillatory behavior of the photon-number distribution.

B. Quadrature squeezing of the field

We now study the quadrature squeezing of the field in the multiphoton JCM governed by the Milburn equation and discuss effects of the decoherence on the squeezing. We introduce the two slowly varying Hermitian quadrature components of the field $\hat{X}_1$ and $\hat{X}_2$ defined by, respectively,

$$\hat{X}_1 = \tfrac{1}{2}(\hat{a}e^{i\omega t} + \hat{a}^+ e^{-i\omega t}),$$
$$\hat{X}_2 = \frac{1}{2i}(\hat{a}e^{i\omega t} - \hat{a}^+ e^{-i\omega t}), \tag{61}$$

where $\hat{a}$ and $\hat{a}^+$ are the annihilation and creation operators, respectively, and ω is the frequency of the cavity field. The commutation of $\hat{X}_1$ and $\hat{X}_2$ is $[\hat{X}_1,\hat{X}_2]=i/2$. The variances $\langle(\Delta X_i)^2\rangle \equiv \langle \hat{X}_i^2\rangle - (\langle \hat{X}_i\rangle)^2$ $(i=1,2)$ satisfy the Heisenberg uncertainty relation $\langle(\Delta\hat{X}_1)^2\rangle\,\langle \Delta\hat{X}_1)^2\rangle \geq \frac{1}{16}$. A state of the field is said to be squeezed when one of the quadrature components $\hat{X}_1$ and $\hat{X}_2$ satisfies the uncertainty relation $\langle(\Delta\hat{X}_i)^2\rangle < \frac{1}{4}$. The degree of squeezing can be measured by the squeezing parameters [22] S_{X_i} $(i=1,2)$ defined by

$$S_i = \frac{\langle(\Delta\hat{X}_i)^2\rangle - \frac{1}{2}|\langle[\hat{X}_1,\hat{X}_2]\rangle|}{\frac{1}{2}|\langle[\hat{X}_1,\hat{X}_2]\rangle|}. \tag{62}$$

Then, the condition for squeezing in the quadrature component can simply be written as $S_i<0$.

In terms of the annihilation and creation operators of the field, we readily find that

$$S_1 = 2\langle \hat{a}^+\hat{a}\rangle + 2\,\mathrm{Re}\langle \hat{a}^2 e^{(i2\omega t)}\rangle - 4(\mathrm{Re}\langle \hat{a}e^{i\omega t}\rangle)^2, \tag{63}$$

$$S_2 = 2\langle \hat{a}^+\hat{a}\rangle - 2\,\mathrm{Re}\langle \hat{a}^2 e^{(2\omega t)}\rangle - 4(\mathrm{Im}\langle \hat{a}e^{i\omega t}\rangle)^2. \tag{64}$$

Expectation values for any function $F(\hat{a}^+,\hat{a})$ are calculated by the usual manner,

$$\langle F(\hat{a}^+,\hat{a})\rangle = \mathrm{Tr}_{\mathrm{field}}[\rho_F(t)F(\hat{a}^+,\hat{a})] = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\left[\frac{t}{\gamma}\right]^k \frac{1}{k!}[\langle n|\hat{\mathcal{M}}_{11}^{(k)}(t)F(\hat{a}^+,\hat{a})|n\rangle + \langle n|\hat{\mathcal{M}}_{22}^{(k)}(t)F(\hat{a}^+,\hat{a})|n\rangle]. \tag{65}$$

