Optical bistability and lasing without inversion in a system of driven two-level atoms with incoherent injection

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We have studied a laser system in which initially incoherent atoms are injected into a cavity and directly driven by a coherent pump field. Here we show that such a system can display optical bistability in the absence of population inversion. Two disconnected branches of the solutions are obtained for lasing action. They correspond to the inversion branch and inversionless branch, respectively. Also, a mechanism is proposed here to produce higher-power lasing by the inversionless laser.

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In recent years, lasing without inversion (LWI) has drawn a great deal of attention [1–18], and various schemes have been proposed. Based on the dressed states, Agarwal [1], Lu, and Berman [2] have studied LWI for a two-level system, where atomic coherence effects play an important role in achieving lasing action without population inversion. Also, atomic coherence effects are used in other models that involve more transition levels [3–6]. Other very interesting proposals advanced by Harris and others [7–18] are based on the differences between the emission and absorption spectra of lifetime-broadened discrete levels of atoms due to Fano interference effects [19]. Scully and collaborators [14,15] have investigated the possibility of LWI within the framework of the quantum-beat laser, which was originally advanced as a means of quenching spontaneous emission noise. Most recently, some interesting experiments on LWI have been made [16–18].

The motivation of the present paper is twofold. First, although the atomic coherence effects and dressed state basis are important for most commonly discussed schemes for LWI (e.g., [1–9]), the initial atomic coherence and dressed state basis are not essentially needed for achieving LWI [4(d),6]. It could be shown that LWI can also be achieved in the bare atomic state basis and in the incoherent initial atomic states. Second, as is well known, while the bistability can be obtained for a laser system and many other systems [20–24] driven by an external injected field, no description showing optical bistability without population inversion has, to our knowledge, been reported so far. Therefore, our question is whether these two properties can be achieved together. In other words, can the optical bistability be obtained in the absence of population inversion?

The present paper studies a two-level system driven by a coherent pump field. Although a driven two-level atomic system was studied some time ago [25,26], and most recently, corresponding investigations on Mollow side-band for this system were reported [27–29], our model is different from those. In our model, a two-level atomic beam is injected into the cavity and atomic states are initially incoherent. The driven pump field is perpendicular to both the atomic beam and the cavity axis (Fig. 1). For this system, we have shown that it is indeed possible to obtain optical bistability in the absence of population inversion. The lasing action takes place in both the population inversion region (where inversion parameter $D=\rho_{22}-\rho_{11}>0$) and the inversionless region ($D<0$). The solutions of the lasing field are shown to be divided into two branches: the inversion branch (curve *a* in Fig. 2) and the inversionless branch (curve *b* in Fig. 2). The inversion branch cannot transit *continuously* either to the

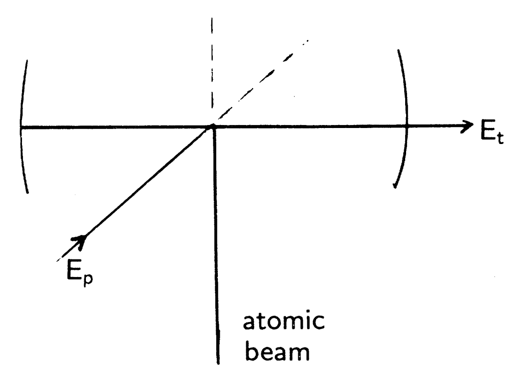

FIG. 1. Schematic outline of optical bistability for LWI, where injected N two-level atoms are directly driven by a pump field Ep.

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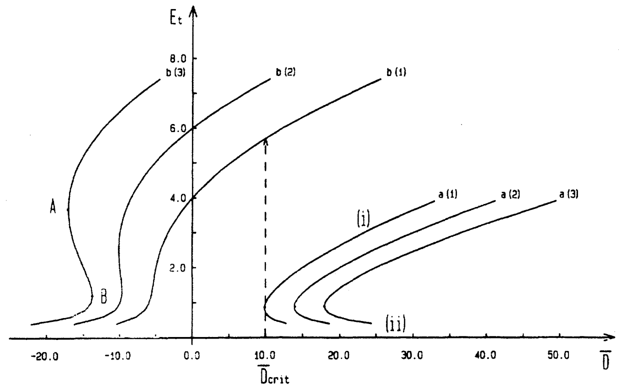

FIG. 2. Variation of normalized total field E_t given by Eqs. (12) and (13) with the normalized inversion parameter $\bar{D}$ [see Eqs. (14) and (15) for its definition]. Curves $a(1), a(2), a(3)$ represent the solutions of the inversion branch determined by Eq. (12), where curve (i) (with the positive slope) is stable, and curve (ii) (with the negative slope) is unstable. Curves $b(1), b(2), b(3)$ show the optical bistability in the inversionless region given by Eq. (13). The parameter $\bar{g}_p$ in both curves a and b is chosen to be 2 for $a(1)$, $b(1)$; 3 for $a(2)$, $b(2)$; 4 for $a(3)$, $b(3)$.