Generally, it is not easy to calculate the expectation value of an arbitrary function $F(\hat{a}^+,\hat{a})$ for the field in the JCM. For the expectation value $\langle \hat{a}e^{i\omega t}\rangle$, through a tedious calculation we find that

$$\langle \hat{a}e^{i\omega t}\rangle = \tfrac{1}{4}\sum_{n=0}^{\infty} Q_n Q_{n-1}^* \exp\left\{-\frac{t}{2\gamma}[S(n,+)+S(n-1,+)]\right\} \times \left\{(\sqrt{n}+\sqrt{n+m})T_1 \exp\left[\frac{t}{\gamma}\varphi_n(+)\varphi_{n-1}(+)\right] + (\sqrt{n}-\sqrt{n+m})T_2\exp\left[\frac{t}{\gamma}\varphi_n(+)\phi_{n-1}(+)\right] + (\sqrt{n}-\sqrt{n+m})T_3\exp\left[\frac{t}{\gamma}\varphi_{n-1}(+)\phi_n(+)\right] + (\sqrt{n}+\sqrt{n+m})T_4 \times\exp\left[\frac{t}{\gamma}\phi_n(+)\phi_{n-1}(+)\right]\right\}, \tag{66}$$

where

$$T_1 = [C_n^{(+)} - iS_n^{(+)}][C_{n-1}^{(+)} + iS_{n-1}^{(+)}][X_n^{(+)} - iY_n^{(+)}][X_{n-1}^{(+)} - iY_{n-1}^{(+)}], \tag{67}$$

$$T_2 = [C_n^{(+)} - iS_n^{(+)}][C_{n-1}^{(+)} - iS_{n-1}^{(+)}][X_n^{(+)} - iY_n^{(+)}][X_{n-1}^{(+)} + iY_{n-1}^{(+)}], \tag{68}$$

$$T_3=[C_n^{(+)}+iS_n^{(+)}][C_{n-1}^{(+)}+iS_{n-1}^{(+)}][X_n^{(+)}+iY_n^{(+)}][X_{n-1}^{(+)}-iY_{n-1}^{(+)}] , \tag{69}$$

$$T_4=[C_n^{(+)}+iS_n^{(+)}][C_{n-1}^{(+)}-iS_{n-1}^{(+)}][X_n^{(+)}+iY_n^{(+)}][X_{n-1}^{(+)}+iY_{n-1}^{(+)}] , \tag{70}$$

where these functions $C_n^{(+)}$, $S_n^{(+)}$, $X_n^{(+)}$, and $Y_n^{(+)}$ are obtained by replacing the number operator $\hat{n}$ in their corresponding operators which are defined in Eqs. (A10), (A11), (A13), and (A14) in the Appendix by the number n.

Substituting the explicit expressions of the functions $C_n^{(+)}$, $S_n^{(+)}$, $X_n^{(+)}$, and $Y_n^{(+)}$ into Eqs. (67)–(70), through a lengthy simplification from Eq. (66) we obtain the final result,

$$\begin{aligned}\langle \hat{a}e^{i\omega t}\rangle=\tfrac{1}{4}\sum_{n=0}^{\infty} Q_nQ_{n-1}^* \Bigg\{ &(\sqrt{n}+\sqrt{n+m}\,)\exp[i\lambda a_-(n)t]\exp\left[-\frac{b_-(n)}{2\gamma}t\right] \\ &+(\sqrt{n}-\sqrt{n+m}\,)\exp[-i\lambda a_-(n)t]\exp\left[-\frac{b_+(n)}{2\gamma}t\right] \\ &+(\sqrt{n}-\sqrt{n+m}\,)\exp[i\lambda a_+(n)t]\exp\left[-\frac{c_-(n)}{2\gamma}t\right] \\ &+(\sqrt{n}+\sqrt{n+m}\,)\exp[-i\lambda a_+(n)t]\exp\left[-\frac{c_+(n)}{2\gamma}t\right]\Bigg\} ,\end{aligned} \tag{71}$$

where

$$a_\pm(n)=\left[\frac{(n+m-1)!}{(n-1)!}\right]^{1/2}\pm\left[\frac{(n+m)!}{n!}\right]^{1/2} , \tag{72}$$

$$b_\pm(n)=\left\{\omega\pm\lambda\left[\left[\frac{(n+m-1)!}{(n-1)!}\right]^{1/2}-\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\right\}^2 , \tag{73}$$

$$c_\pm(n)=\left\{\omega\pm\lambda\left[\left[\frac{(n+m-1)!}{(n-1)!}\right]^{1/2}+\left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\right\}^2 . \tag{74}$$

Similarly, the expectation value $\langle \hat{a}^2e^{i2\omega t}\rangle$ can be written as

$$\begin{aligned}\langle \hat{a}^2e^{i2\omega t}\rangle=\tfrac{1}{4}\sum_{n=0}^{\infty} Q_nQ_{n-2}^*\exp\left\{-\frac{t}{2\gamma}[S(n,+)+S(n-2,+)]\right\} \\ \times\Bigg\{&[\sqrt{n(n-1)}+\sqrt{(n+m)(n+m-1)}]T_1'\exp\left[\frac{t}{\gamma}\varphi_n(+)\varphi_{n-2}(+)\right] \\ &+[\sqrt{n(n-1)}-\sqrt{(n+m)(n+m-1)}]T_2'\exp\left[\frac{t}{\gamma}\varphi_n(+)\phi_{n-2}(+)\right] \\ &+[\sqrt{n(n-1)}-\sqrt{(n+m)(n+m-1)}]T_3'\exp\left[\frac{t}{\gamma}\varphi_{n-2}(+)\phi_n(+)\right] \\ &+[\sqrt{n(n-1)}+\sqrt{(n+m)(n+m-1)}]T_4'\exp\left[\frac{t}{\gamma}\phi_n(+)\phi_{n-2}(+)\right]\Bigg\} ,\end{aligned} \tag{75}$$

where

$$T_1'=[C_n^{(+)}-iS_n^{(+)}][C_{n-2}^{(+)}+iS_{n-2}^{(+)}][X_n^{(+)}-iY_n^{(+)}][X_{n-2}^{(+)}-iY_{n-2}^{(+)}] , \tag{76}$$

$$T_2'=[C_n^{(+)}-iS_n^{(+)}][C_{n-2}^{(+)}-iS_{n-2}^{(+)}][X_n^{(+)}-iY_n^{(+)}][X_{n-2}^{(+)}+iY_{n-2}^{(+)}] , \tag{77}$$

$$T_3'=[C_n^{(+)}+iS_n^{(+)}][C_{n-2}^{(+)}+iS_{n-2}^{(+)}][X_n^{(+)}+iY_n^{(+)}][X_{n-2}^{(+)}-iY_{n-2}^{(+)}] , \tag{78}$$

$$T_4'=[C_n^{(+)}+iS_n^{(+)}][C_{n-2}^{(+)}-iS_{n-2}^{(+)}][X_n^{(+)}+iY_n^{(+)}][X_{n-2}^{(+)}+iY_{n-2}^{(+)}] . \tag{79}$$

Making use of Eqs. (A10), (A11), (A13), and (A14) in the Appendix, from Eqs. (75) to (79) we find that

$$\langle \hat{a}^2 e^{i2\omega t}\rangle = \frac{1}{4}\sum_{n=0}^{\infty} Q_n Q_{n-2}^* \Bigg\{ [\sqrt{n(n-1)}+\sqrt{(n+m)(n+m-1)}]\exp[i\lambda a'_-(n)t]\exp\left[-\frac{b'_-(n)}{2\gamma}t\right]$$
$$+[\sqrt{n(n-1)}+\sqrt{(n+m)(n+m-1)}]\exp[-i\lambda a'_-(n)t]\exp\left[-\frac{b'_+(n)}{2\gamma}t\right]$$
$$+[\sqrt{n(n-1)}-\sqrt{(n+m)(n+m-1)}]\exp[i\lambda a'_+(n)t]\exp\left[-\frac{c'_-(n)}{2\gamma}t\right]$$
$$+[\sqrt{n(n-1)}-\sqrt{(n+m)(n+m-1)}]\exp[-i\lambda a'_+(n)t]\exp\left[-\frac{c'_+(n)}{2\gamma}t\right]\Bigg\}, \tag{80}$$