inversionless branch or to the inversionless region. The inversionless branch cannot transit to the inversion branch at all. However, it can transit continuously from the inversionless region to the inversion region corresponding to the population inversion parameter, and *vice versa*. Though there are two possible steady states in the inversion region, they are not optical bistable states in a common sense, which will be explained later. However, the two steady states in the inversionless region are optical bistable states in a common sense. They form a hysteresis.

The Hamiltonian of the system in the rotating-wave approximation is given by

$$H=\hbar\omega_c a^\dagger a+\sum_j \frac{\hbar}{2}\omega_a\sigma_{zj}+\hbar\sum_j\theta(t-t_j)(ga\sigma_j^\dagger+g^*a^\dagger\sigma_j)+\hbar\sum_j\theta(t-t_j)(g_p e^{-i\omega_p t}\sigma_j^\dagger+g_p^* e^{i\omega_p t}\sigma_j)+H_{\text{loss}}\ , \quad (1)$$

where $a^\dagger$ and a are creation and annihilation operators for the cavity field, respectively; $\sigma_j^\dagger$, σ_j and σ_{zj} are Pauli pseudospin operators for the jth atom; g and g_p are coupling constants of atoms with the cavity and with the pump field, respectively; ω_c, ω_a, and ω_p are frequencies for the cavity, atoms, and pump field, respectively; and the step function $\theta(t-t_j)$ is one for $t\geq t_j$ (injection time of the jth atom) and zero otherwise.

In a rotating frame,

$$\tilde{a}(t)=ae^{i\omega_p t},\quad \tilde{\sigma}_j(t)=\sigma_j e^{i\omega_p t}\ , \quad (2)$$

the Heisenberg equations of motion for the mean values of the cavity mode and of the atoms are

$$\langle\dot{\tilde{a}}\rangle=-\left[\frac{\gamma_c}{2}+i\Delta_c\right]\langle\tilde{a}\rangle-ig^*\sum_j\theta(t-t_j)\langle\tilde{\sigma}_j\rangle\ , \quad (3)$$

$$\langle\dot{\tilde{\sigma}}_j\rangle=-(\Gamma+i\Delta_a)\langle\tilde{\sigma}_j\rangle+ig\theta(t-t_j)\langle\tilde{a}+g_r\rangle\langle\tilde{\sigma}_{zj}\rangle\ , \quad (4a)$$

$$\langle\dot{\sigma}_{zj}\rangle=-\Gamma\langle\sigma_{zj}\rangle+2i\theta(t-t_j)[g^*\langle\tilde{a}^\dagger+g_r^*\rangle\langle\tilde{\sigma}_j\rangle-g\langle\tilde{a}+g_r\rangle\langle\tilde{\sigma}_j^\dagger\rangle]\ , \quad (4b)$$

where $\Delta_c=\omega_c-\omega_p$, $\Delta_a=\omega_a-\omega_p$, $g_r=g_p/g$; γ_c and Γ are the cavity decay rate and the atomic decay rate, respectively. In the above equations, use have been made of the semiclassical approximation $\langle\tilde{a}^\dagger\tilde{\sigma}_j\rangle=\langle\tilde{a}^\dagger\rangle\langle\tilde{\sigma}_j\rangle$, $\langle\tilde{a}\tilde{\sigma}_j^\dagger\rangle=\langle\tilde{a}\rangle\langle\tilde{\sigma}_j^\dagger\rangle$, which implies that $|\langle\tilde{a}\rangle|\gg 1$.

To solve (3) and (4), we note that the total field of the system is

$$\langle b\rangle=\langle\tilde{a}+g_r\rangle=r_0e^{i\Phi_0}\ , \quad (5)$$

where r_0 and Φ_0 are the amplitude and the phase of the total field, respectively. Consequently, Eq. (3) becomes

$$\langle\dot{b}\rangle=-\left[\frac{\gamma_c}{2}+i\Delta_c\right]\langle b\rangle+\left[\frac{\gamma_c}{2}+i\Delta_c\right]g_r-ig^*\sum_j\theta(t-t_j)\langle\tilde{\sigma}_j\rangle\ . \quad (3')$$