where

$$a'_\pm(n) = \left[\frac{(n+m-2)!}{(n-2)!}\right]^{1/2} \pm \left[\frac{(n+m)!}{n!}\right]^{1/2}, \tag{81}$$

$$b'_\pm(n) = \left\{2\omega \pm \lambda\left[\left[\frac{(n+m-2)!}{(n-2)!}\right]^{1/2} - \left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\right\}^2, \tag{82}$$

$$c'_\pm(n) = \left\{2\omega \pm \lambda\left[\left[\frac{(n+m-2)!}{(n-2)!}\right]^{1/2} + \left[\frac{(n+m)!}{n!}\right]^{1/2}\right]\right\}^2. \tag{83}$$

So far we have completed calculations of all expectation values needed in the squeezing parameters Eqs. (63) and (64). It is straightforward to obtain the two squeezing parameters S_1 and S_2 through the simple substitution of the expectation values (59), (71), and (80) into Eqs. (63) and (64). We would not like to write their explicit expressions since they are too long.

We now analyze effects of the intrinsic decoherence on the quadrature squeezing of the field. It is a well-known fact that the field exhibits quadrature squeezing in the conventional JCM governed by the von Neumann equation [23,24]. Hence, in the JCM governed by the Milburn equation there exists the quadrature squeezing of the field when $\gamma \rightarrow +\infty$ since the Milburn equation reduces to the von Neumann equation in this limit. Then, taking into account Eqs. (59), (71), and (80), from Eqs. (63) and (64) we can see that the additional term in the Milburn equation leads to the appearance of decay factors in each term in the expressions of the squeezing parameters. Thus, each term of S_1 and S_2 decays with the decrease of the decoherencing parameter of γ. In particular, for a given small quantity γ, at time $2\omega^2 t/\gamma \gg 1$, we find that

$$S_1 = S_2 \doteq 2|z|^2 + m > 0 , \tag{84}$$

which means that the quadrature squeezing of the field vanishes with the time evolution. Therefore, we can conclude that the intrinsic decoherence suppresses the quadrature squeezing of the field in the JCM.

C. Photon antibunching effect

Photon antibunching [25–27] is one of the best known nonclassical effects of the light field. To understand the effect of the intrinsic decoherence on the photon antibunching in the JCM, we examine the behavior of the second-order zero-time coherence function

$$g^{(2)}(0) = \frac{\langle \hat{n}^2\rangle - \langle \hat{n}\rangle}{(\langle \hat{n}\rangle)^2} , \tag{85}$$

where $\hat{n} = \hat{a}^+\hat{a}$. The photon antibunching exists whenever $g^{(2)}(0)$ is less than 1. To establish if the system is photon antibunching or bunching, we must calculate the two expectation values $\langle \hat{n}\rangle$ and $\langle \hat{n}^2\rangle$. Here $\langle \hat{n}\rangle$ has been evaluated in Eq. (59). And the expectation value $\langle \hat{n}^2\rangle$ can be calculated by the manner,

$$\langle \hat{n}^2\rangle = \sum_{n=0}^{\infty} n^2 p(n,t) , \tag{86}$$

where $p(n,t)$ is the photon distribution function given by Eq. (57).

From Eqs. (57) and (86), we find that

$$\langle \hat{n}^2\rangle = \bar{n}^2 + (m+1)\bar{n} + m^2 - \frac{m}{2} e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{(2n+m)}{n!}(\bar{n})^n \times \exp\left[-\frac{2\gamma^2 t}{\gamma}\frac{(n+m)!}{n!}\right] \times \cos\left[2\lambda t\left[\frac{(n+m)!}{n!}\right]^{1/2}\right], \tag{87}$$

where $\bar{n} = |z|^2$.

It is easy to get the exact expression of $g^{(2)}(0)$ through substituting Eqs. (59) and (87) into Eq. (85), but the result is given as infinite terms that cannot be calculated analytically.

From Eqs. (59) and (86) we can see that both $\langle \hat{n}\rangle$ and $\langle \hat{n}^2\rangle$ decay with the decrease of the decoherence parameter γ. In particular, for a given small quantity γ, at time $2\lambda^2 t/\gamma \gg 1$, we have $\langle \hat{n}\rangle \doteq \bar{n} + m/2$ and $\langle \hat{n}^2\rangle \doteq \bar{n}^2 + (m+1)\bar{n} + m^2$, and we find that

$$g^{(2)}(0)=\frac{2m(2m-2)}{(2\bar{n}+m)^2}+1\geq 1 \ , \tag{88}$$

which means that the suppression of quantum coherences leads to deterioration of the photon antibunching effect.

V. SUMMARY

We have studied the influence of the intrinsic decoherence in the atom-field interaction on nonclassical effects in the multiphoton JCM. We have found the exact solution of the Milburn equation for the resonant multiphoton Jaynes-Cummings Hamiltonian and given the explicit form of the solution in the two-dimensional atomic basis. We have investigated in detail collapses and revivals of atomic inversion, oscillatory behaviors of the field, and photon antibunching effect in the JCM under the influence of intrinsic decoherence. We have shown that in the process of time evolution of the quantum system the intrinsic decoherence in the atom-field interaction suppresses these nonclassical effects in the multiphoton JCM.

ACKNOWLEDGMENTS

We would like to thank our colleagues at Theoretical Physics Division, Nankai Institute of Mathematics for their useful discussions and comments. This research was supported by the National Natural Science Foundation of China.

APPENDIX

In this appendix, we present the derivation of the exact solution (19) of the Milburn equation for the multiphoton Jaynes-Cummings Hamiltonian (5).

Taking into account the definitions of the superoperators $\hat{S}$ and $\hat{T}$ and the initial condition (12), we find that

$$\begin{aligned}\hat{\rho}_2(t)&=\exp(\hat{S}t)\exp\left[-\frac{1}{2\gamma}\hat{H}^2t\right]\hat{\rho}(0)\exp\left[-\frac{1}{2\gamma}\hat{H}^2t\right]\\&=\exp(-i\hat{H}_It)\exp\left[-\frac{t}{2\gamma}\hat{B}\right]\hat{\rho}_1(t)\exp\left[-\frac{t}{2\gamma}\hat{B}\right]\exp(i\hat{H}_It)\ ,\end{aligned} \tag{A1}$$

where we have used the property that these operators $\hat{H}_0$, $\hat{H}_I$, $\hat{A}$, and $\hat{B}$ are commutable with each other.

Making use of the expressions,

$$\exp\left[-\frac{t}{2\gamma}\hat{A}\right]=\begin{bmatrix}\exp\left[-\frac{t}{2\gamma}\hat{S}(\hat{n},+)\right] & 0\\ 0 & \exp\left[-\frac{t}{2\gamma}\hat{S}(\hat{n},-)\right]\end{bmatrix}, \tag{A2}$$

$$\exp(-i\hat{H}_0t)=\begin{bmatrix}\exp\left[-i\omega t\left[\hat{n}+\frac{m}{2}\right]\right] & 0\\ 0 & \exp\left[-i\omega t\left[\hat{n}-\frac{m}{2}\right]\right]\end{bmatrix}, \tag{A3}$$

we can write (A1) in the simple form

$$\hat{\rho}_2(t)=\exp(-i\hat{H}_I)\exp\left[-\frac{t}{2\gamma}\hat{B}\right]\hat{\rho}_1(t)\exp\left[-\frac{t}{2\gamma}\hat{B}\right]\exp(i\hat{H}_I)\ , \tag{A4}$$

where the auxiliary operator $\hat{\rho}_1(t)$ is defined by

$$\hat{\rho}_1(t)=\begin{bmatrix}|\Psi(t)\rangle\langle\Psi(t)| & 0\\ 0 & 0\end{bmatrix}, \tag{A5}$$

where

$$|\Psi(t)\rangle=\exp\left[-\frac{t}{2\gamma}\hat{S}(\hat{n},+)\right]|ze^{-i\omega t}\rangle\ . \tag{A6}$$