Together with Eq. (5), Eq. (4) can be solved without resorting to adiabatic elimination. The solution is

$$\langle\tilde{\sigma}_j(t)\rangle=\exp[-\Gamma(t-t_j)]\left\{\langle\tilde{\sigma}_j(t_j)\rangle\cos^2[|g|r_0(t-t_j)]+\frac{g^2}{|g|^2}\langle\tilde{\sigma}^\dagger(t_j)\rangle e^{i2\Phi_0}\sin^2[|g|r_0(t-t_j)]+i\frac{g}{2|g|}\langle\sigma_{zj}(t_j)\rangle e^{i\Phi_0}\sin[2|g|r_0(t-t_j)]-i\frac{\Delta_a}{2|g|r_0}\langle\tilde{\sigma}_j(t_j)\rangle\sin[2|g|r_0(t-t_j)]+\frac{g\Delta_a}{2|g|^2r_0}\langle\sigma_{zj}(t_j)\rangle e^{i\Phi_0}\sin^2[|g|r_0(t-t_j)]\right\}\ . \quad (6)$$

Because of the incoherent injection of atoms, the initial atomic conditions in (6) are supposed to be

$$\langle\tilde{\sigma}_j(t_j)\rangle=\tilde{\rho}_{21}(t_j)=0,\quad \langle\tilde{\sigma}_j^\dagger(t_j)\rangle=\tilde{\rho}_{12}(t_j)=0\ ,$$
$$\langle\sigma_{zj}(t_j)\rangle=\rho_{22}-\rho_{11}\ . \quad (7)$$

Before we insert (6) and (7) into (3′), we assume that the atoms have a time-independent injection rate R. The summation in (3′) can be replaced with an integration over the injection time t_j [2(b)],

$$\sum_j\theta(t-t_j)\langle\tilde{\sigma}_j(t)\rangle\to R\int_{-\infty}^{t}dt_j\langle\tilde{\sigma}_j(t)\rangle\ , \quad (8)$$

which can be integrated by using Eqs. (6) and (7). The final result will lead to an expression for $\langle \dot{b} \rangle$,

$$\langle \dot{b} \rangle = \frac{1}{1+\frac{4g^2r_0^2}{\Gamma^2}} e^{i\Phi_0} \left[\frac{Rg^2r_0}{\Gamma^2}(\rho_{22}-\rho_{11}) - i\left[\frac{Rg^2\Delta_a}{2\Gamma^3}\right] \times \left[\frac{2gr_0}{\Gamma}\right](\rho_{22}-\rho_{11})\right] - \left[\frac{\gamma_c}{2}+i\Delta_c\right] r_0 e^{i\Phi_0} + \left[\frac{\gamma_c}{2}+i\Delta_c\right] g_r \ . \tag{9}$$

Without a loss of generality, we have chosen the coupling constants g and g_p to be real in Eq. (9). Using Eq. (5) and separating Eq. (9) into a real part and an imaginary one, we find an equation for the effective total field amplitude $E_t = 2gr_0/\Gamma$ to be

$$\dot{E}_t = \frac{g_c}{2}\frac{E_t}{1+E_t^2}(\rho_{22}-\rho_{11}) - \frac{\gamma_c}{2}E_t + \frac{2g_p A}{\Gamma}\cos(\Phi_0-\sigma) \ , \tag{10}$$

and an equation for phase Φ_0 to be

$$\dot{\Phi}_0 = -\frac{g_c\Delta_a}{2\Gamma}\frac{1}{1+E_t^2}(\rho_{22}-\rho_{11}) - \Delta_c + \frac{2g_p A}{\Gamma E_t}\sin(\sigma-\Phi_0) \ , \tag{11}$$

where $g_c = 2g^2R/\Gamma^2$, $A = \sqrt{(\gamma_c/2)^2+\Delta_c^2}$, $\tan\sigma = \Delta_c/\gamma_c/2$.

Equations (10) and (11), for various possible choices of parameters, can be dealt with by numerical computations. But in this paper, for our main concern, we will focus on a particular and also important case of both a cavity field and a pump field resonating with the atom, namely, detuning $\Delta_a = \Delta_c = 0$ and $\sigma = 0$, $A = \gamma_c/2$. In this case, it is easy to show that steady-state solutions for the phase are $\Phi_0 = 0, \pi$; and the corresponding time-independent field amplitude satisfies the following equations:

$$\frac{E_t D}{1+E_t^2} = \frac{1}{\bar{g}_c}E_t + \frac{2\bar{g}_p}{\bar{g}_c} \tag{12}$$

for $\Phi_0 = \pi$, and

$$\frac{E_t D}{1+E_t^2} = \frac{1}{\bar{g}_c}E_t - 2\frac{\bar{g}_p}{\bar{g}_c} \tag{13}$$

for $\Phi_0 = 0$, where

$$D = \rho_{22}-\rho_{11}, \quad \bar{g}_p = g_p/\Gamma, \quad \bar{g}_c = g_c/\gamma_c = \frac{2Rg^2}{\Gamma^2\gamma_c} \ , \tag{14}$$

where D is the inversion parameter, and $\bar{g}_p$ and $\bar{g}_c$ are dimensionless coupling constants for the pump field atom and the cavity atom, respectively. It is appropriate and convenient to introduce the normalized inversion parameter

$$\bar{D} = D\bar{g}_c = (\rho_{22}-\rho_{11})\frac{g_c}{\gamma_c} \ , \tag{15}$$

which possesses the same sign as the inversion parameter D.