If we notice that the powers of the off-diagonal operator $\hat{B}$ can be written as

$$\left[\frac{\hat{B}}{2\lambda\omega}\right]^{2k}=\begin{bmatrix}\left[\left(\hat{n}+\frac{m}{2}\right)\sqrt{\hat{a}^m\hat{a}^{+m}}\right]^{2k} & 0\\ 0 & \left[\left(\hat{n}-\frac{m}{2}\right)\sqrt{\hat{a}^{+m}\hat{a}^m}\right]^{2k}\end{bmatrix},\tag{A7}$$

$$\left[\frac{\hat{B}}{2\lambda\omega}\right]^{2k+1}=\begin{bmatrix}0 & \hat{a}^m\dfrac{\left[\left(\hat{n}-\frac{m}{2}\right)\sqrt{\hat{a}^{+m}\hat{a}^m}\right]^{2k+1}}{\sqrt{\hat{a}^{+m}\hat{a}^m}}\\ \dfrac{\left[\left(\hat{n}+\frac{m}{2}\right)\sqrt{\hat{a}^m\hat{a}^{+m}}\right]^{2k+1}\hat{a}^{+m}}{\sqrt{\hat{a}^{+m}\hat{a}^m}} & 0\end{bmatrix},\tag{A8}$$

then we can write the operator $\exp[(-t/2\gamma)\hat{B}]$ in the form

$$\exp\left[-\frac{t}{2\gamma}\hat{B}\right]=\begin{bmatrix}\hat{X}_n^{(+)}(t) & -\hat{a}^m\dfrac{1}{\sqrt{\hat{a}^{+m}\hat{a}^m}}\hat{Y}_n^{(-)}(t)\\ -\hat{a}^{+m}\dfrac{1}{\sqrt{\hat{a}^m\hat{a}^{+m}}}\hat{Y}_n^{(+)}(t) & \hat{X}_n^{(-)}(t)\end{bmatrix},\tag{A9}$$

where

$$\hat{X}_n^{(+)}(t)=\cosh\left[\frac{\lambda\omega t}{\gamma}\left(\hat{n}+\frac{m}{2}\right)\sqrt{\hat{a}^m\hat{a}^{+m}}\right],\quad \hat{X}_n^{(-)}(t)=\cosh\left[\frac{\lambda\omega t}{\gamma}\left(\hat{n}-\frac{m}{2}\right)\sqrt{\hat{a}^{+m}\hat{a}^m}\right]\tag{A10}$$

and

$$\hat{Y}_n^{(+)}(t)=\sinh\left[\frac{\lambda\omega t}{\gamma}\left(\hat{n}+\frac{m}{2}\right)\sqrt{\hat{a}^m\hat{a}^{+m}}\right],\quad \hat{Y}_n^{(+)}(t)=\sinh\left[\frac{\lambda\omega t}{\gamma}\left(\hat{n}-\frac{m}{2}\right)\sqrt{\hat{a}^{+m}\hat{a}^m}\right].\tag{A11}$$

Similarly, we can write the operator $\exp(-i\hat{H}_I t)$ in the two-dimensional atomic basis as

$$\exp(-i\hat{H}_I t)=\begin{bmatrix}\hat{C}_n^{(+)}(t) & -i\hat{a}^m\hat{S}_n^{(-)}(t)\dfrac{1}{\sqrt{\hat{a}^{+m}\hat{a}^m}}\\ -i\hat{a}^{+m}\hat{S}_n^{(+)}(t)\dfrac{1}{\sqrt{\hat{a}^m\hat{a}^{+m}}} & \hat{C}_n^{(-)}(t)\end{bmatrix},\tag{A12}$$

where

$$\hat{C}_n^{(+)}(t)=\cos(\lambda t\sqrt{\hat{a}^m\hat{a}^{+m}}),\quad \hat{C}_n^{(-)}(t)=\cos(\lambda t\sqrt{\hat{a}^{+m}\hat{a}^m})\tag{A13}$$

and

$$\hat{S}_n^{(+)}(t)=\sin(\lambda t\sqrt{\hat{a}^m\hat{a}^{+m}}),\quad \hat{S}_n^{(-)}(t)=\sin(\lambda t\sqrt{\hat{a}^{+m}\hat{a}^m}).\tag{A14}$$

Then, from Eqs. (A9) and (A12) it follows that

$$\exp(-i\hat{H}_I t)\exp\left[-\frac{t}{2\gamma}\hat{B}\right]=\begin{bmatrix}\hat{R}_n^{(+)}(t) & -\hat{a}^m\dfrac{\hat{V}_n^{(-)}(t)}{\sqrt{\hat{a}^{+m}\hat{a}^m}}\\ -\hat{a}^{+m}\dfrac{\hat{V}_n^{(+)}(t)}{\sqrt{\hat{a}^m\hat{a}^{+m}}} & \hat{R}_n^{(-)}(t)\end{bmatrix},\tag{A15}$$

where

$$\hat{R}_n^{(\pm)}(t)=\hat{C}_n^{(\pm)}(t)\hat{X}_n^{(\pm)}(t)+i\hat{S}_n^{(\pm)}(t)\hat{Y}_n^{(\pm)}(t),\tag{A16}$$

$$\hat{V}_n^{(\pm)}(t)=\hat{C}_n^{(\pm)}(t)\hat{Y}_n^{(\pm)}(t)+i\hat{S}_n^{(\pm)}(t)\hat{X}_n^{(\pm)}(t).\tag{A17}$$

Substituting Eqs. (A5) and (A15) into Eq. (A4), we obtain an explicit expression for the operator $\hat{\rho}_2(t)$ as follows:

$$\hat{\rho}_2(t)=\begin{bmatrix}\hat{\Psi}_{11}(t) & \hat{\Psi}_{12}(t)\\ \hat{\Psi}_{21}(t) & \hat{\Psi}_{22}(t)\end{bmatrix}, \tag{A18}$$

where we have used the following symbol:

$$\hat{\Psi}_{ij}(t)=|\Psi_i(t)\rangle\langle\Psi_j(t)| \quad (i,j=1,2)\ , \tag{A19}$$

with

$$|\Psi_1(t)\rangle=\hat{R}_n^{(+)}(t)|\Psi(t)\rangle, \quad |\Psi_2(t)\rangle=-\frac{\hat{V}_n^{(-)}(t)}{\sqrt{\hat{a}^{+m}\hat{a}^{m}}}\hat{a}^{+m}|\Psi(t)\rangle\ , \tag{A20}$$

where $|\Psi(t)\rangle$ is given by Eq. (23).

Taking into account the definition of the superoperator $\hat{R}$, it is straightforward to obtain the action of the operator $\exp(\hat{R}_t)$ on the "density" operator $\hat{\rho}_2(t)$ as follows:

$$\hat{\rho}=\sum_{k=0}^{\infty}\left[\frac{t}{\gamma}\right]^k\frac{1}{k!}\hat{H}^k\hat{\rho}_2(t)\hat{H}^k\ , \tag{A21}$$

where the Hamiltonian $\hat{H}$ and the operator $\hat{\rho}_2$ are given by Eqs. (5) and (A18), respectively. This is the operator form of the exact solution of the Milburn Eq. (4) for the multiphoton JCM in the resonant case.