By investigating Eqs. (12) and (13), we find that they reveal some interesting and significant properties of the system. First, we can see that Eq. (12) admits a solution only for $D > 0$ (notice that $E_t \geq 0$ by the definition). Consequently, the solution is called the inversion branch. The solution for this branch is depicted in Fig. 2 (curve *a*). From this curve one can see that two solutions are possible on this branch for a given $\bar{D}$. But only one of them is stable (i), and the other is unstable (ii). A detailed analysis shows that in order to achieve lasing action for the inversion branch, the normalized inversion parameter must be greater than a threshold value that has a minimum magnitude of unity and increases monotonically with pump coupling parameter $\bar{g}_p$ (this tendency can also be seen from curve *a* of Fig. 2) and that the lasing acting is favored in the case of a larger population inversion, a stronger cavity-atom coupling, and a smaller decay.

The solution to Eq. (13) is called the inversionless branch because the lasing action exists not only for $D > 0$ but also for $D < 0$. It is graphically shown in Fig. 2 (curve *b*). From this figure, one can see that the optical bistability can take place on the inversionless branch. To be more specific, we can show that no bistability is possible when the pump coupling parameter $\bar{g}_p$ is less than the critical value $\bar{g}_{pc} = 2.598$, and the system can exhibit optical bistability in the region of lasing without population inversion (i.e., $\bar{D} < 0$) for $\bar{g}_p > \bar{g}_{pc}$. This interesting result of the possibility of the bistability in a laser system without population inversion is first demonstrated here. Although there have been a large number of papers discussing optical bistability [20–24, and references therein], to our knowledge no one has verified that it exists in the situation of LWI.

From Eqs. (12) and (13) one sees that the external pump field plays a key role. If the pump field is absent ($g_p = 0$), then not only is LWI impossible, but also bistability disappears. In this case Eqs. (12) and (13) merge into one equation, from which laser intensity is obtained.

$$I = E_t^2 = (g_c/\gamma_c)[D - \gamma_c/g_c] \ . \tag{16}$$

This equation shows obviously the requirement of population inversion, i.e., $D > \gamma_c/g_c$ or $\bar{D} = \bar{g}_c D > 1$. As a matter of fact, the laser becomes an ordinary one in this case [2(b)]. Moreover, we can further show that the inversion branch in the presence of the external pump field corresponds to the previous results about the pump laser.

In summary, we have investigated the behaviors of a laser system of driven two-level atoms with incoherent injection. It is shown that lasing without inversion can indeed be achieved in the incoherent initial atomic states and in the bare atomic state basis. What is more, it is found that the optical bistability can exist in a laser system without inversion, though the bistability in a laser system with inversion is a widely known phenomenon. The transition between the bistable states involves a sudden change in the amplitude of the field but not its phase. Besides the above-mentioned bistability in the region of $D<0$ (i.e., without inversion), our results also show that there may exist bistable states in the region of $D>0$ (i.e., with inversion). In this situation, one of the bistable states belongs to the inversionless branch (curve b in Fig. 2) and another is on curve a of Fig. 2. It is interesting to note that the transition from curve a to curve b can take place, but the reverse process is forbidden. These two steady states in the inversion region do not form a hysteresis. Let us now consider the above transition. Suppose the system is initially in the state on curve a. A sudden jump to the state on curve b will take place as the normalized inversion parameter $\bar{D}$ slowly decreases to a critical value $\bar{D}_{\rm crit}$, corresponding to the nose-type point on curve a [for instance , $\bar{D}_{\rm crit}\simeq 10$ for $\bar{g}_p=2$. See curve ($a1$) in Fig. 2]. This kind of transition involves the sudden change for both the phase $\Phi_0(\pi\rightarrow 0)$ and the amplitude E_t of the field with the typical laser intensity ($\sim E_t^2$) gain of the order of about ten or more (see Fig. 2). This property, together with the fact that the laser system remains in the state on curve b for slowly increasing $\bar{D}$, suggests a mechanism to realize the higher-power laser operation in the inversion region.

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