[1] G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986).
[2] G. C. Ghirardi, P. Pearle, and A. Rimini, Phys. Rev. A **42**, 78 (1990).
[3] L. Diosi, Phys. Rev. A **40**, 1165 (1989).
[4] G. C. Ghirardi, R. Grassi, and A. Rimini, Phys. Rev. A **42**, 1057 (1990).
[5] J. Ellis, S. Mohanty, and D. V. Nanopaulos, Phys. Lett. B **221**, 113 (1989).
[6] J. Ellis, S. Mohanty, and D. V. Nanopaulos, Phys. Lett. B **235**, 305 (1990).
[7] C. M. Caves and G. J. Milburn, Phys. Rev. D **36**, 5543 (1987).
[8] G. J. Milburn, Phys. Rev. A **44**, 5401 (1991).
[9] G. J. Milburn, Phys. Rev. A **47**, 2415 (1993).
[10] B. W. Shore and P. L. Knight, J. Mod. Opt. **40**, 1195 (1993).
[11] B. Buck and C. V. Sukumar, Phys. Lett. **A81**, 132 (1981); E. I. Aliskendernov, K. A. Rustamov, A. S. Shumovsky, and T. Quang, J. Phys. A **20**, 6265 (1987).
[12] S. Singh, Phys. Rev. A **25**, 3206 (1982).
[13] V. G. Kadyshevsky, Nucl. Phys. **B141**, 477 (1978); S. Golden, Phys. Rev. A **46**, 6805 (1992).
[14] H. Moya-Cessa, V. Bužek, M. S. Kim, and P. L. Knight, Phys. Rev. A **48**, 3900 (1993).
[15] Le-Man Kuang and Xin Chen, J. Phys. A **27**, L663 (1994); Xin Chen and Le-Man Kuang, Phys. Lett. A **191**, 18 (1994).
[16] M. D. Srinivas and E. B. Daries, Opt. Acta **28**, 981 (1981).
[17] U. Ueda, Phys. Rev. A **41**, 3875 (1990); M. Ueda, N. Imoto, and T. Ogawa, *ibid.* **41**, 389 (1990).
[18] W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973), Chap. 5.
[19] S. J. D. Phoenix and P. L. Knight, Ann. Phys. **186**, 381 (1988); Phys. Rev. A **44**, 6023 (1991).
[20] J. Gea-Banacloche, Phys. Rev. A **44**, 5913 (1991).
[21] V. Bužek, H. Moya-Cessa, P. L. Knight, and S. J. D. Pheonix, Phys. Rev. A **45**, 8190 (1992).
[22] V. Bužek, A. Vidiella-Barranco, and P. L. Knight, Phys. Rev. A **45**, 6570 (1992).
[23] C. C. Gerry and P. J. Mayer, Phys. Rev. A **38**, 5665 (1988); A. S. Shumovsky, V. L. Kien, and E. I. Aliskenderov, Phys. Lett. **A124**, 351 (1987); P. Goldberg and L. C. Harrison, Phys. Rev. A **43**, 376 (1991).
[24] P. Meystre and M. S. Zubairy, Phys. Lett. **A89**, 390 (1982); J. S. Peng and G. X. Li, Phys. Rev. A **47**, 3167 (1993); D. Cohen, Y. Ben-Aryeh, and A. Mann, *ibid.* **49**, 2040 (1994).
[25] P. Alsing and M. S. Zubairy, J. Opt. Soc. Am. B **4**, 177 (1987); M. S. Zubairy and J. J. Yeh, Phys. Rev. A **21**, 1624 (1980); H. Voigt, A. Bandilla, and H. H. Ritze, Z. Phys. B **36**, 295 (1980).
[26] B. E. A. Saleh and M. C. Teich, Opt. Commun. **52**, 429 (1985); M. J. Kimble, M. Danganis, and L. Mandel, Phys. Rev. Lett. **39**, 691 (1977); R. Short and L. Mandel, *ibid.* **51**, 384 (1983); A. Aspect, P. Grangier, and G. Roger, Europhys. Lett. **1**, 173 (1986).
[27] N. B. Narozhny, J. J. Sánchez-Mondragón, and J. H. Eberly, Phys. Rev. A **23**, 236 (1981).